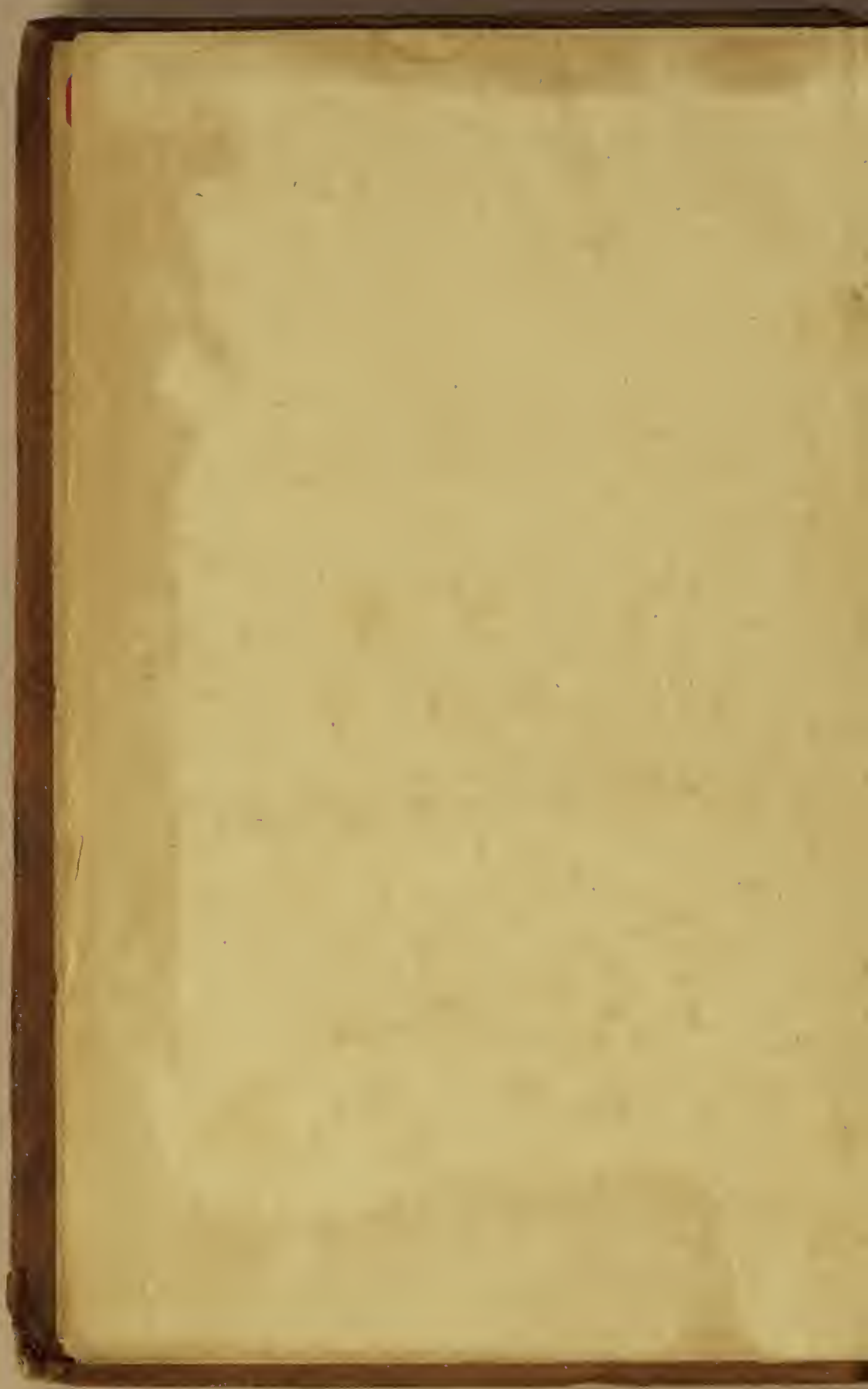
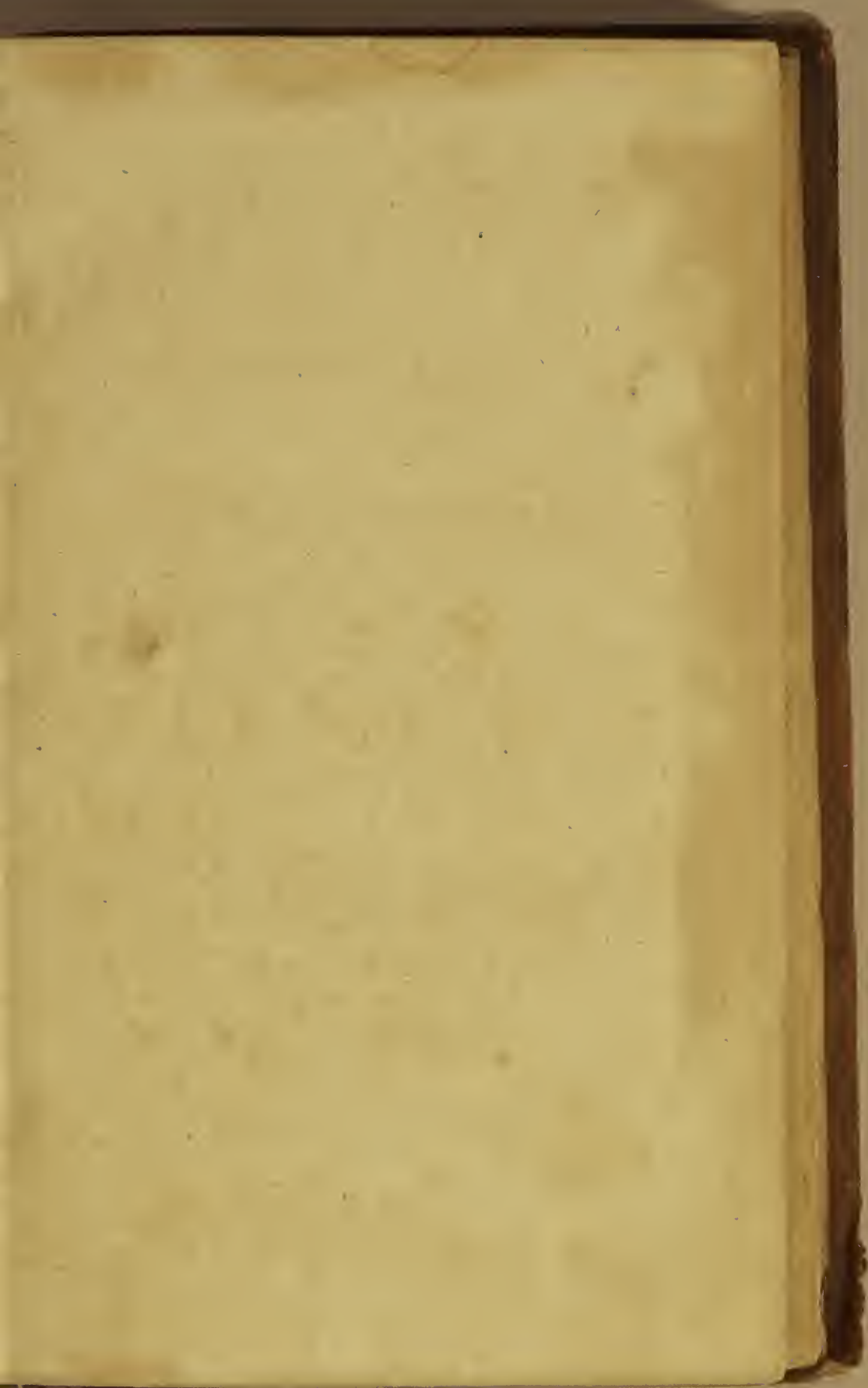
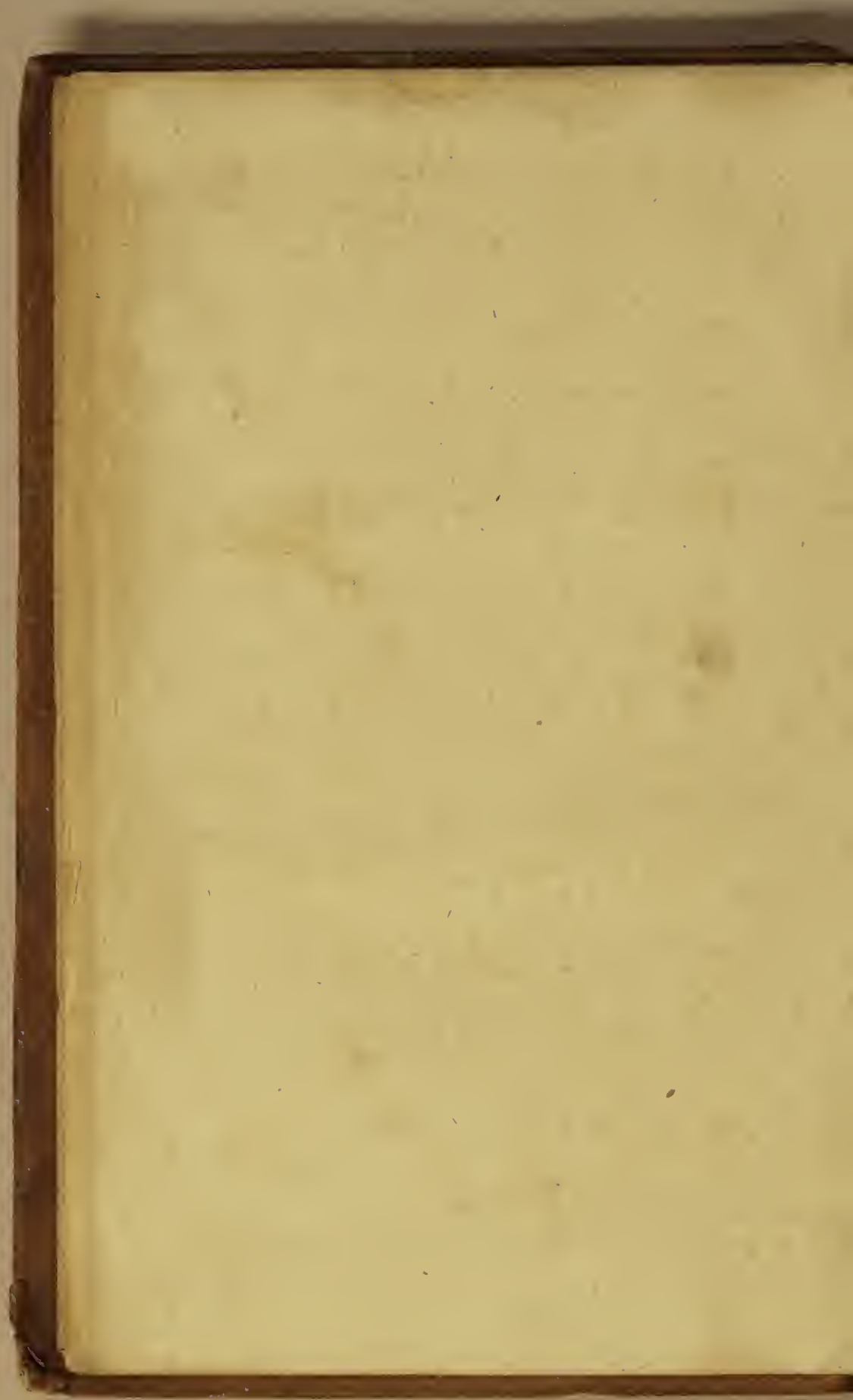


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Philosophia Bryannica:
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A NEW and COMPREHENSIVE

S Y S T E M

OF THE

Newtonian PHILOSOPHY,

ASTRONOMY and GEOGRAPHYS,

IN A

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CONTAINING

The PHYSICAL, MECHANICAL, GEOMETRICAL, and
EXPERIMENTAL PROOFS and ILLUSTRATIONS of
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N A T U R A L S C I E N C E :

A L S O

A particular Account of the INVENTION, STRUCTURE,
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With *new Calculations* relating to their

NATURE, POWER, and OPERATION.

The Whole collected and methodized from all the principal Authors,
and public Memoirs to the present Year;

And embellish'd with EIGHTY-ONE COPPER-PLATES.

By B. MARTIN.

Quæ toties Animos veterum torfere Sophorum

Quæque Scholas frustra rauco Certamine vexant,

Obvia conficimus, Nubem pellente Mathesi. HAL. in NEWT. Prin.

The SECOND EDITION.

In THREE VOLUMES.

VOL. I.

L O N D O N :

Printed for M. COOPER in *Fater-nasser-row*, J. NEWBERRY in *St. Paul's*
Church-yard, S. CROWDER and Co. on *London-bridge*, B. COLLINS at
Salisbury, and J. LEAKE, and W. FREDERICK at *Bath*; and sold by
the AUTHOR at his House in *Fleet-street*. MDCCCLIX.

RPJOB

P R E F A C E.

HAVING read and considered the Design of the several Books hitherto published for the Explanation of the NEWTONIAN PHILOSOPHY, under the Titles of *Commentaries, Courses, Essays, Elements, Systemis, &c.* I observed not one of them all could be justly esteemed a TRUE SYSTEM, or COMPLEAT BODY of this Science.

I would not here be understood to have any Design of depreciating the Labours of those Gentlemen who have gone before me on this Subject; I only would intimate that their Views and Designs were very different from mine; which is to present the World with a PHYSICO-MATHEMATICAL TREASURY of NATURAL KNOWLEDGE; upon the Principles of the *Newtonian Philosophy*; which I have been chiefly enabled to effect from the abundance of Materials supplied by the Authors of the Treatises in the following Catalogue, each of which is excellently good in his Way.

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So that if the Case be truly stated, my Book may be looked upon as a *General Compendium* or *Abridgment* of all theirs. For as they have all lain before me, I have constantly collated them upon every particular Subject, as I proceeded through the Work, and selected the best of every Thing I could find for my own Composition.

HENCE it is, that where some of them have been *very prolix*, my Labour has been to abridge or reduce them to a Compendium of Principles and Essential Propositions only. Thus *Keill* on the Subject of *Motion* and *Astronomy*, *Muschenbroek* on *Cohesion*, Sir *Isaac Newton* on *Optics*, &c. have all been treated.

As Redundancy on the one Hand has been avoided, so their Deficiencies on the other have been supplied. Thus *Keill's* Introduction to *Philosophy* relates to the Mechanical Part only. Professor *Muschenbroek* has wholly omitted the grand Article of *Astronomy*, and the *Physical Causes* of the *Celestial Motions* and *Phænomena*. Dr. *Desaguliers* has thought fit to omit the most essential Science of *Optics*, and is almost wholly taken up on Mechanical and Hydrostatical Subjects. Yea, the best and most general Writer *S'Gravesande*, in his last large Edition

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Edition of his *Elements*, falls far short of a general System or Body of *Philosophy*, in various important Subjects, as will easily appear, by comparing his Table of Contents with mine.

I TAKE no notice of lesser Tracts, since few of them pretend to any thing Systematically; and they who do, propose it with restricted Views, either to the Theory only, or else to some principal Parts or Branches of the Science. Thus *Worster's Principles of Philosophy* amount to little more than a Rehearsal of *Theses*, with scarce any Demonstrations throughout. *Helfham's Lectures* are admirably good as far as they go. And the Lectures, lately publish'd by the *Abbé Nollet*, are but a Part of his Work, and are indeed excellent Philosophical Sermons or Declamations, and strictly in the *French Style and Taste* *.

BUT what others have done is not so much the Design of this Preface to relate, as what I have done Myself in this Treatise, which I shall specify under the following Heads.

* The Lord *Roscommon* observes,
The weighty *Bul* ion of *one Sterling Line*,
Drawn to *French Wire*, would thro' *whole Pages* shine.

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FIRST, I have endeavoured to compile a *Compleat System of Philosophy*, from the great Fund of Materials which the Writings and Inventions of the Moderns have supplied me with. Under this Head I have had regard only to the Choice and Propriety of the Subjects; and their Order and Disposition, or due Arrangement in the Body of the Work. Hence it is that I have admitted nothing that is not of a Physical Nature, and omitted nothing that is so, as far as the Limits of my prescribed Form would admit me to go. Nor are these so strait as not to be comprehensive of every material Branch of natural Science, or considerable Phænomenon, as will easily appear, from a View of the *Contents* and *Index*.

SECONDLY, What has been wrote upon each Subject in every Part of the Science has been carefully examined, and such an Extract made as was judged to give a sufficient Idea or Notion thereof to the Reader. And where I thought a more extensive View could be of Service, I have constantly referred to the best Authors on the Subject.

THIRDLY, If from the whole Account, given by others, of any particular Branch
of

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of Science, I could not collect what sufficed for an entire Sketch, I have endeavoured to supply those Deficiencies, in order to render every Part compleat, and save the Reader the Trouble of turning to several Authors for that Purpose. Remarkable Instances of this may be found under the Articles of *Wheel-Carriages*, *Hydraulics*, *Optics*, &c.

FOURTHLY, If the Method of representing or demonstrating any Subject or Proposition by others has not been judged so clear, concise or natural as was necessary, I have proposed it in my own way, to answer this important End: For the greatest Excellency of a Demonstration is Conciseness and Perspicuity. The Want of this is a great Discouragement to Physical Enquiries, as I am but too well convinced of by my own Experience, and that of many others.

FIFTHLY, According to the different Nature of the Subject, a various Process has been made use of to explain or illustrate the same. Thus some Things require a *Physical*, others a *Mechanical*, some a *Geometrical*, and some an *Experimental Proof*, which have been accordingly applied, sometimes singly, sometimes two or more, and

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sometimes altogether. Thus from *Phænomena* we institute a Ratiocination concerning the Materiality, the Velocity, different Magnitudes, Colours, and other Affections of the Particles of Light. Then the Laws of Reflection and Refraction are explained by *Mechanical Principles* relating to the Motion and Action of Bodies one upon another. Again, The *Principles of Geometry* are call'd in to explain and account for the Effects of Mirrours, Lenses, Optic Instruments, the Rainbow, &c. And, Lastly, those various Phænomena are represented, and the Manner how they happen explained by *Experiments*.

SIXTHLY, I have all along applied the *Newtonian Geometry*, that is, the *Fluxionary Calculus*, for the Demonstrations, which I think is with the greatest Propriety and Justness in Physical Matters, because the Principles of this Geometry are in themselves strictly Philosophical, as I have shewn in its proper Place *. Besides, the Process is not only rendered by this means much more congruous and natural, but more evident and concise than by the common Methods of Demonstration it could be. Of this we

* See Vol. I. of *Mathematical INSTITUTIONS*, just now published in the GENERAL MAGAZINE of ARTS and SCIENCES.

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have a signal Instance in the Calculation of the Angle, which the Incident Ray makes with the last refracted Ray in a Drop of Water, as will appear by comparing what is here contained in *Scholium* to *Annot.* CXXIV. with what you find on this Head in other Authors.

SEVENTHLY, As the Bounds of this Science have been of late Years vastly extended, so I have been very careful to treasure up every useful Invention, and given a short historical Account of the same, and the Names of the Authors. If in any of them I found an Improvement could be made, I have done it, as in the *Air-Pump*, the *Microscope*, &c. or hinted it to the Reader, as in Case of the *Air* or *Steam-Gun*.

EIGHTHLY, I find by common Experience nothing tends more to the Conception and enlivening an Idea, and fixing it in the Memory, than a good Delineation of the Figure or Form thereof. Hence it is that the Reader will here find a greater Number of Copper-Plate Figures than in any other Treatise of this Kind, of its Size. And if I tell him they were all done by the ingenious Mr. BOWEN (GEOGRAPHER to his MAJESTY) it may likewise prepossess him in Favour of their Goodness; which though an indirect and collateral, is yet no inconsiderable

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derable Argument to recommend a Book of this Kind.

NINTHLY, I have delivered the Whole by way of LECTURES, and ANNOTATIONS thereto, (a Method of late Years the most used and approved of) not only because the Reader is by this Means led through a SYSTEM of PHILOSOPHY, without being confined to the Irksomeness of a precise and regular Path, and a simultaneous Prospect or View of the Whole at once; but also because that vast Variety of Things, which here present themselves, may have the Pleasure that attends Variety heighten'd and encreased by emerging fresh to his View as he passes along, and regaling him with something still novel and unexpected. *Annotations* seem to answer the End of *Scenes* in a Play; they present the Whole most agreeably in Parts, which thus more immediately affects us, and gives us the greatest Pleasure and Entertainment.

THUS much for the Oeconomy of the Work; and as it is manifestly calculated for a general Good, so I believe none will find fault with the Design. As to the Manner of executing it, I have done it as well as I could; and if any one thinks me deficient in any thing, let him qualify his Criticism

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ticism by considering that I pretend not to Infallibility, but only where I give a plain and clear Demonstration. In this, if I am not understood sometimes, I am not perhaps to be blamed; for I'll appeal to Envy itself, if the Manner of the Demonstration, where most difficult, be not as plain and as obvious as the Nature of the Thing will admit. And here it will be proper to observe, that although I have made use of *Geometry*, *Algebra*, and *Fluxions*, to investigate the Truth of all the principal Propositions in Philosophy, yet as I cannot suppose Readers in general qualified to understand it in that Manner, I have been careful to express the Result of every Process in plain *English*; so that every one may know what is the Truth, though he cannot be so properly said to understand it. For Instances of what I now speak of see the several Articles of *Annotat.* XXVII, XXXIV, and XLVIII.

I do not know that I have advanced any thing here, which I have not one Way or another proved to be true; however, I desire that nothing but what is demonstrated may be accounted to deserve the Title of NEWTONIAN PHILOSOPHY. Our Great Author seemed quite averse to *Hypotheses*;
and

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and though he indulged them to others under proper Restrictions, yet did himself never make use of this *fallacious Way of Reasoning*, though he has been unjustly charged therewith by some, who, by *National Instinct*, scorn to subscribe to any System but of their own making. Hence it is we often hear them babbling about *Newton's Hypotheses*, &c. and making unsufferable Comparisons between him and *Descartes* and *Leibnitz*. One would think they who knew but little of Philosophy would yet see no Reason for a Parallel between a System of Philosophy that is founded on nothing but *Demonstration*, and is every Way comportant with *Phænomena*, and one that is wholly *hypothetical*, and, instead of accounting for, runs counter to every thing that appears! I think it is little Glory, (not to say *Vain-Glory*) for a Man to boast that his Philosophy is *not that of Newton* *, till he has first satisfied the World his Abilities and Discernment into the Nature of Things are greater than those of *Sir Isaac*. This I am well assured of, that the greater Share of *Mathematical Knowledge* any Man has, the more readily he embraces *Sir Isaac's* Prin-

* See the Preface of Abbot NOLLET's *Leçons de Physique*.
ciples ;

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ciples; and I scarce ever found any that oppugn'd Sir *Isaac*, but who were either grossly ignorant of what they despised, or were actuated by some particular Views. In short, nothing but *Pride, Prejudice, and Ignorance*, will ever stir up Enemies to the *Newtonian Philosophy*.

THEY who depreciate this Philosophy by the Term *Hypothesis*, seem to me not to have read (at least not considered) what Sir *Isaac* has wrote, when he tells us, "That in Experimental Philosophy, Hypotheses are not to be regarded.—What I call ATTRACTION may be performed by IMPULSE, or by some OTHER MEANS unknown to me. I use that Word to signify *only in general* ANY FORCE by which Bodies tend towards one another, whatsoever be the Cause *." After our illustrious Author had thus expressly disclaimed all Hypotheses, and so fully explained his Sentiments about the Power which he calls *Attraction*, is it not very wonderful to hear a Set of People charging him with founding his Philosophy on the Hypothesis of Gravity? Will these very sagacious Gentlemen allow any Power at all

* OPTICS, Qu. 31. See also the Definitions of his *Principia*.

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is concerned in the *Tendency* of one Body to another? If they do, is it not because it is impossible to consider Bodies acting at a Distance upon each other without? If so, is not the Existence of such a Power a Reality in Nature? And can such a Force then be called an *Hypothesis*? Surely not. Again, their accusing Sir *Isaac* for accounting for Things by *Attraction*, is meer cavilling about Words, and begging him guilty; he declares he knows not the Manner in which that Force acts, whether by *Attraction* or *Impulsion*, or otherwise; but as some Name must be given it, why may not he call it by that of *Attraction* or *Gravity*, as well as by any other? Not that this great Man did even this without a Reason; for 'tis evident from a thousand Experiments and Phænomena, that this is much more like the Effect of one Body's *drawing* than *impelling* another towards it.

BUT so far is Sir *Isaac* from *supposing* that Bodies tend to each other by *ATTRACTION*, that he once condescended to form an *Hypothesis* to shew that it might be done by *IMPULSION*, and that of a very subtle Elastic Spirit or *Æther*. But as he was not able to prove the Existence of any such Thing, he seemed not at all delighted with
the

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the Thought, nor ever laid any Strefs upon it; and accordingly we have not admitted it as a Principle of the *Newtonian* Philosophy. Besides, suppose this could be proved, we should still be at a Loss to account for the *material Cause* of this very great Elastic Power between the Particles of this *Æther*; we should indeed advance three or four Steps higher on the infinite Ladder, but should be equally *non-plus'd* with the mechanical Cause of *Elasticity* then, as we are at present with that of *Gravity*. But after all, 'tis evident by Experiments that this Power which actuates Bodies, whatever it be, is not either *Attractive* or *Repulsive* solely; but it produces both the different Effects in *different Circumstances of Distances*. Thus the Loadstone at one Distance moves the Needle *towards itself*, at another Distance it causes it *to recede* or move *the contrary Way*. This Power in Iron is attractive, if touched upon the Stone in one Direction; but if in a contrary it becomes repulsive. The same ambiguous Properties of this Power obtain in *Electricity*, and undoubtedly in all other Sorts of Attractions. Therefore, before we set ourselves about finding out a *Power, Spirit, or Æther*, that shall move Bodies either by *Attraction* or
Repulsion,

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Repulsion, it is in the first Place necessary to find out an Agent that shall do both, for such a Principle is at present the *Desideratum* to our further Advances in the Research of natural Causes.

It is sometimes surprizing, to observe, how very fond People seem of this *subtle Æther*; one accounts for the *Cause of Gravity* thereby, another for *Muscular Motion*, a third for *Electricity*, a fourth from hence derives *Animal Spirits*, a fifth *Elementary Fire*: And, in short, we want nothing but the *Proof of such an Æther* to account for almost every Thing by it. But is it not a preposterous Proceeding to account for any Thing by a Principle in itself unaccountable, and utterly unknown?

HOWEVER, we are arrived at great Dexterity since Sir *Isaac's* Time; for we can now almost prove the Existence of this *Æther* by the Phænomena of Electricity; and then we find it very easy to prove that Electricity is nothing but this very *Æther* condensed and made to shine. But, I believe, when we enquire into the Nature and Properties of this *Æther* and Electricity, we shall find them so very different and dissimilar, that we cannot easily conceive how they should thus mutually prove each other. For according

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According to Sir *Isaac*, this *Æther* is the most subtle of all Bodies, and consists of Particles (of course) very much smaller than those of Light, and which have a much greater Velocity; this *Æther* therefore pervades with the greatest Freedom all Bodies, actuates their Parts, &c. But on the other hand, we find by Experiments that the Fire of Electricity is a very gross Substance; for it dashes against the Surface of Glass like so much Water, and does not enter the Pores, as Light does, that is so much grosser than *Æther*. This is still made more plain and demonstrable by electrifying a Wire in an exhausted Receiver, where the Electricity issues out in much greater abundance, and in divers Jets, striking against the Sides of the Glass, and appearing to run down by the Sides in all respects like *Liquid Phosphorus*; and is no ways reflected or refracted like common Light, or even like the Light of a Candle: For if a Candle be viewed through a Prism, the Light coming from every Part thereof is refracted, and the whole Flame is coloured and distorted; but it is quite otherwise with the *Flame of Electrical Fire*, for I often made that much larger than the Flame of a Candle, and viewed it through the Prism, but no Co-

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lours were thereby produced, no different Form of Flame, but every way the same as it appears to the naked Eye. Indeed the denser Part, just upon the Tip of the Iron whence the Blaze diverged, appeared a little tinged with Colours; which I take to proceed from some common *Solar Light* intermixed with that of Electricity. From hence (and for many other Reasons I could name) I see no cause to believe that the Matter of Electricity is any thing like the Idea we ought to have of the *Spiritus subtilissimus* of Sir *Isaac*. Nor indeed can it be the same with the Matter of common Light, which is differently reflexible and refrangible; whereas this appears not to be so, but is a particular Sort of Light, and nearly the same as *Phosphorus*, which I could never observe to exhibit the least Appearance of Colours through the Prism. The Smell also of *Electrical Fire* is so very much like that of *Phosphorus*, that one may be easily induced to believe a great Part of the Composition of both is the same.

As to the Doctrine of ELEMENTS and *Elementary Fire*, we leave that to others, and expatiate only on the noble *Theory of Light and Fire* left us by our incomparable Author, who justly considers them as they
are

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are in Nature, illustrates all their Properties and Qualities, and applies them to the Explication of a great Number of the most interesting and important Phænomena of Nature. The *Newtonian* Doctrine of *Elements* is of the *Singular Number*: *One Sort of Matter only* makes the Substance of all the infinite Variety of Bodies we behold. Thus our Author speaks: “ It seems probable to me, That God in the Beginning formed Matter in solid, massy, hard, impenetrable, moveable Particles, of such Sizes and Figures and with such other Properties, and in such Proportion to Space, as most conduced to the End for which he formed them.—Now by Virtue of certain active Principles or Powers, all material Things seem to have been composed of the above-mentioned Particles, variously associated in the first Creation, by the Counsel of an intelligent Agent.”

To conclude: It was an Observation worthy of so great a Genius as Mr. *Cotes*, that the *Newtonian Philosophy* was our (*mutinissimum præsidium adversus Atheorum impetus*) best Defence against the Efforts and Assaults of Atheists.—That herein we more nearly view the Majesty of Nature, and enjoy it

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in the most grateful Contemplation; at the same time we are excited more intensely to worship and revere the CREATOR and LORD of all Things. He ought to be reputed *blind*, who cannot immediately behold, in the best and wisest Structures of Things, the infinite Goodness and Wisdom of the Omnipotent Architect; and *mad*, who will not confess it.

A C A T A

A
CATALOGUE

OF THE

Principal BOOKS

Made use of in compiling the following WORK.

1. **P**HILOS. Nat. Principia Mathematica, Auctore I. NEWTONO, Eq; Aurato Ed. Tertia.
2. Eadem Principia illustrata Perpetuis Commentariis, communi Studio PP. THOM. LE SEUR & FRAN. JACQUIER. Genevæ. 1739.
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5. *Essai de Physique*, par M. P. VAN MUSCHENBROEK, Tome i. & ii. *Leyden*, 1739.
6. *A Course of Experimental Philosophy*, by Dr. DESAGULIERS, 2 Vols. *London*, 1734 and 1744.
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8. *The Philosophical Transactions abridged*, by Mr. LOWTHORP, JONES, MARTYN, EAMES, &c. Seven Vols.
9. *A Compleat System of Optics*, by Dr. SMITH, Two Vols. *Cambridge*, 1738.
10. *Lectiões Opticæ*. Auctore I. NEWTONO, Eq; *Aurato*.
11. *Optics, or a Treatise of Reflections, Refractions, Inflections, and Colours of Light*. By Sir I. NEWTON, Knt. *Lond.* 1718.
12. *An Essay on Distinct and Indistinct Vision*. By Dr. JURIN.
13. *Architecure Hydraulique*, par M. BELIDOR, Tom. i. & ii. *A Paris*.
14. *Hydrostatical and Pneumatical Lectures*, by Mr. COTES, with Notes by Dr. SMITH. *London*, 1738.
15. *The Motion of Fluids Natural and Artificial, particularly Air and Water*. By Mr. CLARE. *London*, 1737.
16. *Astro-*

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6. *Astronomy*, in five Books. By Dr. GELONG. Cambridge, 1742.
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18. *Astronomical Lectures*. By Mr. W. WHISTON. London, 1728.
19. *The Chronology of Ancient Kingdoms amended*. By Sir I. NEWTON. London, 1728.
20. *The Elements of Algebra*, in Ten Books. By Dr. SAUNDERSON. Cambridge, 1740.
21. Sir I. NEWTON's *Two Treatises of the Quadrature of Curves and Analysis*, by Equations of an infinite Number of Terms explained. By Mr. J. STEWART. London, 1745.
22. ROHAULT's *Physics*, with Dr. CLARKE's Annotations. London, 1710.
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24. *A System of the Conic-Sections*, with the Doctrine of Fluxions. By JOHN MULLER. London, 1736.
25. *A Treatise of Fluxions*, in Two Books. By COL. MACLAURIN. Edinburgh, 1742.
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- DE L'HOSPITAL, *with Additions* by Mr. STONE. London, 1730. 1s.
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29. *A Compendious and Methodical Account of the Principles of Natural Philosophy.* By Mr. BENJAMIN WORSTER. London, 1730.
30. *A Course of Lectures in Natural Philosophy.* By Dr. HELSHAM, with Additions by Dr. ROBINSON. London, 1739.
31. *Philosophiæ Mathematicæ Newtonianæ illustratæ Tomi Duo.* A. GEOR. PETRO DOMCKIO. Londini, 1730.
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35. *Vegetable Statics.* By Dr. HALES. London, 1738.
36. *Hæmestatics.* By the same Author. London, 1733.
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42. *A Course of Lectures in Chemistry.* By Dr. SHAW.

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LECTURE

L E C T U R E I.

Of EXPERIMENTAL PHILOSOPHY in general.
Of the NEWTONIAN METHOD and RULES
of Philosophizing. Of MATTER in gene-
ral, and its essential Properties. Of the
CHEMICAL ELEMENTS or PRINCIPLES of
Natural Bodies. Of ATTRACTION and
REPULSION. The Attraction of COHE-
SION. Of the COHESION of various Sorts
of Bodies. The Phænomena of CAPILLARY
TUBES. Of a VACUUM. The Rationale
of various Processes of CHEMISTRY, and
other Arts. Of the principal Properties of
ELECTRICAL Attraction and Repulsion.
Of MAGNETISM and its Laws; of the va-
rious Properties of the MAGNET in relation
to the Needle; its VARIATION and INCLI-
NATION.

THE Business of Experimental Philosophy (the Subject of this Course of Lectures) is to enquire into and investigate the Reason and Causes of the various Appearances (or *Phænomena*) of Nature; and to make the Truth or Probability thereof ob-
B
vious

Of the Properties of BODIES.

vious and evident to the Senses, by *plain, undeniable, and adequate Experiments*, representing the several Parts of the grand Machinery and Agency of Nature.

IN our Enquiries into Nature we are to be conducted by those Rules and Maxims which are found to be genuine, and consonant to a just Method of Physical Reasoning; and these Rules of Philosophizing are, by the greatest Master in this Science, (the incomparable Sir *Isaac Newton*) reckon'd Four; which I shall give, from his *Principia*, as follows:

RULE I. *More Causes of natural Things are not to be admitted, than are both true and sufficient to explain the Phænomena.* For Nature does nothing in vain, but is simple, and delights not in superfluous Causes of Things.

RULE II. *And therefore of natural Effects of the same Kind the same Causes are to be assigned as far as it can be done.* As of Respiration in Man and Beasts: Of the Descent of Stones in *Europe* and *America*: Of Light in a culinary Fire and in the Sun: And of the Reflection of Light in the Earth and in the Planets.

RULE III. *The Qualities of natural Bodies which cannot be increased or diminish'd,*
and

Of the Properties of BODIES.

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and agree to all Bodies in which Experiments can be made, are to be reckoned as the Qualities of all Bodies whatsoever. Thus, because Extension, Divisibility, Hardness, Impenetrability, Mobility, the *Vis Inertiæ*, and Gravity, are found in all Bodies which fall under our Cognizance or Inspection, we may justly conclude they belong to all Bodies whatsoever; and are therefore to be esteemed the Original and universal Properties of all natural Bodies.

RULE IV. *In Experimental Philosophy, Propositions collected from the Phænomena by Induction are to be deem'd (notwithstanding contrary Hypotheses) either exactly or very nearly true, till other Phænomena occur by which they may be render'd either more accurate, or liable to Exception.* This ought to be done, lest Arguments of Induction should be destroy'd by Hypotheses.

IF according to these Rules we take a Survey of the visible World, and strictly examine the Nature of particular Bodies, we shall find Reason to conclude, that they all consist of *one and the same Sort of Matter or Substance*; and that all the Diversity or Difference we observe among them arises only from the various Modifications and

Of the Properties of BODIES.

different Connection or Adhesion of the
same primigenial Particles of Matter (I).

(I) These four Rules of Philosophizing are premised by Sir *Iaac Newton* to his third Book of the *Principia*; and more particularly explained by him in his *Optics*, where he exhibits the Method of proceeding in Philosophy in the following Words:

1. As in Mathematics, so in Natural Philosophy, the Investigation of difficult Things by Way of *Analysis*, ought always to precede the Method of *Composition*. This Analysis consists in making Experiments and Observations, and in drawing general Conclusions from them by Induction (*i. e.* Reasoning from the Analogy of Things by natural Consequence) and admitting no Objections against the Conclusions but what are taken from Experiments, or other certain Truths. And altho' the arguing from Experiments and Observation by Induction be no Demonstration of general Conclusions; yet it is the best Way of arguing which the Nature of Things admits of, and may be looked upon as so much the stronger by how much the Induction is more general. And if no Exception occur from Phænomena, the Conclusion may be pronounced generally. But if at any Time afterwards, any Exceptions shall occur from Experiments, it may then be pronounced with such Exceptions. By this way of *Analysis* we may proceed from Compounds to Ingredients, and from Motions to the Causes producing them; and in general from Effects to their Causes; and from particular Causes to more general ones, till the Argument ends in the most general. This is the Method of Analysis. And that of *Synthesis* (or Composition) consists in assuming Causes discover'd and establish'd as Principles, and by them explaining the *Phænomena* proceeding from them, and proving the Explanations.

2. That Matter or Substance is *one and the same* in all Bodies, and that all the Variety we observe arises from the various Forms and Shapes which it puts on, is I think very probable, and may be concluded from a general Observation of the Procedure of Nature in the Generation and Destruction of Bodies. Thus, for Instance,

MATTER

Of the Properties of BODIES.

5

MATTER, thus variously modified and configured, constitutes an infinite Variety

Water rarified by Heat, becomes *Vapour*; great Collections of Vapours form *Clouds*; these condensed descend in Form of *Rain*, or *Hail*; Part of this collected on the Earth constitutes *Rivers*; another Part mixing with the Earth enters into the Roots of Plants, and supplies Matter to, and expends itself into various Species of *Vegetables*. In each Vegetable it appears in one Shape in the *Root*, another in the *Stalk*, another in the *Flowers*, another in the *Seeds*, &c. From hence various Bodies proceed; from the Oak, *Houses*, *Ships*, &c. from Hemp and Flax, we have *Thread*; from thence our various Kinds of *Linen*; from thence *Garments*; these degenerate into *Rags*, which receives, from the Mill, the various Forms of *Paper*; hence our *Books*; which by Fire are converted partly into Water, partly into Oil, another Part into Air, a fourth Part into Salt, and a fifth into Earth; which are call'd the Elements of Bodies, and which mix'd with common Earth, are again refuscitated in various Forms of Bodies.

3. The ELEMENTS, or Principles to which all Bodies are ultimately reduced, are the five above mentioned, viz. (1.) WATER, or *Phlegm* (as 'tis call'd) which generally rises and goes off first, as in the Chemical Analysis of a Plant in a Retort by Fire. (2.) AIR, which escapes unseen in great Quantities from all Bodies; and tho' it has not till lately been known to make a Part of natural Bodies in a fix'd State (and therefore never taken Notice of as an Element of natural Bodies) yet that it is so in a very remarkable Degree, (even so far as to make half the Substance of some Bodies) I shall give sufficient Proof when I come to treat of artificial or factitious Air. (3.) OIL, which appears swimming on the Top of the Water. (4.) SALT, which is either *Volatile*, or rises in the Still, as that of Animal Substances; or *Fix'd*, as in Vegetables, and which is extracted by dissolving them in Water from a Lixivium of the Ashes, and afterwards by evaporating the Moisture to a Pellicle, and setting the Salt to shoot into *Chrystals*. (5.) EARTH, or what is

of Bodies, all which are found to have the following *common Properties*, viz.

EXTENSION, or that by which it possesses or takes up some Part of universal Space; which Space is call'd the PLACE of that Body. For all Bodies are *extended* either (1.) into *Length* only, and then it is called a LINE; or (2.) into *Length* and *Breadth*, which is call'd a SUPERFICIES; or (3.) into *Length*, *Breadth*, and *Depth*, which then is call'd a SOLID. These are the *three Dimensions*, according to the Quantity of which the *Magnitude* or *Bulks* of Bodies are estimated (II).

call'd the *Caput Mortuum*, being what remains of the Ashes after the Salt is extracted, which can be no farther alter'd by any Art whatsoever.

(II) THE MAGNITUDE of Bodies is the Quantity of their Dimensions express'd in some common or *standard* Measure, as an *Inch*, a *Foot*, a *Yard*, &c. and it is thus estimated:

1. When Bodies have but one Dimension, as Lines, then it is express'd by the Number of Inches and Parts of an Inch contained in their Length: Thus the Dimension of the Line A B is 3 Inches; of C B 2 Inches; and the Comparison of the Length of A B to C B is call'd the Proportion or *Ratio*; thus the Ratio of A B to C B is that of 3 to 2, or (as it is usually express'd) $AB : CB :: 3 : 2$. And because this Comparison consists of *only one Ratio*, it is said to be a *simple Ratio*.

2. Those Bodies which have two Dimensions, as Superficies, have their Magnitude express'd by the Rectangle under both, or the Product of their Length by their Breadth; thus if any Surface ABCD has its Length

Fig. 2. $AB=4$, and its Breadth $BC=2$; then its Dimension

SOLIDITY,

Of the Properties of BODIES.

7

SOLIDITY, sometimes called the IMPENETRABILITY of Matter, is that Proper-

(which in this Case is called the *Area*, or *Superficial Content*) is thus expressed, $AB \times BC = 4 \times 2 = 8$; that is, there are eight small Squares, which are square Inches, square Feet, &c. according to the Measure of the Sides.

3. If this Surface be compared with any other, as EG; whose Length is $EF = 3$, and Breadth $FG = 2$; then their Magnitudes will be to each other as $AB \times BC$ to $EF \times FG$, that is, as 8 to 6, or as 4 to 3. And because in this Comparison each Term consists of two Parts, or there is a twofold Ratio of Length to Length, and Breadth to Breadth, therefore this is said to be a *Duplicate Ratio*; and so all Surfaces are to each other in the *Duplicate Ratio* of their Sides.

Plate I.
Fig. 3.

4. In like Manner all Solid Bodies, which have three Dimensions, have their Magnitudes express'd by the Product of their Length, *Breadth*, and *Depth* together. Thus if there be one Solid AG, whose Length $AB = 4$, its Breadth $AE = 2$, and Depth $AD = 3$; and another Solid HO, whose Length $HI = 3$, Breadth $HM = 1$, and Depth $HL = 2$; then will their Magnitudes be to each other as $AB \times BE \times AD$ to $HI \times HM \times HL$, that is $4 \times 2 \times 3 = 24$ to $3 \times 1 \times 2 = 6$, or as 24 to 6. And therefore Solids are said to be in a triplicate Ratio, *viz.* of their Length, Breadth, and Depth, or Thickness; and the Standard Measure in this Case is call'd a *Cubic Inch*, *Foot*, &c. because of its being in the Form of a Dye, or *Cube*, which Figure is contain'd under 6 equal and rectangular Planes.

Plate I.
Fig. 4, 5.

5. And here it will be necessary to advertise the Reader, that any Quantity is generally denoted by a single Letter, as A, B, &c. and the Square of that Quantity by $A \times A$ or AA , or A^2 , B^2 , &c. and the Cube by A^3 or B^3 . And when we express the Ratio of two Quantities A and B, it is usually thus, $A : B$; and when we compare this Ratio with any other, as of C to D, we express it thus $A : B :: C : D$. If both Terms of the Ratio increase or decrease together, the Ratio is said to be direct, and express'd as before: But if one Quantity

Of the Properties of BODIES.

ty by which a Body excludes all others from the Place which itself possesseth ; for it would be absurd to suppose two Bodies could possess one and the same Place at the same Time. From this Definition it follows, that the *softest Bodies* are equally *solid* with the *hardest* (III).

increases while the other decreases, and *vice versa*, in each Ratio ; then they are expressed in a different Manner, viz. $A : B :: C : D$, or $A : C :: B : D$. And the Addition of one Ratio $A : B$ to another $C : D$ is performed by multiplying the first Terms in each Ratio together, and also the last ; and the Ratio of those Products $AC : BD$ is the Sum of both the other. To square any Ratio is to multiply it by itself $A : B \times A : B = A^2 : B^2$, and so the Cube of any Ratio, $A : B$ is $A^3 : B^3$. If any would know more of the Nature of Ratios, they may consult Dr. Sanderfon's Algebra, or my *Logarithmologia*.

(III) FIGURABILITY is as necessary to Matter as any of the Properties abovemention'd ; for since Matter is not infinite, it must be circumscribed within certain Limits and Bounds on every Part, which constitute the Figure of the Body ; and as the Particles of Matter may exist together in any Manner of Situation, so the Figures or Form of Bodies, which they compose, may be infinitely various and different from each other.

2. On this Property several Things of Moment depend ; thus according to the several Figures of the Corpuscles, they will touch by a greater or lesser Quantity of Surface, and so will cohere more or less firmly together ; from hence will arise various Qualities of Bodies, which are the Foundation of most of the considerable Phænomena of Nature, as will be hereafter taken Notice of.

3. Hence we may observe likewise, that Bodies of different Figures contain, under the same Quantity of

DIVISI.

Of the Properties of BODIES.

9

DIVISIBILITY is that Property by which the Particles of Matter in all Bodies are capable of a Separation or Difunion from each other. Hence the *Resolution* or *Dissolution* of Bodies into their constituent Corpuscles, as in many Operations of Chemistry. How far this may actually obtain in Nature is not easy to say : But that Matter is infinite-

Surface, different Magnitudes or Bulks ; thus a Circle contains a greater Area under the same Length of Periphery, than any other figured Superficies ; and a *Sphere*, under the same Quantity of Surfaces, contains a greater Bulk or Space, than any other Solid whatsoever.

4. Of all the infinite Variety of Forms or Figures, which Matter is liable to, there are only *five* which will admit the Particles of Matter, placed together, to fill the Space between them compleatly, or so as to leave no Pore, Vacuity, or Interstice between them : and Bodies of these Forms have been known by the general Name of the *five Platonic Bodies*, and are as follows :

1. *Tetrahedron*, which has *four equal* triangular Sides.
2. *Hexahedron*, of *six equal* square Sides, viz. a Cube.
3. *Octahedron*, of *eight equal* pentagonal Sides.
4. *Dodecahedron*, of *twelve equal* triangular Sides.
5. *Icosahedron*, of *twenty equal* triangular Sides.

5. Hence the Asserters of a *Plenum* are necessitated to prove, that tho' Matter is liable to an Infinity of Forms indifferently, yet all its Particles must have but some one of the five abovementioned ; otherwise it would be as evident as an *Axiom*, that the Particles of Matter have a different Form, and must therefore admit Vacuities or Pores in the Composition of every kind of Bodies. Tho' this Argument for a Vacuum has not been observed, yet perhaps there is none more evident, geometrical and conclusive.

ly

ly divisible in a mathematical Consideration, is demonstrable various Ways (IV).

MOBILITY is that Property which all Bodies have, of being moveable or capable of changing their Situations or Places. This

(IV) 1. The easiest Way to demonstrate the infinite Divisibility of Matter, I think, is the following. Let AB be the Length of a Particle to be divided; through each
 Plate I. Extreme, A and B, let there be drawn the Lines CD,
 Fig. 6. EF parallel to each other: In the Line CD, between A and C, let there be taken any Point G; and in the Line EF, from B towards F, let there be taken any finite Number of Points, H, I, K, L, &c. If now from the Point G we draw the Lines GH, GI, GK, GL, &c. they will each of them cut off a Part of the given Particle AB; and yet after all a Part PA will remain; and since this will be the Case for any finite Number, 'tis plain the Particle AB contains a Number of Parts greater than any finite Number, and therefore infinite.

2. Another Demonstration equally obvious is the following, viz. Let AB be the Length of the given Particle; thro' A draw CD, and in the Point E, at right
 Plate I. Angles, draw the indefinite right Line EF, in which let
 Fig. 7. there be any finite Number of Points taken, as G, H, I, K, &c. on these, as Centers, describe the several Arches of Circles EL, EM, EN, EO, &c. to cut the Particle AB, each of these will take off a Part, and yet there will a Part AP remain; therefore, &c. as before.

3. Tho' we have no Possibility of an Instance of this infinite Divisibility of Matter, yet Nature proceeds towards it to inconceivable Lengths in many of her wonderful (tho' common) Operations. How minutely is Matter divided by Fire, as a Piece of Tallow, for Instance, in a lighted Candle! What a prodigious Sphere or Space does the least Flame fill with Particles of Light, or a Grain of *Assa fœtida* with odorous Particles! How numerous and small the Particles or Vapours arising from the Surface of Fluids! Or the Particles of Copper, which from a single Grain shall be diffused thro' the Sub-

Property

Of the Properties of BODIES.

II

Property of Matter is evident to all our Senses; and what nobody has confessedly deny'd (V).

THE VIS INERTIÆ, (as Sir *Isaac* call'd it) or the *Inactivity* of Matter, is that Property of it, by which it endeavours to continue in its State either of *Motion* or *Rest*, or by which it resists the Actions and Impressions of all other Bodies which tend to generate or destroy Motion therein (VI).

stance of many Millions of Times its own Quantity of Water, and tinge it of a different Colour! But Instances of this Kind are endless.

(V) *Des Cartes* and his Followers, who deny a *Vacuum*, do virtually deny Motion; for since Motion is the Translation of a Body from one Part of Space to another, if there be no *void Space*, such a Translation must be made thro' a Space absolutely fill'd with Matter, or not at all; if the first be asserted, it will follow, that two Bodies may be in one and the same Place at once, which is absurd; therefore there can be no Motion at all upon their Principles. To say a Body may move in a *Plenum*, as a Bird moves thro' a Body of Air, or a Fish thro' Water, is saying nothing to the Purpose; because both Air and Water abound with Vacuities, (as is well known from the Nature of each,) whereas a *Plenum* admits of no vacuous Space, and therefore of no Motion of the Particles, nor consequently of any Body between them. Besides, neither the Bird nor Fish would be able to continue in the Air or Water, were they not specifically heavier than those Fluids; but upon the Supposition of a *Plenum*, all specific Gravity is taken away (as will be shewn) and so the Case is not parallel, and therefore the Argument fallacious.

Plate I.
Fig. 8.

(VI) This Power or Force of Matter arises wholly from its Sluggishness or Inactivity, and not at all from its Gravity, as some (of no small Name) have mista-

ATTRAC-

ATTRACTION is a Property that we find all Matter endued with in a greater or a lesser Degree: By this Property the Particles of Matter attract each other by a Power which causes them mutually to accede to, or approach each other. This Tendency of one Body to another is call'd

kenly conceived of it. To make this Matter plain, suppose a Body A at rest in the Point A, it would not of itself move out of that Point, unless by some external Force impress'd. *First*, Suppose the Body absolutely light, and it be required to move it from A to B, in a certain Time; to this End some Force must be applied acting in the Direction AB, by which, at the End of that Time, the Body will be found in B; and this Force once impress'd will continue in the Body, and in every such Particle of Time will carry it thro' a Space equal to AB. *Secondly*, Suppose now Gravity begins to act, and give the Body Motion, and that by the Power of Gravity the Body be carried from A to D in a Line, passing thro' the Earth's Centre, in the same Time as it before moved from A to B; draw BC and DC equal and parallel to AD and AB; and it is evident all the Effect of Gravity will be only to cause the Body during that Time, to descend through Space equal to AD from the Line AB, and so in the End of that Time instead of being at B it will be in C; but since the Point C is as far from D as B is from A, it follows, that whether Gravity acts or not, the same Space is described in the same Time in the Direction AB, and so the same Force is required to give it the Motion, which therefore has no Relation to or Dependance on Gravity; but arises solely from the Nature of inactive Matter, and is on that Account ever proportional to the Number of Particles or Quantity of Matter in Bodies.

Of the Properties of BODIES.

13

GRAVITATION, or, in the Abstract, the GRAVITY or WEIGHT of Bodies (VII).

REPULSION seems to be a Property belonging to the small Particles of Matter universally; for they do not more evidently *attract* in some Circumstances, than they *repel each other* in others, as will be evident by Experiments, hereafter to be exhibited (VIII).

(VII) This Power, or Virtue, was originally communicated to all Matter by the Omnipotence of the Deity, and is no ways necessary to its Existence; therefore I apprehend when Sir *Isaac* says, he is ignorant of the Cause of Gravity, he can only mean the particular Manner in which it is exerted from one Body, and in which it acts upon another, and not that he was ignorant of the Source or Principle of Power whence it could be derived. But as the Manner in which it is exerted, so the Means by which it is propagated from one Body to another, is as yet unknown. To account for the Propagation of this Virtue, as of Light, &c. 'tis supposed that there is an exceeding fine imperceptible Medium or ethereal Spirit, diffused thro' all the System, and pervading the Pores of all Bodies; that this Medium is extremely elastic, and that by its Tremours and Undulations, which are almost instantaneous, the Emanations of the attracting Virtue, Light, &c. are propagated with an immense Velocity, much after the same Manner as Sounds are convey'd by the undulatory Motion of the Air. By Means of this most subtle Spirit, 'tis supposed, likewise, that electric Bodies exert their Power of attracting and repelling light Bodies at greater Distances; that Light is emitted, reflected, refracted, inflected, and heats Bodies; that all Sensation is excited; and animal Motion perform'd at Will; namely, by the Vibrations of this Spirit mutually propagated along the solid Filaments of the Nerves from the outward Organs of Sense to the Brain, and from the Brain to the Muscles.

(VIII) *Attraction* and *Repulsion* differ in no other Re-

THESE

Of the Properties of BODIES.

THESE are the general Properties of Matter, which we must regard in our Explications of the various Phænomena of Nature, in the Sense we have defined them. These are the several *Data*, or fundamental Principles on which the Science of Philosophy depends, and which will each of them afford an ample Field both in the *speculative and practical Part*. We shall (for the sake of Method) begin with the *Attraction* of Bodies, consider its several Species, and prove their Existence and the Properties of each by Experiments.

THE *Power of Attraction*, or Cause of *Gravity*, we presume not to define, or say what it is, but only that it is, or does exist; and the Laws of its Action we shall

spect than this, that the attracting Virtue in the first Case carries Bodies *towards* the attracting Body, and in the latter, it carries them *from* it. In each Case, the Particles are moved in the same Manner among themselves, by the *Attracting, Electric, or Magnetic Power*. Some very eminent Philosophers of late think different from Sir Isaac Newton, who always asserted, that Attraction and Repulsion were two different and opposite Powers, but that the latter begun where the former ended; but these Gentlemen assert, they both belong to or begin originally from the Particles of Matter themselves, and that those which once attract or repel, always do so; however this Principle requires to be supported by more Experiments, and better Reasons than we have yet seen in its Favour.

endea-

endeavour to assign by what may be discover'd by Reason and Experiment. To this End we must consider, that *any Kind of Power or Virtue, proceeding or propagated from a Body in Right Lines every Way as from a Centre, must decrease in its Energy or Strength as the Squares of the Distances from the Body increase*; for 'tis evident, the Force will be every where as the Number of Particles issuing from the central Body on a given Space, which Number of Particles will decrease as the Squares of the Distances increase. Thus the Number of Particles which at *one Distance* AB, from a Point in the Body at A, falls on a Square Inch BEFG, will be *four times* as great as the Number which falls on a Square Inch CHIK at *twice* that Distance AC; and *nine times* as great as the Number which falls on the square Inch DLMN at *three times* that Distance AD; and so on, as is evident from the Diagram.

Plate III.
Fig. 1.

HENCE, since we have no Reason to doubt but that all Kinds of Attraction consist in fine imperceptible Particles or invisible *Effluvia*, which proceed from every Point in the Surface of the attracting Body, in all right-lined Directions every way, which in their Progress lighting on other
Bodies

Bodies urge and solicit them towards the superior attracting Body; therefore the Force or Intensity of the attracting Power in general must always decrease as the Squares of the Distances increase.

HENCE also we may observe, by the way, that *Light* and *Heat*, *Odours* and *Perfumes*, which consist of Particles or *Effluvia* that proceed every way from luminous, heated, and odoriferous Bodies, as from a Centre, have always their Forces abated according to the above Law. The Force of *Sounds* also decreases in the same Proportion, for Reasons that will be hereafter assign'd.

By virtue of attracting Power, the grand *Machinery* of the *Solar System*, and doubtless of all the others in the Universe, is effected, established, and conserved. It is therefore of the greatest Consequence, to be acquainted with the different Species of this universal Power or Agent, and to learn by Experiments the peculiar Nature, Laws, or Manner of Action in each. Naturalists generally reckon *Four* different Sorts of Attraction, viz.

I. THE *Attraction* of COHESION, which is peculiar to Corpuscles or primigenial Particles of Matter, of which larger Bodies are composed, by the Accretion and firm

Adhesion of these Particles, arising from their strong attractive Power.

II. THE *Attraction of* ELECTRICITY, which is peculiar to some Kinds of Bodies, as Glass, Amber, Sealing-wax, &c. which are therefore called *Electrical*.

III. THE *Attraction of* MAGNETISM, or of the Loadstone, which is peculiar to, and mutual between the Loadstone and Iron.

IV. THE *Attraction of* GRAVITATION, which is observable only in the larger Compositions and Systems of Matter; as in the *Earth* and *Moon*, and the *Sun* and *Planetary Bodies* which compose the solar System. Of each of these Species of Attraction in Order; and first,

OF the *Attraction of* COHESION: The Laws and Properties of this Attraction are the following. (1.) It is very discernible and most powerful in Corpuscles, or the smallest Particles of Matter. (2.) It is mutually exerted between those Particles; or they mutually attract, and are attracted by each other. (3.) The Sphere of Attraction, or Extent of this Power, is greater in some Particles of Matter than in others, but very small at the utmost: For (4.) This Power is insensible in solid Bodies in the least sensible Distance, acting as it were

Of the Attraction of COHESION.

only on Contact; and therefore, (5.) It must be nearly proportional to the Quantity of contiguous Surfaces: or the Parts of Bodies cohere most strongly, whose touching Surfaces are largest. (6.) This Power must decrease as the Squares of the Distances increase; because it must be supposed to issue from each Particle in right-lined Directions. (7.) Where the Sphere of Attraction ends, there a *Repelling* Power begins, by which the Particles, instead of *attracting*, *repel* and fly from each other. (8.) By this Power the small Portions or Drops of a Fluid conform themselves to a spherical Figure (IX).

Plate I. (IX) To illustrate this Matter of Corpuscular Attraction and Repulsion: Let A, B, (in *Fig. 9, 10, 11, 12.*) represent two small Bodies or Corpuscles of Matter; and their Spheres of Attraction, (or Extent of their Attracting Power every way) be CDE, and EFG. Then it is evident,

1. That the Force with which the Bodies attract each other will be greatest in the Case where they touch each other.

Plate I. 2. If the Bodies are of a Spherical Figure (as *Fig. 9.*) they can touch but in a single Point; and therefore the Power with which they stick or cohere together will be the slightest or weakest possible.

3. But if the Bodies are bounded by plain Surfaces (as *Fig. 12.*) the Power of Cohesion will, be so much the greater, as the Number of Particles, or Area of the Surface by which they touch is larger.

4. If the Corpuscles A, B, touch not, but are yet within the Sphere of each other's Attraction, (as *Fig. 10.*) they will then be easily moveable or voluble about each other,

THE first and second of these Properties are made manifest by various Experiments; as the sudden Union of two contiguous *Drops of Mercury, Water, &c.* The strong Adhesion of two *Leaden Balls*, which touch by polish'd Surfaces (X); as also of *Glass*

other, and yield to the least Impression of external Force; and this will be the Case of all Particles, whatever be their Figures or Forms.

5. If the Corpuscles A, B, are separated to such a Distance as to be without each other's Sphere of Attraction (as in *Fig. 11.*) then will they fly from, or repel each other, with a Power that extends to a considerable Distance, and is exerted every way equally from each Particle.

6. If there be a great Number of these Particles thus disengaged from each other, but yet confin'd within certain Bounds, they will, by this repulsive Power, be all situated at equal Distances from each other through the whole Space, if the confining Power be on every Side equal.

7. This Power, while *attractive*, extends to but very small Distances from the Corpuscles, and then becomes *repulsive*, for strictly speaking, there are no Bodies in Nature which repel *absolutely*, but only at certain Distances; thus the Loadstone does not at all Distances repel the Needle, but only at a definite Distance; for either Pole will attract either End of the Needle at a very near Distance, or upon Contact, as is easy to experiment.

(X) This Adhesion of *Leaden Balls* is so very considerable, that with two (not weighing above a Pound each, nor touching upon more than $\frac{1}{100}$ of a square Inch Surface) I have lifted above 200 *lb.* weight. In order to this, the Surfaces by which they touch must be finely planed with the Edge of a sharp Pen-knife, and equally press'd together with a considerable Force, with a gentle Turn of the Hand at the same Time; and thus two common

Of the Attraction of COHESION.

Planes, and Crystal Buttons: The Ascent of Water between *Glass Planes*, and in *Capillary Tubes*: The rising of Water by the

leaden Bullets will adhere so firmly together, as to require upwards of 80 *lb.* to separate them.

2. In polish'd Surfaces that are very hard, as Glass, Brass, &c. it is impossible to bring them into such close Contact as to cohere without the Interposition of Water, or something humid, to fill the Pores by expelling the Air contained therein, which prevents the planes coming together while dry; the Humidity in this Case proves a Cement which holds the Planes together by all its Force of Attraction on either Side.

3. This Force of Attraction between the Brass Planes is greater with Oil than with Water; and greater with any Sort of Grease or Fat that will harden with Cold than with Oil. I never yet could meet with four Men strong enough to separate Planes thus put together, which are but $4\frac{1}{4}$ Inches Diameter; and therefore they cohere with a Force much superior to the Force or Pressure of Air on such a Surface, which contains about 14 square Inches, and allowing 15 *lb.* for Pressure to every Inch it will amount to but 210 *lb.* which is not half the Strength of four Men pulling against each other to the best Advantage. I have often seen six Men endeavour to separate them in vain.

4. Professor *Muschenbroek* has made many Experiments to shew the Force of Cohesion between Planes of various Substances, and about 2 Inches Diameter, well polished, having first heated them in boiling Water, and then besmear'd them first with a cold Tallow-Candle, and afterwards with boiling Grease, and the Weights to separate them were as in the following Table:

		<i>Cold Grease.</i>	<i>Hot Grease.</i>
Planes	of Glass	— 130 <i>lb.</i>	300 <i>lb.</i>
	of Brass	— 150	800
	of Copper	— 200	850
	of Marble	— 225	600
	of Silver	— 150	250
	of Iron	— 300	950

Sides

Of the Attraction of COHESION.

21

Sides of a Glass Vessel, and into Tubes of Sand, Ashes, Sugar, Sponge, and all porous Substances.

5. These Planes adhere by other Sorts of Matter with Forces as in the Table below, where the Weights necessary to separate them are specified :

With Water —————	12 Ounces.
With Oil —————	18 Ounces.
With Venice Turpentine —	24 Ounces.
With Rosin —————	850 Pounds.
With Tallow Candle —	800 Pounds.
With Pitch —————	1400 Pounds.

Though these Experiments would not always give the same Numbers, yet they sufficiently shew the vast attractive Forces, and the very great Difference between them.

6. After this he gives us an Account of his Experiments made to find the Force with which Bodies cohere naturally, or what is the absolute Force of Cohesion in various Bodies, which he estimates by the Weights required to pull them asunder, drawing according to their Length ; this he tried first in Wood, and afterwards in Metal. His Pieces of Wood were of a long square Form, of which each Side was $\frac{26}{100}$ of an Inch, and by Weights suspended they were drawn asunder, according to the several Sorts, as mention'd below :

Wood of Linden-Tree —	1000 lb.
of Alder —————	1000
of Fir —————	600
of Oak —————	1150
of Elm —————	950
of Beech —————	1250
of Ash —————	1250

7. The Tryal he made with Metals was of Weights suspended to Wires of each Sort, whose Diameter was $\frac{1}{10}$ of a Rhinland Inch ; or because the Rhinland Foot is to ours as 139 to 135, the Wire was $\frac{135}{139}$ Parts of an Inch *English*. The Metals and Weights were as follow :

Of the Attraction of COHESION.

THE third of these Properties is evident by the Experiments of Water rising above the common Level, and Mercury's sinking below it, in Capillary Tubes: By the sticking or adhering of Water to common Substances, which by Mercury are left dry.

Of Copper	_____	299 $\frac{1}{4}$ lb.
Of yellow Brass	_____	360
Of Gold	_____	500
Of Silver	_____	370
Of Iron	_____	450
Of Tin	_____	40 $\frac{1}{4}$
Of Lead	_____	29 $\frac{1}{4}$

8. These Experiments shew the *absolute Force of Cohesion* in Bodies. That which he calls the *Relative Force* is that by which any Body resists the Force of any other Body acting upon it in a Direction perpendicular to its Length; and this he estimated in the same Pieces of Wood as before, by putting one End into a square Hole of a Metal Plate, and hanging Weights towards the other End sufficient to break each Piece of Wood at the Hole. These Weights and Distances from the Hole were in his Experiments as follow:

	Distance.	Weights.
Fir	9 Inches	40 Ounces.
Oak	8 $\frac{1}{2}$	48
Elm	9	44
Pine	9 $\frac{1}{2}$	36 $\frac{1}{2}$
Alder	9 $\frac{1}{4}$	48
Beech	7	56 $\frac{1}{2}$

Such as would see much more on this Subject, may consult the Author's admirable *Essai de Physique*; and his Treatise of the Cohesion of Bodies, altogether on the Subject.

THE fourth and fifth Properties are evinced by the Experiments of the different Heights to which Fluids ascend between Glafs Planes unequally inclined, and in Capillary Tubes of different Bores: Also, by the accelerated Motion of a Drop of Oil between two inclined Planes: And likewise by the *Hyperbolical Curve* form'd by the Superficies of a Fluid ascending between Glafs Planes touching each other on one Side.

THE sixth Property is evident. The seventh seems evident between *fat and oily Particles of Matter*, and those of an *aqueous Nature*: But is most manifest from the Elastic Property of the Air, whose Particles compress'd together restore themselves by this repellent Power to their first State: Also by the Ascent of *Steam* or *Vapour* from humid or fluid Bodies.

THE eighth Property of this Attraction is manifest by Drops of Water falling on Dust; by Drops of Dew gathering on the Tops of Grass; and lastly, by *Quicksilver* divided into small Portions, which always form themselves into perfect Spherules or Globules (XII).

(XII) 1. In order to account for the Phænomena of Glafs Capillary Tubes, &c. it will be necessary first to

FROM this Account of the *Attraction of Cohesion* we learn a rational Solution of several very curious and surprizing Phæno-

Plate I.
Fig. 13.

premise, that there is a greater Attraction between the Particles of Glass and Water, than there is between the Particles of Water themselves: For if it were not so, the least Quantity or Drop of Water C applied to the under Side of a Glass Tube, in a Position parallel to the Horizon, as A B, would not adhere to it, but immediately fall down by its Gravity; but we see that it does not, till its Bulk and Gravity be so far increased as to overcome the Attraction of the Glass, and then it falls off.

2. Since we find such a strong attractive Power in the Surface of Glass, it will be easy to conceive how sensibly such a Power must act on the Surface of a fluid (not viscid) as Water, contained within the small Cavity or Bore of a Glass Tube; as also that it will be in Proportion stronger as the Diameter of the Bore is smaller; for that the Efficacy of the Power follows the inverse Proportion of the Diameter is evident from hence, that only such Particles as are in Contact with the Fluid, and those immediately above the Surface, can affect it.

3. Now those Particles form a Periphery, or rather a very narrow *Annulus* or Ring contiguous to the Surface, the upper Part of which attracts and raises the Surface, and the lower Part, which is in Contact with it, supports and holds it up; so that neither the Thickness nor Length of the Tube avails any thing, only the said Periphery of Particles, which is always proportional to the Diameter of the Bore.

4. The Quantity of the Fluid raised will therefore be as the Surface of the Bore which it fills; that is, as the Diameter, since the Effect would not be otherwise proportional to the Cause.

5. Since the Quantities follow the Ratio of the Diameters, the Heights to which the Fluid will rise in different Tubes will be inversely as the Diameters, which is thus demonstrated:

mena

mena of Nature: As why the Parts of Bodies adhere and stick so firmly toge-

Let $\begin{cases} Q, q, \text{ represent the Quantities of Matter raised;} \\ D, d, \text{ the Diameters of the Tubes Bores;} \\ H, h, \text{ the Heights to which the Fluid rises in the} \end{cases}$ Plate I.
Fig. 15.
Tubes.

Then since Q, q , represent the Contents of two Cylinders of the Fluid (suppose GL and IM) it will be $Q : q :: DDH : ddh$, from the Nature of a Cylinder; and from the Nature of this Attraction, it is $Q : q :: D : d$ (as was just now shewn;) therefore we have, As $D^2 H : d^2 h :: D : d$; and so $D^2 H d = d^2 D h$, that is $DH = dh$; consequently, $D : d :: h : H$.

6. The Velocity with which it begins to rise in the Tube is exceeding great, but presently abates, and so continues by increasing the Weight of the Fluid that is raised; and it will continue to rise till an Equilibrium be made between the Gravity and the attracting Force of the Glass.

7. The Pressure of the Air neither helps nor hinders the rise of the Fluids, for the Effect is the same in *Vacuo* as in the Air. But the more viscid the Fluid, the less apt it is to rise in the Tube.

8. The Power by which the Fluid is raised will keep it in the Tube for any Time without Exhaling or Evaporation, as I have tried by hanging several Capillary Tubes thus charged with their Fluids for seven or eight Months together in the Summer Sun, whose Heat did not appear to diminish the Fluids in the least sensible Degree.

9. By the same Power the Water rises between Glass Planes, so as to form the celebrated HYPERBOLIC CURVE, when they are so placed in Water as to touch on one side, and be separate or opened to a small Distance on the other. Thus suppose A^BCD the Surface of Water, in which are placed the two Planes $f m O$ Plate I. and $e m O$, so as to touch all along the Side $m O$, and yet Fig. 14. open on the other Part, so as to contain a small Angle $f O e$; then forming the Rectangles $O d h p$, $O c i o$, $O b k n$, $O a l r$, &c. and measuring very exactly, they will alter;

ther; why some are *hard*, others *soft*; some *fixed*, others *fluid*; some *elastic*, others *void*

ways be found equal to each other; and this is a well known Property of the Hyperbolic Space between the Curve and its Legs *Oe* and *Om*.

10. If two long Glass Planes are first smear'd over with Oil, and then set together at their Ends, and inclined to each other under a very small Angle, and a Drop of the Oil so placed between them as to touch both Planes, it will immediately begin to move towards the touching Ends, or Angles of the Planes, and that Motion will be continued with an accelerated Velocity, by reason of the increasing Attraction of the Planes, on account of the decreasing Distance between them, and the larger Portion of touching Surface on each Side the Drop; concerning all which a great deal has been wrote to very little Purpose.

11. If Glass be applied to any other Fluid, whose Particles attract each other more strongly than Glass attracts them, all the Phænomena of such a Fluid in Capillary Tubes, and between Glass Planes, will be just the Reverse of those now mentioned of Water. Now *Quick-silver* is such a Fluid, and therefore it will stand lower within a Capillary Tube than without; the Surface *convex*, and not *concave*, as in Water; and between the Planes it will move a contrary Way, &c.

12. But if a Basin or Dish be made of Copper or Brass, and polished well within Side, and then tinn'd all over, Mercury put into such a Vessel will every where unite with the Tin, and may be properly said to *wet* it, as Water does Glass; and the Mercury, if clean, put into this *Mercurial Basin*, will be attracted and rise all round the Sides, and have the same *Phænomena* with *Water* put into a *wet Glass*, Transparency only excepted.

13. Here I think proper to mention an Experiment, with which I have often entertained myself and others, and that is, pouring Mercury into the abovemention'd Dish or Basin form'd into a true Spherical Figure, and well

of

of any elastic Quality; which all arise from the different Figure of the Particles, and

planish'd, it leaves the concave Surface so nicely silver'd, that it makes a most delightful Mirrour, and reflects the Light so perfectly, as to produce the strongest and most distinct Image of an Object that can possibly be effected by Art. How far the Nature and Application of Mirrours may be improved by this Method, must be left to farther and future Experience: And also whether it be not practicable by this Means to make a very large and most effectual Burning-Glass, at a very easy and cheap Rate.

14. A late *French* Author (*L'Abbé Nollet*) has thought fit to call in question the Cause we have here assigned for the Rise of Fluids in Capillary Tubes, and has almost sneer'd Dr. *Furin's* learned Dissertations on that Subject; but I see nothing that could oblige him to do this, from what he has said to explode it; no Man can be convinced by his *ipse dixit*, that this is a mere Hypothesis; the *Phænomena* so plainly indicate the Cause to be Attraction, and agree in every Particular so nicely to the Theory, that 'tis not to be expected we should give it up till we see better reason for it than this reverend Father has given in his prolix Digression on this Subject. See his *Leçons de Physique Experimentale*. Tom. II. Page 421, &c.

15. But though the Attraction of the Glass is the indubitable Cause of the Ascent of Fluids, yet it must be allow'd that the Nature and Genius of the Fluid is to be regarded in most of the *Phænomena*, which are not proportion'd to the attracting Power of the Glass only, but to that Power conjointly with the various Disposition of Fluids to yield thereto; nor is the Density of the Fluid of primary Consideration, it being evident by Experiment, that some lighter Fluids will rise to a less Height than others which are much heavier: Thus *Spirit of Wine* will rise but $4\frac{1}{2}$ Tenths of an Inch in a Tube, where Oil of Tartar will rise 9, as in the Table below. Nor does it at all depend on the Viscidity or
the

Of the Attraction of COHESION.

the greater or lesser degree of Attraction consequent thereupon. Hence the Reason

Tenacity of Parts; for hard white Varnish (very thick and viscid) and Spirit of Wine ascend nearly to the same Height. I am inclined to think from the Experiments which I have made, that the Ascent of Fluids depend greatly upon the Spirit they contain, and that not on the Quantity of it, but on some peculiar Action or Disposition thereof, relative to the attracting Power of the Glafs. So from the Table it appears, that Spirituous Liquors rise to a small Height in general; but Water, which has none, to the greatest; yet Spirit of Hart's-horn and Spirit of Urine rise higher than Ale and Small-Beer, which contain but a small Proportion of Spirit.

16. But to set this Matter in a clear Light, I have very carefully made the Experiment on the following Fluids, which were each of them often repeated, and with the same Success; the Bore of the Tube was about $\frac{1}{30}$ of an Inch in Diameter.

	<i>Altitude.</i>
Common Spring Water —	12 Tenths of an Inch.
Spirit of Urine ———	11
Tincture of Galls ———	11
Recent Urine ———	10
Spirit of Salt ———	9
<i>Ol. Tar. per Deliquium</i> —	9
Vinegar ———	$9\frac{1}{2}$
Small Beer ———	9
Strong Spirit of Nitre —	$8\frac{1}{2}$
Spirit of Hart's-horn —	$8\frac{1}{2}$
Cream ———	8
Skimm'd Milk ———	8
<i>Aqua fortis</i> ———	$7\frac{1}{2}$
Red Wine ———	$7\frac{1}{2}$
White Wine ———	$7\frac{1}{2}$
Ale ———	$7\frac{1}{2}$
<i>Ol. Sulph. per Campanam</i> —	$6\frac{1}{2}$
Oil of Vitriol ———	$6\frac{1}{2}$
Sweet Oil ———	6

why

why Flies walk on the Surface of the Water, and wet not their Feet. On this

	Altitude.
Oil of Turpentine —	$5\frac{1}{2}$ Tenths of an Inch.
Geneva ———	$5\frac{1}{2}$
Rum ———	5
Brandy ———	5
White hard Varnish —	5
Spirit of Wine ———	$4\frac{1}{2}$
Tincture of Mars ———	$4\frac{1}{2}$

17. From other Experiments it appears that Heat and Cold are not concerned in the Ascent of Fluids, the hottest Water standing at the same Height as the Cold. Also Salts of any kind dissolved in Water makes but little Alteration in the Heights, for when Water has taken up all it can in the Solution. it will rise 10 or $10\frac{1}{2}$, which small Deficiency is owing undoubtedly to the increased Gravity of the Fluid.

18. There is one Phenomenon of Capillary Tubes very extraordinary, which is this; if a large bored Tube Plate I. N P be at one End P drawn out into a fine Capillary as Fig. 15. at O, such a Tube fill'd with Water, and then inverted into a Vessel of Water, as A B C D, will retain all the Water suspended through its whole Length, if it does not exceed the Height to which a Capillary of the same Bore, as at O, will attract it. To see the Reason of this, we must consider the Motion of a small Quantity of Water C D in a tapering Capillary A B in an horizontal Position. And here we must assign two Principles of Motion in the Fluid C D; one is the *attracting Power* of the Glass at C and D, the other is the *Velocity* of the moving Fluid in these Places. For the *Momentum* of any Body, whether Solid or Fluid, is always proportion'd to the Intensity of the moving Power, and the Velocity of Motion. (As is shewn fully hereafter.) But the moving Power or Attraction at C is to that at D as the Diameter of the Bore at C to the Diameter thereof at D; Or, putting a , A, for the two Attractions in D and C, and d , D, for the Diameters, we have $a : A :: d : D$.

Principle

Principle we account for the Manner how Plants imbibe the nutritive Juice or Mois-

19. Again, the Velocities (V and v) of the Water in those Points are as the Squares of the Diameters inversely (as will be shewn hereafter;) that is, $V : v :: DD : dd$. Therefore $V \times a : v \times A :: D^2 d : d^2 D :: D : d$. That is, *the whole Motion of the Water at D is to that at C as the Diameter at C to that at D; consequently the Water will move towards the End of the Tube B.*

20. Because when the Water moves in the Tube, the Velocity at D is greater than that at C, and therefore the Spaces described in equal Time by the Extremities of the Fluid unequal, it follows that the Water will move from D towards B with an accelerated Velocity, the Difference between the Diameters d and D continually increasing. This likewise shews the Reason of the accelerated Motion of the Drop of Oil between the two inclined Glass Planes, as mentioned *Art. 10.*

Plate I.

Fig. 17.

21. The Motion of the Water once begun at D, will continue till it has with great Rapidity reached the End B; and if then the End B be raised above the horizontal Line A F, yet will the Water be drained at B, and not descend, till the perpendicular Altitude BF is equal to the Height to which the Water would rise in a Capillary of the same Bore as at B.

Plate I.

Fig. 18.

22. Hence also if the Tube be held in an upright or perpendicular Position, as A B, the Water will not descend, unless its Weight be such as when added to the Power acting at C towards A, the Sum shall be greater than the Power acting upwards at B; which Power arises from the Attraction of the Tubes, the Velocity of the Fluid, and the Cohesion of its Particles; for unless we consider this Cohesion of the Particles, the Weight of the Water, when the Vessel is large, will be too great to suffer a Suspension of the Fluid by the other Forces in the Capillary Part.

23. Nor is this Cohesion of the Parts a *Petitio Principii*. but what we know by Experiment to be the Cause of such a *Phænomenon* when nothing else can be so;

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ture of the Earth by the Fibres of the Roots: Also for the Rise of the Sap in Vegetables, and for the whole Oeconomy of Vegetation in general. Hence the *Rationale* of the various Secretions of Fluids by the Glands of an Animal Body, and their wonderful Circulation through the fine Capillary Vessels. Hence also the Reason of Soldering and Gilding of Metals; also of Melting or Fusion of Heat. Hence also the Exhalation of Vapours by the Heat of the Sun or Fire; the Aggregation of Aqueous Particles in the Air, forming the Drops of Rain. We hence see the Reason of

thus we find the Particles in a Drop of Water hang together by Cohesion, till the Gravity of the Whole draws it from the Part to which it coheres. Without this Cohesion there could be formed no Drop at all.

24. Hence likewise it is, that a Column of Mercury well purged of Air, will be suspended by Cohesion to the Height of 70 or 75 Inches, when the Air alone will sustain it to no greater Altitude than that of 31 Inches at most. Dr. *Furin* supposes the Pressure of an elastic subtle Medium (which can penetrate the Pores of Glass and Water) to have some Part in producing this very odd Phænomenon. But if all we can learn from Experience be thought insufficient, I shall leave the Reader to amuse himself with what Conjecture he shall think fit; assuring him in the last Place, that this wonderful Suspension of the Fluid in ever so large a Glass, as A C, whose Top terminates in the Capillary B, is not owing to the Pressure of the Air, because the same thing will happen in an exhausted Receiver on the *Air-pump*. See Appendix, Numb. II. to *Cote's Lectures*.

Plate I.

Fig. 19.

Distil-

Distillation, Filtration, Dissolution, Digestion, Sublimation, Precipitation, Crystallization, and all the other Operations of *Chemistry* and *Pharmacy*, which are no otherways to be accounted for. Lastly, we find *Sir Isaac Newton* (at the End of his *Optics*) gives a beautiful and clear Solution to those wondrous Phænomena of subterranean Accensions and *Explosions*; of *Volcano's* and *Earthquakes*; of *Hot Springs, Damps,* and suffocating *Exhalations* in Mines, &c. on the Principles of this Sort of Attraction and Repulsion (XIII).

(XIII) 1. The Rationale of the several important Particulars here mention'd will easily appear from the preceding Principles or Properties of Matter. Thus HARDNESS of Bodies arise from such Figures of the Particles of Matter as permit them to touch by large Surfaces, and thus to have a great Power of Attraction; and at the same Time, so to fit and fall in with each other, that they prevent any Motion ensuing among themselves by any external Pressure; whereas in another Sort of Figure, the Particles may be admitted to touch by pretty large Surfaces, yet such as not confining each other, will permit the Motion impress'd on the external Parts to proceed among the interior Particles, which will thus easily move among themselves, and make what we call Softness in Bodies.

2. FIXITY is much the same as Hardness, and owing to the same Cause, and is oppos'd to FLUIDITY, which may in some Sense be esteem'd the greatest Degree of Softness, because a fluid Body yields to the least Pressure, and therefore the attractive Power, by which the Particles of a Fluid cohere, must be the least possible; but this results from such a Figure as admits of the fewest Points of Contact possible, that is, a *Spherical Figure*.

And

And hence 'tis evident that Bodies are more or less fluid as their Parts approach more or less to a globular Figure. All which follows from *Annotot.* III. 2.

3. ELASTICITY in Solids arises from the same Principle of corpuscular Attraction; thus if a Steel Spring, or Wire, or Piece of very thin Glass, be bent out of its natural Position, the Particles on the convex Part are forced from that intimate Contact they before had; and on the concave Part they are forced nearer together, or harder upon each other, than in the natural State; in both Cases there will be a considerable Resistance to be overcome, and require a superior Force. During this State of the Particles they may be said to be under a Sort of Tension on one Side, and Compression on the other; and since by this Force they are not drawn out of each other's Attraction, as soon as the Force is remitted or ceases to act, the attractive Power reduces the Particles, and unbends the Wire by restoring it to its *natural State and Site*. Now 'tis well known that many Substances are composed of such fibrous Parts or Filaments, which resemble fine Wire, and are interwoven and disposed in such a Manner (as in *Sponge*, for Instance) that they cannot be compress'd without being bent or wrested from their natural Position, whence all such Bodies will in such a Case exert a Force or Spring, and thus prove *elastic*. This Theory may be applied to Bodies of a closer Texture, as *Cork, Wood, Ivory, Glass, &c.* And such Matter as wants this Texture and Disposition of Parts, will not exert such a Spring or Force, but take the Figure arising from the impress'd Force, and thus prove what we call *Non-Elastic Bodies*.

4. SOLDERING is another notable Effect of this cohesive Power; for when the Surfaces of two Pieces of Metal are made clean, or freed from all intervening Matter besides the Solder, the Solder being then flux'd, its Particles will freely enter the Pores, or come into Contact with those of the polish'd Surfaces of Metal, and by that Means adhere firmly to each, and therefore when the melted Solder becomes cold and fixed, it holds the Surfaces exceeding fast together, in the Nature of a GLUE or CEMENT, which are all of the same

VOL. I. D kind,

kind, but differ in the Degree of the cohesive or attracting Power of the Parts.

5. MELTING or FUSION of fixed Bodies depends upon the same Theory; for when the Particles of a fixed Body are separated by the Action of Heat so far as not to touch, or but very slightly, yet not so far as to be out of each other's Attraction, they then will easily move by each other, and put on all the Appearances of a fluid Body, (as plainly follows, from *Annotat. X. 4.*)

6. EVAPORATION proceeds likewise from hence, that when the Particles are so far separated by Heat as to be without each other's Attraction, they then begin to repel each other, and thus will seem to rise from the Surface of the Fluid in Form of a Vapour, or Body of Particles, which are at equal Distances from each other; and becoming thus specifically lighter than the same Bulk of airy Particles, they will rise in the fluid Body of Air till they come to that Part of it which has the same Gravity, and they will there make what we call CLOUDS, which will move this Way or that, according to the Current of Air in those Regions. See *Annot. X. 5, 6.*

7. The VAPOURS thus raised become the Original Matter of all METEORS, one Degree of Cold condensing them into larger Globules, which fall in Drops of RAIN; a greater Degree producing a Fixedness or Coagulation of the Particles, which shoot like Salts into various curious Forms, and make the *Fleaks* of SNOW; a third and still greater Degree of Cold congeals the Vapours into a harder Substance, greatly variegated in Form and Consistence, and thus produces HAIL. If the Cold condenses the Vapour so that it can't rise high, above the surface of the Earth, it will there hover about, and fill the lower Air with an obscuring FOG or MIST: Or, if the Cold be more intense, it freezes the Mist to every Twig and Blade of Grass in Form of a white Incrustation, which we call a RIME. If the Air be warm, so that the Vapour therein be too fine to be visible in the Day-time, it will yet be condensed by the Coolness of the Evening so far as to descend, and settle upon the Tops of Grass in the Form of DEW: But, lastly, if the Evenings of such a fine Day be cold enough to freeze,

freeze, then instead of a Dew there will appear a WHITE FROST all over the Surface of the Ground.

8. The CAPILLARY SYPHON is a most curious *Phænomenon*, arising from the same Principle; for the Water being raised to the Flexure or bended Part, by the attracting Force of the shorter Leg immersed into it, the same Force continuing to act carries it even into the longer Leg, and there conspiring with the Force of Gravity, the Water precipitately descends and drives the Air before it, and thus would keep running or dropping out till all the Water were exhausted to the Orifice of the immersed Leg, if the Operation were to be continued long enough.

9. The FILTER is to be consider'd as nothing more than a *Compound Capillary Syphon*; for the Threads or Filaments of which it consists, lying very near together, make long and slender Vacuities or Interstices, which represent so many Capillary Syphons, which attract, raise, and decant off the Fluid in a considerable Quantity, proportional to their Number or the Largeness of the *Filter*.

10. After the same Manner it is, undoubtedly, that the Humidity of the Earth is drawn into the Substance of the Roots of Plants and Vegetables, which we know consist of long and very minute Fibres so disposed as to form an infinite Number of tubular Interstices, which act in the Nature of a Filter, and imbibe the Juices and Moisture destin'd for the Nutriment and Growth of the Plant.

11. Thus also it is reasonable to suppose the fine Expansion or Ramification of the *Lacteal*, the *Lymphatic*, and *Sanguiferous Vessels* thro' all the Substance of the Glands and Viscera in the finest Capillary Tubes, does greatly assist, if not wholly promote, the Circulation of the Blood and Juices, in order to the various Secretions of each respective Gland; an Effect too great for the Pulsive Force of the Heart, or larger Vessels, when thus so infinitely and minutely divided, were it not to be joined by the strong Attraction of those Capillary Parts.

12. By this Means it is that Tallow and Oil rise into the Wicks of Candles and Lamps: That Sponge so readily sucks up and retains so great a Quantity of a

Of the Attraction of COHESION.

Fluid: That Tubes of Ashes or Sand will thus raise Water to the Height of several Feet. 'Tis by this Virtue that the Pen retains Ink, and the Paper draws it forth in Writing; or refuses to admit it, if oil'd, by the Repellency of the Particles of Oil.

13. By this Power fix'd Bodies are dissolved by proper Mediums; as *Sugar* and all Kinds of *Salts* by *Water*; *Silver*, *Copp.r*, *Tin*, &c. by *Aqua-Fortis*; *Gold* by *Aqua Regia*; and other Bodies in other Menstruums. The Reason is, that when the Particles of the Solid and Fluid attract one another more strongly, than either those of the Solid or those of the Fluid attract each other separately, then a Separation of the Parts or *Dissolution* must ensue. For since, for Instance, if a Particle of the Fluid be attracted with a greater Force than a contiguous Particle of the Solid, that Fluid Particle will rush upon the other with so great an Impetus, as instantly to displace it, and separate it from the Solid Particle to which it before adhered with a weaker Force; and thus we are to conceive of every other Particle, till the Whole be dissolved, which will then end when each Fluid Particle has fill'd its Sphere of Attraction with those of the Solid; and the Fluid is then said to be *saturated*, and will dissolve no more.

14. PRECIPITATION likewise is a Consequence from the same Principle; for if to the Solution of any Solid some other Solid or Fluid Body be added, whose Particles are attracted by the Fluid with a greater Force than those of the dissolved Solid, then will the former take Place of the latter, which, being disengaged, will fall to the Bottom, and settle in Form of a very fine Powder; thus is *Silver* precipitated by *Copper*, which the *Aqua-Fortis* attracts more powerfully than *Silver*. Thus a Solution of *Copper* is again precipitated by *Iron*, and other Solutions have their respective Precipitants.

15. FERMENTATION is also an Effect of the same Cause, inasmuch as it consists in the internal Agitation and Commotion of the Parts of the fermenting Body, arising from their mutual Action and Attraction so often mention'd. By this means Bodies are either wholly dissolved, as has been said; or have their Parts so far alter'd

ter'd and compounded with other Matter, that they seem to change their Natures, and acquire quite new Properties and Principles. Thus Malt by Fermentation yields a Spirit by Distillation, tho' not like it before; not to mention Putrefaction, and the many notable *Phænomena* thence arising. See *Shaw's Lectures on Chemistry*.

16. The HEAT, EBULLITION, and EXPLOSION of sundry Mixtures are owing to the violent Action and great Rapidity of Motion among the constituent Particles; for all Heat and Accension proceeds from an Attrition or Rubbing of the Particles of Matter one among another. Thus *Aqua-Fortis* and the Particles of Copper act so violently on each other as to cause a great Effervescence and Heat; and Steel-Filings produce a Blast or Explosion instantly. Again, Oil of Caraway-Seed, pour'd on compound Spirit of Nitre, kindles to Flame with such prodigious Force, that *in Vacuo* it has blown up and burst an exhausted Receiver six Inches wide and eight Inches deep.

17. In accounting for THUNDER and LIGHTNING we have recourse to the same Principles, that is, a Fermentation between the sulphureous Steams exhaled from the Earth, and the Nitrous Acids or Salt in the Air, which take Fire, and cause Explosions in the same Manner as in the Experiments just mentioned. And in some such Manner all other *fiery Meteors* are produced.

18. Also that Burning-Mountains, Mineral Coruscations and Suffocating Damps, Earthquakes, hot Springs, &c. are produced by the intestine Commotion arising from the Mixture of divers Nitro-acid and Sulphureous Matters in the Bowels of the Earth, is evident from a common and easy Experiment, which is by taking equal Parts of Sulphur and Steel-Filings, and working them together, with a little Water, to the Consistence of a Paste; then putting it under Ground, it will presently begin to heat, and in a few Hours take Flame and blow up the Earth about it with a considerable Shock, so as to represent an Earthquake, and a Sort of Volcano in Miniature.

19. That Metals dissolved in Fluids diffuse themselves equally through all the Substance, and keep an

equal Distance among themselves, seems owing to a repulsive Virtue. Also the *Reflection* and *Inflexion* of the Rays of Light from Bodies before they come to touch them, as also the Emission of Light in luminous Bodies, seem effected by such a repulsive Power; which (as has been said) commences where the Sphere of Attraction ends.

20. The Spherical Figure, which the Drops of a Fluid affect being left to themselves, is another Consequence of Corpuscular Attraction; for considering one single Particle of the Fluid, its attractive Force being every Way equally exerted to an equal Distance, it must follow that other Fluid Particles are on every Side drawn to it, and will therefore take their Places at an equal Distance from it, and consequently form a spherical or perfectly round Superficies; in like Manner as the Earth and Sea affect a spherical Figure from the central Attraction being every way equally directed.

21. The surprising *Phænomena* of Glass Drops (sometimes called Prince *Rupert's* Drops) are most probably supposed to arise from hence, *viz.* That while the Glass is in Fusion or melted State, the Particles are in a State of Repulsion; but being dropped into cold Water, it so condenses the Particles in the external Parts or Superficies, that they are thereby reduced within each other's Attraction, and by that Means form a Sort of hard Case which keeps confined the aforesaid Particles in a repulsive State; but when this Case is broke by breaking off the Tail of the Drop, the said confined Particles have then Liberty to exert their Force, which they do by bursting the Body of the Drop, and reducing it to a very peculiar Form of Powder. This Theory seems to be corroborated by making the Drop red hot, and letting it cool gently in the open Air, for then it has no such Effect. Yet another Experiment seems to impugn this Hypothesis; and that is, by grinding away any Part of the Drop on a Grind-stone, the remaining Part continues entire; and yet there appears no manifest Reason why it should not break or burst into Dust, if the internal Particles be the Cause of it, since by this Means they must needs be set at Liberty in the most ample Manner possible: Unless it be, that in Grinding, the Vacuities between the internal Particles are fill'd up with

Of the Attraction of ELECTRICITY.

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THE *Second Species of Attraction* is that of ELECTRICAL BODIES, as *Glass, Amber, Sealing-Wax, Jet, &c.* the principal Properties of which are as follow. (1.) Those Bodies attract others which are very light, as Feathers, Hairs, Leaf-Brass, &c. (2.) The Sphere or Extent of this attracting Power is at the Distance of several Feet; but,

with the Matter of the Stone forced in by the hard Attrition, and by this Means, fixing the Particles of the Glass next the Stone, and thus destroying their repulsive Force, constituting, as it were, another Sort of hard external Case, which confines the Particles no less than the other. And this seems to be confirm'd by that Part which touch'd the Stone remaining entire, when all the rest of the Stone is reduced to Dust by breaking off the Tail or small Part, as any one will find to be the Case on Trial.

SCHOLIUM.

By this wonderful and most extensive Power of Corpuscular *Attraction* and *Repulsion*, we account for most or all the *Phænomena* of Nature, arising from the heterogeneous Mixture and various Associations of Particles of different Bodies; but why, in some Cases, those Particles should readily attract, and in others as strangely repel each other, there has as yet been no demonstrative Reason assign'd. The Hypothesis of a *Polarity*, or *Magnetic Power*, in every Particle of Matter, by which they are disposed on one Part to attract, and on the other to repel, in Manner of the Load-stone, is as specious as it is ingenious; but since the *Newtonian Philosophy*, in fundamental Principles especially, is founded entirely on the Basis of experimental Proof, or Demonstration from Geometry, we ought to be careful how we introduce any Conjecture for Truth, unless all Circumstances conclude and urge the same, which I believe is not the Case of that above-mentioned.

40 *Of the Attraction of* ELECTRICITY.

(3.) It varies with the State of the Weather, being greatest in hot and dry Weather, but weaker in warm and moist. (4.) It may be communicated to a great Distance, *viz.* seven or eight hundred Yards, by the Intervention of a proper Body, as Hempen Strings, &c. (5.) This Virtue is excited by Attrition, or hard Rubbing by the Hand or with a Piece of Cloth, but will not be produced by the Warmth of Fire. (6.) It penetrates or pervades the Pores of Glafs; and, (7.) It may be communicated to other Bodies, so as to render them electrical. (8.) If the Sphere of Attraction be interrupted on any Part, it destroys the Efficacy of the Whole. (9.) By this Virtue Bodies are not only attracted, but also repelled alternately to a very sensible Distance, and with a surprising Velocity. (10.) The Body once repell'd from the Tube will not be again attracted by it, till it has first touch'd some other Body. (11.) Bodies attracted, and sticking to the Tube, will be then attracted by other Bodies not electrical; as the Finger, &c. (12.) This Virtue will exert itself *in Vacuo*, as well as in open Air. (13.) It appears lucid, and sparkles like Fire, in a dark Room. (14.) It is also sensible to
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the Ear by a crackling Noise, like a green Leaf in the Fire (XIV).

(XIV.) 1. To the Properties of Electrical Bodies, here enumerated, we shall add the following.—This Quality is of two Sorts, viz. (1.) *Vitreous Electricity*, or that which belongs to *Glass*; and (2.) *Resinous Electricity*, or that which belongs to *Amber, Rosin, Wax, Gums*, and such like Substances. They are different Kinds, because their Effects are contrary; for the light Bodies repell'd by one will be attracted by the other, tho' excited, which cannot happen when both Bodies have the same attractive Power.

2. All Vegetable Substances differ from Animal ones in this respect also; for an Hempen or Flaxen Thread or String will receive the Electricity of Glass, and carry it instantaneously to an incredible Distance, to some non-electric Body, but will not impart it to a Silk or Hair Line by which it is suspended. This is the Reason why several Men standing on Cakes of Rosin or Wax (which will not receive the *Vitreous Electricity*) convey that Virtue from one to the other in a most surprising Manner. Thus also it is that a Feather suspended by a Flaxen Thread will be alternately attracted and repell'd, but if hung by a Silken String will not be again attracted after it is once repell'd; and *vice versa*, with respect to Amber or Sealing-Wax.

3. Hitherto we have not discover'd the Uses of this wondrous Quality; but Mr. Gray, just before he died, hit upon an Experiment of this Kind, which seem'd to indicate that the Attractive Power, which regulates the Motions of the Heavenly Bodies, is of the Electric Kind. The Experiment was thus; he fix'd a large round Iron Ball upon the Middle of a large Cake of Rosin and Wax, and exciting the Virtue strongly in the Cake, a fine Feather suspended by a Thread, and held near the Iron Ball, was carried round it by the Effluvia in a circular Manner, and perform'd several Revolutions; it moved the same Way with the Planets, from West to East, and its Motion (like theirs) was not quite cir-

Of the Attraction of MAGNETISM.

THE *Third Species of Attraction* is that of the MAGNET or LOADSTONE ; the primary Properties whereof are the following. (1.) Every Loadstone has two Points call'd *Poles*, which emit the Magnetic Virtue. (2.) One of those Poles attracts, the other repels *Iron*, but no other Body. (3.) This Virtue is communicated to Iron very copiously by the Touch, which renders it strongly Magnetic. (4.) A Piece of Iron so touch'd by the Loadstone, and nicely suspended on a sharp Point, will be determined to settle itself in a Direction nearly *North* and *South*. (5.) The End of the Needle touch'd by the *South* Pole of the Stone will point *Northwards*; and the contrary. (6.) Needles touch'd by the Stone will dip below the Horizon, or be directed on the touch'd Part to a Point within the

cular but a little *elliptical*. See the whole Account in *Philos. Trans.* N^o 444.

Many other Experiments of this kind have been tried since ; but when they are rightly considered they will be found of no Service in accounting for the Motions of the Heavenly Bodies, which depend on much more rational and certain Principles, as we shall shew in a proper Place.

Since the first Edition of this Work there have been a great Number of new Discoveries and Improvements in ELECTRICITY and MAGNETISM, which we must by no Means suffer to be wanting in a Compleat System of Philosophy, and therefore shall add an Appendix to this Lecture, giving an Account of the same.

Earth's

Earth's Surface. This is called the *Dipping Needle*. (7.) This Virtue is also to be communicated to Iron, by a strong Attrition all one way ; whence Files, Drills, &c. are always found to be Magnetical. (8.) Iron Rods or Bars acquire a Magnetic Virtue by standing long in one Position. (9.) Fire totally destroys this Virtue by making the Stone or Iron red hot. (10.) This Power is exerted sensibly to the distance of several Feet. (11.) It is sensibly continued through the Substance of several contiguous Bodies or Pieces of Iron, as *Keys*, &c. (12.) It pervades the Pores of the hardest Bodies ; and (13.) Equally attracts the Iron *in Vacuo*, as in open Air (XV.)

(XV.) 1. To the above Properties of the Loadstone I shall add the following, viz. *That the same Pole of the Stone will communicate to a Piece of Iron the Power of attracting or repelling the same End of a touch'd Needle, by drawing it different Ways thereon.* Or thus, If a Piece of Iron be drawn to the Right Hand and attracts, it will repel if drawn to the Left, which is not a little wonderful.

2. I have oftentimes, by a smart Stroke of a Hammer on the untouch'd End of the Dipping-Needle, caused the Magnetic Virtue to come all to that End from the other, so as to make it dip on that Side as much as it did on the other Side before : And on the contrary, I have by such a Stroke sometimes made it dip much more on the touch'd End than before. Again, sometimes by striking it, the Needle, which dipp'd

THESE

THESE, and many others are the Properties of a Body, not more wonderful than useful to Mankind.

before, will be restor'd to its Equilibrium, as if the Virtue had made its Escape, or were uniformly diffus'd over all the Needle. So capricious are the Phænomena of this amazing Power.

Plate II.
Fig. 1.

3. There is a very curious Method of rendering visible the Directions which the Magnetic Effluvia take in going out of the Stone: Thus, let A B, C D, be the Poles of the Stone; about every Side gently strew some Steel-Filings on a Sheet of white Paper; these small Particles will be affected by the Effluvia of the Stone, and so posited as to shew the Course and Direction of the Magnetic Particles on every Part. Thus in the Middle of each Pole between A B, and D C, it appears to go nearly strait on; towards the Sides it proceeds in Lines more and more curved, till at last the Curve-Lines from both Poles exactly meeting and coinciding, form numberless Curves on each Side, nearly of a circular Figure, as represented in the Diagram. This seems to shew that the Magnetic Virtue emitted from one Pole circulates to and enters the other.

4. The Cause of the Variation of the Needle has remain'd hitherto without any demonstrative Discovery; yet since its Declination, and Inclination, (or Dipping) do both of them manifestly indicate the Cause to be somewhere in the Earth, it has given Occasion to Philosophers to frame Hypotheses for a Solution, which make the Earth a large and general *Magnet* or Loadstone, of which all the lesser ones are but so many Parts or Fragments, and, being possess'd of the same Virtue, will, when left to move freely, have the same Disposition, and Similarity of Position, and other Circumstances.

5. The most considerable of these Hypotheses is that of the late sagacious Dr. *Halley*, which is this; *The Globe of the Earth is one great Magnet, having four Magnetical Poles or Points of Attraction, near each Pole of the Equator two; and that in those Parts of the World which*
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lie near adjacent to any one of these Magnetical Poles, the Needle is chiefly govern'd thereby, the nearest Pole being always predominant over the more remote one. Of the North Poles, that which is nearest to us, he supposes to be in the Meridian of the Land's End, and about seven Degrees from the North Pole of the World, which governs the Variations in *Europe*, *Tartary*, and the *North Sea*; the other he places in a Meridian passing thro' *California*, about fifteen Degrees from the North Pole of the World, which governs the Needle in *North America*, and the Oceans on either Side. In like Manner he accounts for the Variations in the Southern Hemisphere. See *Philos. Transf.* N^o 148.

6. The Variation of the Needle from the North and South Points of the Horizon not being the same, but variable in different Years and in a diverse Manner in different Parts of the Earth, made the *Doctor* farther conjecture, that two of the Magnetic Poles were fix'd, and two moveable; and in order to make this out, he supposes the external Part of the Earth to be a Shell or *Cortex*, containing within it a Magnetic moveable *Nucleus*, of a globular Form, whose Centre of Gravity is the same with that of the Earth, and moveable about the same Axis.

7. Now if the Motions of both the Shell and the *Nucleus* were the same, the Poles of each would always have the same Position to each other; but he supposes the Motion of the *Nucleus* to be a very small Matter less than that of the Shell, which yet is scarce sensible in 365 Revolutions; and if so, the Magnetic Poles of the *Nucleus* will by slow Degrees change their Distance from the Magnetic Poles of the Shell, and thus cause a Variation in that Needle's Variation, which is govern'd by the moveable Pole of the *Nucleus*; while that Variation which respects the fix'd Poles of the Magnetic Shell remains more constant; as in *Hudson's Bay* the Change is not observed to be near so fast as in these Parts of *Europe*. And it appears from some late Accounts, that in some of the northernmost Parts of the Bay the Needle loses its Magnetism, and which has been recovered again by laying the Needle by a Fire, and keeping it in a warm Room.

8. What

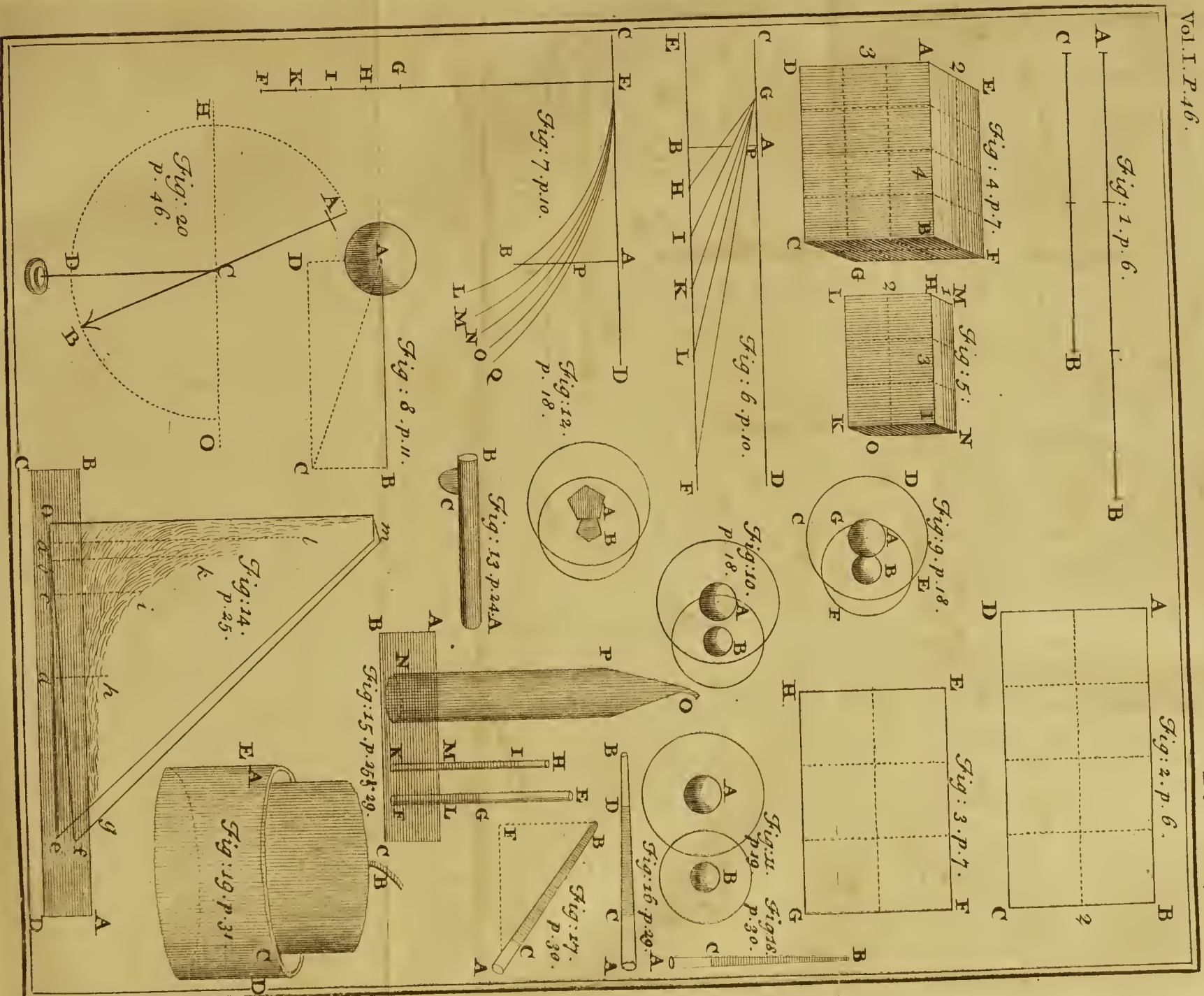
Plate I.
Fig. 20.

8. What seems a little strange is, that the *Doctor* has no where, that I know of, undertook to account for the Dipping of the Needle by this Hypothesis; tho' the Invention of this (by Mr. *Blagrove*) was before that of the Change of the *Needle's Variation* (by Mr. *Gellibrand*); nor do I see at present which way this *Phænomenon* is explicable by it. But we have not yet so many accurate observations of the *Needle's Inclination* as we have of its *Variation*, which is its only useful Property. By several Experiments Mr. *Graham* very accurately made it appear, that the Quantity of the Needle's Inclination to the horizontal Line was an Angle of about 74 or 75 Degrees: That is, suppose *AB* a touch'd Needle, supported on the Point *C* of the Pin *CD*, it will contain an Angle *ACH* or *BCO* with the horizontal Line *HO* of 74 or 75 Degrees.

Plate II.
Fig. 20.

9. The Variation of the Needle has within a Century past undergone a remarkable Alteration; for at *London* it was observed by Mr. *Burrows* in the Year 1580 to be $11^{\circ} 15'$. East; that is, if *N*, *S* represent the North and South Points of the Horizon, and *E*, *W*, the East and West Points, the Needle then had the Situation *AB*, so that the Arch *NB* = $11^{\circ} 15'$. After that, in the Year 1622, it was observed by Mr. *Gunter* to be but 6° East. In the Year 1634 Mr. *Gellibrand* observed it to be $4^{\circ} 5'$. East. In 1657, it was observed by Mr. *Bond* to be nothing at all, that is, the Needle placed itself then in the Situation *SN*, and pointed directly to the North. After this, in the Year 1672, Dr. *Hally* observed it to be $2^{\circ} 30'$. Westward; and again in the Year 1692, he found it 6° . West. Since then, in the Year 1722, Mr. *Graham*, by most accurate Experiments, found it to be $14^{\circ} 13'$. And at present it is about $17\frac{1}{2}$ Degrees; and in some Places I have found it 18 Degrees Westward, as in *Mount's Bay* by *Pensance*.

10. The Variation of the Declination and Inclination of the Needle is variable and subject to no regular Computation. What the Quantity of both Sorts of Variation is in the several Parts of the World, will be shewn in Dr. *Hally's* Map of the World, improved from the
Obser-





Of the Attraction of MAGNETISM.

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Observations of Mr. Pound, which is inserted at the End of the LECTURE ON WIND and SOUND.

11. I think the Law of Magnetic Attraction is not yet ascertain'd; Sir Isaac Newton supposes it to decrease nearly in the *Triplicate Ratio* of the Distance: But Dr. Hefham, trying the Experiment with his Loadstone, found it to be as the *Squares of the Distances* inversely. And the Power of my Loadstone decreases in a different Manner from either, it being in the *Sesquiplicate Ratio* of the Distances inversely. For Exactness, I made a square Bar of Iron just $\frac{1}{4}$ of an Inch thick, and then provided three Pieces of Wood of the same Form and Thickness precisely. Then poising the Loadstone very nicely at the End of a Balance which would turn with less than a Grain, I placed under it the Iron with first one Piece of Wood, then two Pieces, and lastly all three Pieces upon it, by which means the Steel Points of the Pole were kept at $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ of an Inch from the Iron, and in those Distances the Weights put into the opposite Scale to raise the Loadstone from the Wood (which it touch'd while the Beam was horizontal) were as follows:

		<i>Ratio of</i>		<i>Ratio of Sesquip.</i>	
		<i>Grains.</i>	<i>the Squares.</i>	<i>the Cubes.</i>	<i>Ratio.</i>
Dis- tances	$\frac{1}{4}$	— 156	— 156	— 156	— 156
	$\frac{1}{2}$	— 58	— 39	— 19	— 56
	$\frac{3}{4}$	— 28	— 17	— 6	— 30

12. Whence it appears, that the Numbers of Grains to counter-act the Power of the Loadstone in these Distances approach very near, and are almost the same with those which are in the *Sesquiplicate Ratio*, but are widely different from those which are in the *Duplicate* or *Triplicate Ratio*; and this Experiment I tried several times for each Distance, and with scarce any Variation in the Success.

13. The Power or Force of Magnets is generally greater in small than in large ones, in Proportion to their Bulk. It is very rare that very large ones will take up more than 3 or 4 times their own Weight, but a small one is but tolerably good that will take up no more than 8, 10, or 12 times its Weight. The Honourable Mr. Berkley, at Bruton, has one whose Weight

Weight (with the Armature) is but 43 Grains, which will take up 1032 Grains, which is 24 times its Weight. But that of Mr. *Newton* (which he wears in his Ring, instead of a Diamond of less Value) which weighs scarce THREE Grains, will take up 746 Grains, which is 250 times its Weight, which is by far the strongest and best of any I have seen; and therefore, as a great Curiosity, I have given a Print of the RING and STONE with the *Iron* hanging to it.

Fig. 3.

14. Iron Bars by standing long in the same Position will acquire the Magnetic Virtue to a surprizing Degree. I have seen one about 10 Feet long and 3 Inches thick, (supporting the *Summer Beam* of a Room) which has been able to turn the Needle at 8 or 10 Feet distance, and exceeded a Loadstone of $3\frac{1}{2}$ Pounds Weight. From the Middle Point upwards it was a North Pole, and downwards a South Pole; and I have by me an Iron Rod which has several Poles or Parts which change the Position of the Magnetic Needle: It is the Beam of a large Steelyard, which let A E represent; then at A, and from thence to B, the North End of the Needle is attracted; at B, and from thence to C, the North End is repell'd and the South End attracted; from C to D, the North End is again attracted and the South End repell'd, and from D to E the South End is attracted, and the North End repell'd; lastly, at the End E begins another Pole, and there the Needle stands nearly parallel with the Beam, with neither End attracted.

Fig. 7.

15. It will be proper here to acquaint the Reader with the several Methods of communicating the Magnetic Virtue to Iron, in so great a Degree, that such a Piece of Iron or Steel is deservedly call'd an *Artificial Magnet* or *Loadstone*; and as it possesses all the Properties, so it is used to all the Purposes of the Natural Loadstone.

16. The first Way of making an Artificial Magnet is by preparing several Steel *Lamineæ*, or thin Plates, about 6 or 8 Inches long, and half an Inch broad; these well polished, and properly touch'd on a very strong Loadstone, at each End, and then put together and arm'd with Steel Points like the common Stone, will become an Artificial Magnet.

17. A

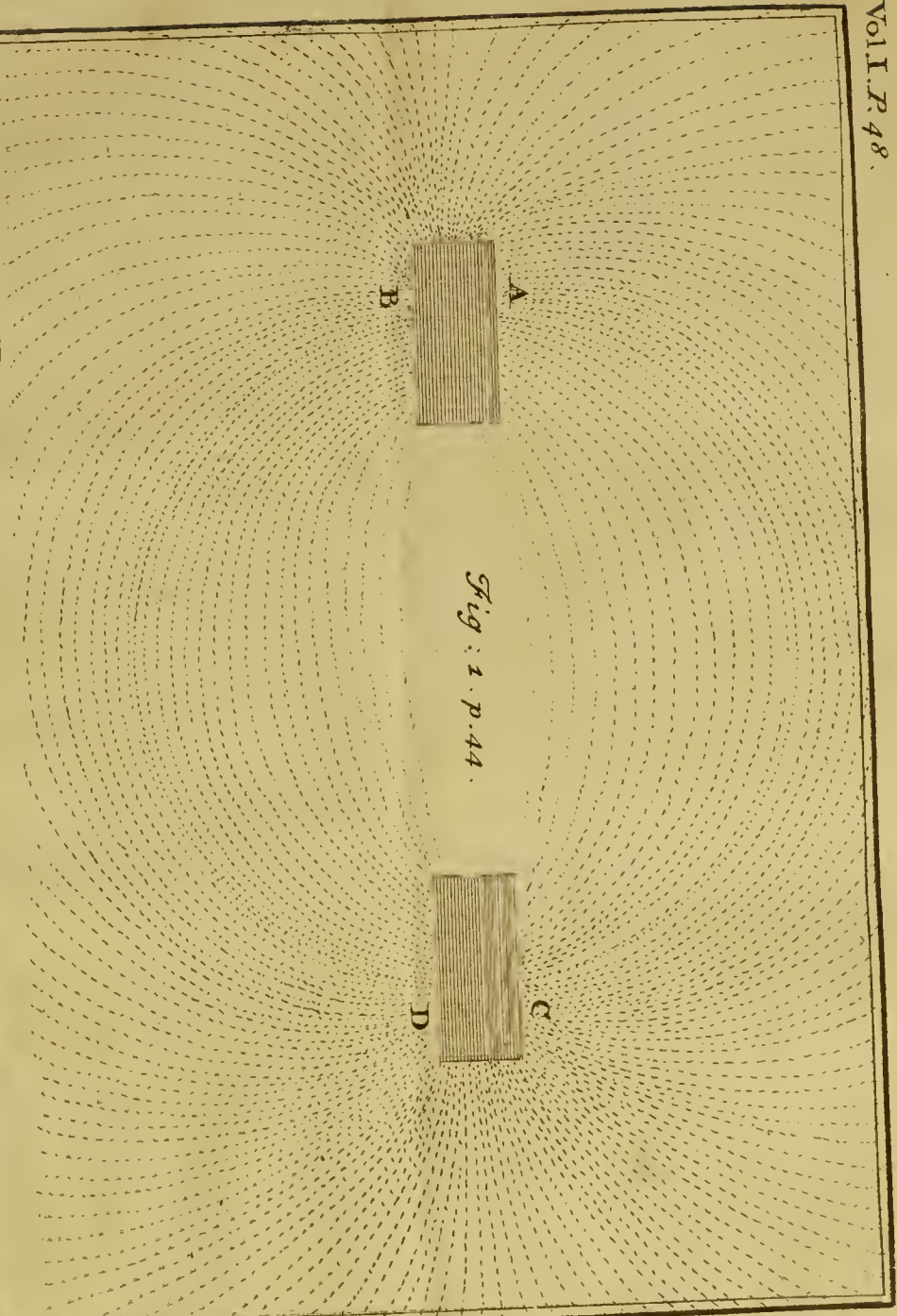


Fig. 1. p. 44.

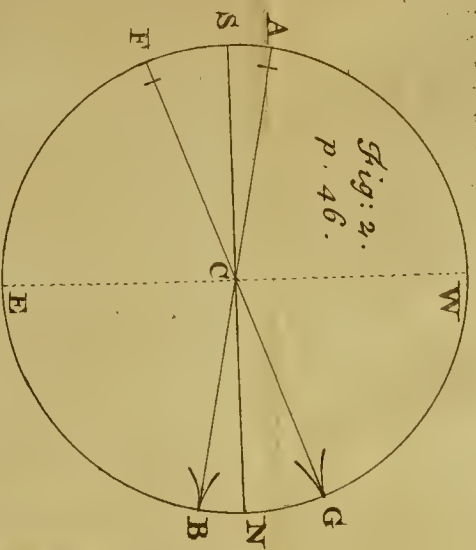


Fig. 2.
p. 46.

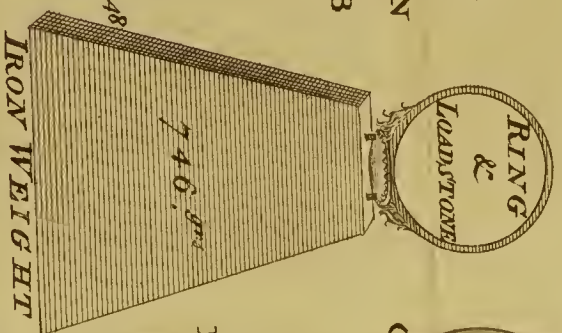


Fig. 3. p. 48

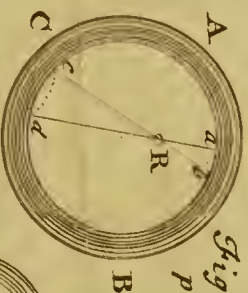


Fig. 4.
p. 86.



Fig. 5. p. 87.

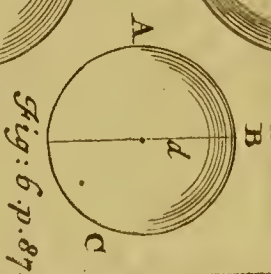


Fig. 6. p. 87

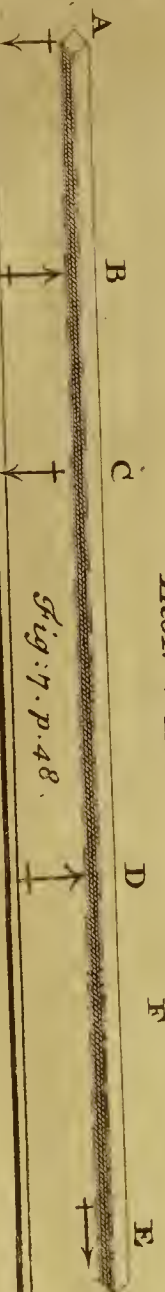
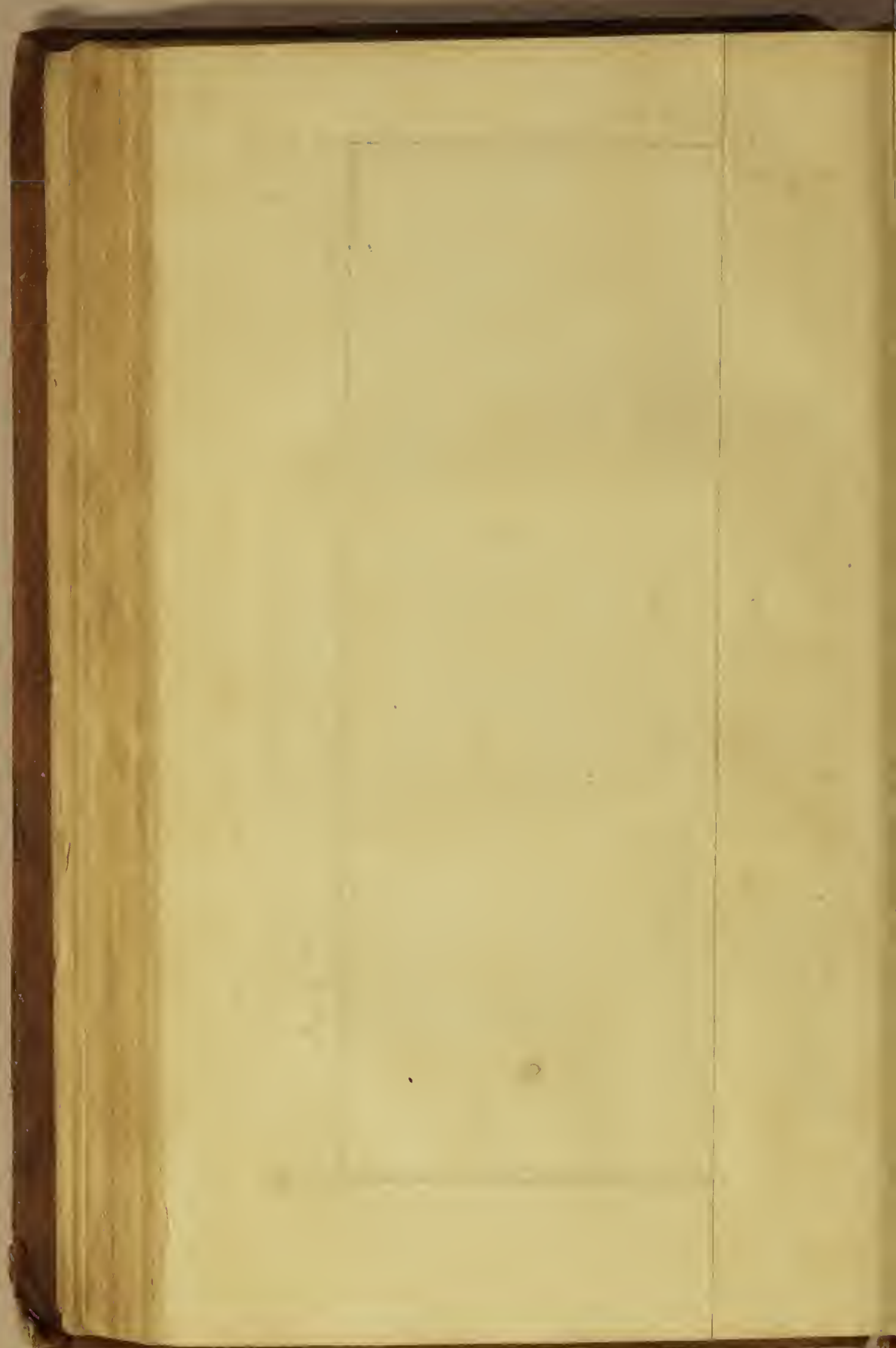


Fig. 7. p. 48.



17. A second Way is by preparing solid Pieces of Steel, and having wrought them to a proper Figure, polish'd, and arm'd them with Steel Armour; if then one of them be properly held in the Magnetic Meridian Line, and the other which is arm'd be drawn several Times along it, it will conceive a great Degree of Magnetism; if then the Armour be removed from this and placed on the other, and that in like Manner touch'd on this, it will become Magnetical. But to describe the particular Manner, with all its Circumstances, would be too tedious; I shall therefore refer the Reader to the Account given by the reputed Inventor, Mr. *Savery* *, who was the first that publish'd any considerable Discoveries relative to *Artificial Magnetism*; but I have heard Mr. *Lovelace Senr.* affirm, that he was before that Time possessed of the Secret, and communicated it to Mr. *Savery*.

18. A third Way is to provide a Piece of Steel, and continue rubbing it *very hard all one Way* with a polish'd Steel Instrument, which will by this Sort of Friction conceive so great a Degree of Virtue, as to become an *Artificial Magnet*, of greater Force, very often, than the natural ones. A small piece of Wire thus order'd will be as tender, and point as nicely Northwards, as any one touch'd by the Natural Magnet.

19. A fourth Way is to heat a Piece of Steel red hot, and then quench it in Water, being held therein in such a Position as the Dipping-Needle takes when suspended on a sharp Point. This Method will succeed only in the Hand of those who by frequent Trials have acquired a peculiar Sagacity this Way; and indeed the same may be said of all the others. 'Tis Experience only that gives a Person the Ingenuity necessary for succeeding in all such Affairs.

20. A fifth Way is by setting a Bar of Iron or Steel in a proper Place, and there letting it stand for a long Time in a Position unalter'd, and it will acquire a very great Magnetic Virtue, as has been already taken Notice of; a perpendicular Position is best, and the larger the Bar, and the longer it stands, the better or stronger it will be.

* *Philos. Transf.* N^o 414.

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21. A sixth Method of giving this Virtue to Iron in a surprising Degree, and also to the Loadstone itself, yea of altering or reversing its Poles, and dispossessing it of its Virtue, almost in any Manner, was long a Secret in the Hands of a few, but is now well known; and for the Reader's practical Entertainment, as well as Instruction, we shall give a Plate in the following Appendix, representing a particular Method of communicating this Virtue in the greatest Degree, as invented by a Gentleman well known for a peculiar *Enchiræsis* and great Success in *Magnetical and Electrical Experiments*.

N. B. Since the following APPENDIX ON ELECTRICITY was printed off, I have been apprised of a *New Discovery* highly conducing to the Illustration of that hitherto almost unintelligible Subject. A Stone (said to be brought from the Isle of *Ceylon*, and is called *Tour Malin*) has lately been found to be possessed of the Electrical Property in a very singular and surprising Manner. It is the *first Stone*, or hard and unelastic Body, I believe, that was ever experienced to be electrical. It is the *first Subject* that has been found to have the electric Virtue excited *without Friction*; for *Heat* or Warmth alone is sufficient to render it very sensibly electrical. It is the *first Substance* found to possess a twofold Species of Electricity; *Glass* has only one Sort, and Amber only one other, contrary to the former; but this wonderful Stone possesses *both Sorts*, the Vitreous on one Side, and the Resinous on the other; in short, all Kinds of electrical Experiments are shewn by *it alone*; which can be said of no other Body. It is supposed this Discovery will tend greatly to illustrate, if not wholly confirm, the new Doctrine of *Electrical Thunder and Lightning*.

A P P E N D I X

To L E C T U R E I.

P A R T I.

*Concerning the Improvements and new
Experiments in ELECTRICITY.*

THE Improvements in Electricity have been so great and numerous, that it would be almost in vain to attempt even the epitomizing of them all. We must therefore refer our inquisitive Readers to the several Authors who have wrote on this Subject; but still we think it necessary in a Treatise of this Kind, that there should be a larger Account of this wonderful Power than what was given in the first Edition of this Work; and therefore we have added by Way of Appendix, two Papers of Experiments by Mr. CANTON, as we find them in the Transactions of the Royal Society. The first of which is contained in N° LIII. of Vol. XLVIII. Part I. The Title of which is,

Electrical Experiments, with an Attempt to account for their several Phænomena; together with some Observations on Thunder-Clouds, by John Canton, M. A. and F. R. S.

Experiment 1.

FROM the Cieling, or any convenient Part of a Room, let two Cord-balls, each about the Bigness of a small Pea, be suspended by Linen Threads of eight or nine Inches in Length, so as to be in contact with each other. Bring the excited glass Tube under the Balls, and they will be separated by it, when held at the Distance of three or four Feet; let it be brought nearer, and they will stand farther apart; intirely withdraw it, and they will immediately come together. This Experiment may be made with very small Brass Balls hung by Silver Wire; and will succeed as well with Sealing-wax made electrical, as with Glass.

Experiment 2.

IF two Cork-Balls be suspended by dry Silk Threads, the excited Tube must be brought within eighteen Inches before they will

will repel each other; which they will continue to do, for some time, after the Tube is taken away.

As the Balls in the first Experiment are not insulated, they cannot properly be said to be electrified: But when they hang within the Atmosphere of the excited Tube, they may attract and condense the electrical Fluid round about them, and be separated by the Repulsion of its Particles. It is conjectured also, that the Balls at this time contain less than their common Share of the electrical Fluid, on Account of the repelling Power of that which surrounds them; though some, perhaps, is continually entering and passing through the Threads. And if that be the Case, the Reason is plain, why the Balls hung by Silk, in the second Experiment, must be in a much more dense Part of the Atmosphere of the Tube, before they will repel each other. At the Approach of an excited Stick of Wax to the Balls, in the first Experiment, the electrical Matter is supposed to come through the Threads into the Balls, and be condensed there, in its Passage towards the Wax: for, according to Mr. *Franklin*, excited glass *emits* the electrical fluid, but excited Wax *receives* it.

Experiment 3.

LET a Tin Tube, of four or five Feet in Length, and about two Inches in Diameter, be insulated by Silk; and from one End of it let the Cork-balls be suspended by Linen Threads. Electrify it, by bringing the excited Glass Tube near the other End, so as that the Balls may stand an Inch and a half, or two Inches apart: Then, at the Approach of the excited Tube, they will by Degrees lose their repelling Power, and come into Contact; and as the Tube is brought still nearer, they will separate again to as great a Distance as before: In the Return of the Tube they will approach each other till they touch, and then repel as at first. If the Tin Tube be electrified by Wax, or the Wire of a charged Phial, the Balls will be affected in the same Manner at the Approach of excited Wax, or the Wire of the Phial.

Experiment 4.

ELECTRIFY the Cork-balls as in the last Experiment by Glass; and at the Approach of an excited Stick of Wax their Repulsion will be increased. The Effect will be the same,

same, if the excited Glafs be brought towards them, when they have been electrified by Wax.

THE bringing the excited Glafs to the End, or Edge of the Tin-Tube, in the third Experiment, is supposed to electrify it positively, or to add to the electrical Matter it before contained; and therefore some will be running off through the Balls, and they will repel each other. But at the Approach of excited Glafs, which likewise *emits* the electrical Fluid, the Discharge of it from the Balls will be diminished; or Part will be driven back, by a Force acting in a contrary Direction; and they will come nearer together. If the Tube be held at such a Distance from the Balls, that the Excess of the Density of the Fluid round about them, above the common Quantity in Air, be equal to the Excess of the Density of that within them, above the common Quantity contained in Cork; their Repulsion will be quite destroyed. But if the Tube be brought nearer; the Fluid without, being more dense than that within the Balls, it will be attracted by them, and they will recede from each other again.

WHEN the Apparatus has lost Part of its natural Share of this Fluid, by the Approach of excited Wax to one End of it, or is electrified negatively; the electrical Fire is attracted and imbibed by the Balls to supply the Deficiency; and that more plentifully at the Approach of excited Glass, or a Body positively electrified, than before; whence the Distance between the Balls will be increased, as the Fluid surrounding them is augmented. And in general, whether by the Approach or Recess of any Body; if the Difference between the Density of the internal and external Fluid be increased or diminished; the Repulsion of the Balls will be increased, or diminished accordingly.

Experiment 5.

WHEN the insulated Tin Tube is not electrified, bring the excited Glass Tube towards the Middle of it, so as to be nearly at Right Angles with it, and the Balls at the End will repel each other; and the more so, as the excited Tube is brought nearer. When it has been held a few Seconds, at the Distance of about six Inches, withdraw it, and the Balls will Approach each

each other till they touch; and then separating again, as the Tube is moved farther off, will continue to repel when it is taken quite away. And this Repulsion between the Balls will be increased by the Approach of excited Glass, but diminished by excited Wax; just as if the Apparatus had been electrified by Wax, after the Manner described in the third Experiment.

Experiment 6.

INSULATE two Tin Tubes, distinguished by *A* and *B*, so as to be in a Line with each other, and about half an Inch apart; and at the remote End of each, let a Pair of Cork Balls be suspended. Towards the Middle of *A*, bring the excited Glass Tube; and holding it a short time at the Distance of a few Inches, each Pair of Balls will be observed to separate: Withdraw the Tube, and the Balls of *A* will come together, and then repel each other again; but those of *B* will hardly be affected. By the Approach of the excited Glass Tube, held under the Balls of *A*, their Repulsion will be increased: But if the Tube be brought, in the same Manner, towards the Balls of *B*, their Repulsion will be diminished.

IN

IN the fifth Experiment, the common Stock of electrical Matter in the Tin Tube, is supposed to be attenuated about the Middle, and to be condensed at the Ends, by the repelling Power of the Atmosphere of the excited Glass Tube, when held near it. And perhaps the Tin Tube may lose some of its natural Quantity of the electrical Fluid, before it receives any from the Glass; as that Fluid will more readily run off from the Ends or Edges of it, than enter at the Middle: And accordingly, when the Glass Tube is withdrawn, and the Fluid is again equally diffused through the Apparatus, it is found to be electrified negatively: For excited Glass brought under the Balls will increase their Repulsion.

IN the sixth Experiment Part of the Fluid driven out of one Tin Tube enters the other; which is found to be electrified positively, by the decreasing of the Repulsion of its Balls, at the Approach of excited Glass.

Experiment 7.

LET the Tin Tube, with a Pair of Balls at one End, be placed three Feet at least from any Part of the Room, and the Air rendered very dry by Means of a Fire: Electrify

Electrify the Apparatus to a considerable Degree; then touch the Tin Tube with a Finger, or any other Conductor, and the Balls will, notwithstanding, continue to repel each other; though not at so great a Distance as before.

THE Air surrounding the Apparatus to the Distance of two or three Feet, is supposed to contain more or less of the electrical Fire than its common Share, as the Tin Tube is electrified positively, or negatively; and when very dry, may not part with its Overplus, or have its Deficiency supplied so suddenly, as the Tin; but may continue to be electrified, after that has been touched for a considerable time.

Experiment 8.

HAVING made the Torricellian Vacuum about five Feet long, after the Manner described in the *Philosophical Transactions*, Vol. xlvii. p. 370. if the excited Tube be brought within a small Distance of it, a Light will be seen through more than half its Length: Which soon vanishes, if the Tube be not brought nearer; but will appear again, as that is moved farther off. This may be repeated several times, without exciting the Tube afresh.

THIS

THIS Experiment may be considered as a Kind of ocular Demonstration of the Truth of Mr. *Franklin's* Hypothesis; that when the electrical Fluid is condensed on one Side of thin Glafs, it will be repelled from the other, if it meets with no Resistance. According to which, at the Approach of the excited Tube, the Fire is supposed to be repelled from the Inside of the Glafs surrounding the Vacuum, and to be carried off through the Columns of Mercury; but, as the Tube is withdrawn, the Fire is supposed to return.

Experiment 9.

LET an' excited Stick of Wax, of two Feet and an half in Length, and about an Inch in Diameter, be held near its Middle. Excite the Glafs Tube, and draw it over one half of it; then, turning it a little about its Axis, let the Tube be excited again, and drawn over the same half; and let this Operation be repeated several times: Then will that half destroy the repelling Power of Balls electrified by Glafs, and the other half will increase it.

By this Experiment it appears, that Wax also may be electrified positively and negatively. And it is probable, that all Bodies
whatsoever

whatsoever may have the Quantity they contain of the electrical Fluid, increased, or diminished. The Clouds, I have observed, by a great Number of Experiments, to be some in a positive, and others in a negative State of Electricity. For the Cork Balls, electrified by them, will sometimes close at the Approach of excited Glass; and at other Times be separated to a greater Distance. And this Change I have known to happen five or six Times in less than half an Hour; the Balls coming together each Time, and remaining in Contact a few Seconds, before they repel each other again. It may likewise easily be discovered, by a charged Phial, whether the electrical Fire be drawn out of the Apparatus by a negative Cloud, or forced into it by a positive one: And by whichever it be electrified, should that Cloud either part with its Overplus, or have its Deficiency supplied suddenly, the Apparatus will lose its Electricity: Which is frequently observed to be the Case, immediately after a Flash of Lightning. Yet when the Air is very dry, the Apparatus will continue to be electrified for ten Minutes, or a Quarter of an Hour, after the Clouds have passed the Zenith; and sometimes

times till they appear more than Half-way towards the Horizon. Rain, especially when the Drops are large, generally brings down the electrical Fire: And Hail, in Summer, I believe never fails. When the Apparatus was last electrified, it was by the Fall of thawing Snow; which happened so lately, as on the 12th of November; that being the twenty-sixth Day, and sixty-first Time, it has been electrified, since it was first set up; which was about the Middle of May. And as *Fahrenheit's* Thermometer was but seven Degrees above freezing, it is supposed the Winter will not entirely put a Stop to Observations of this Sort. At *London*, no more than two Thunder-Storms have happened during the whole Summer: And the Apparatus was sometimes so strongly electrified in one of them, that the Bells, which have been frequently rung by the Clouds, so loud as to be heard in every Room of the House (the Doors being open), were silenced by the almost constant Stream of dense electrical Fire, between each Bell and the Brass Ball, which would not suffer it to strike.

I SHALL conclude this Paper, already too long, with the following Queries:

I. MAY

1. MAY not Air, suddenly rarefied, give electrical Fire to, and Air suddenly condensed, receive electrical Fire from, Clouds and Vapours passing through it?

2. Is not the *Aurora Borealis*, the Flashing of electrical Fire from positive, towards negative Clouds at a great Distance, thro' the upper Part of the Atmosphere, where the Resistance is least?

As the ingenious Author of the foregoing Paper has given no Plate to illustrate those Experiments with Figures; and as he has since contrived to shew them in a more neat and elegant Manner by Means of *Drinking-Glasses*, we shall therefore represent the Experiments this Way, as it may be easily practised by any private Person for his own Amusement. Therefore, in *Fig. 1.* let A B and C D represent two Plate I. to this Appendix. Pieces of Wood about six or eight Inches long, half an Inch wide, and $\frac{1}{10}$ of an Inch thick; and at the End of each, let two fine Flaxen Threads be fixed, of about six Inches in Length, with two round Balls (about the Size of a white Pea, made of the Pith of Elder) be suspended; then if the excited Tube be brought under the Balls at B, they will separate, and repel each other, while the

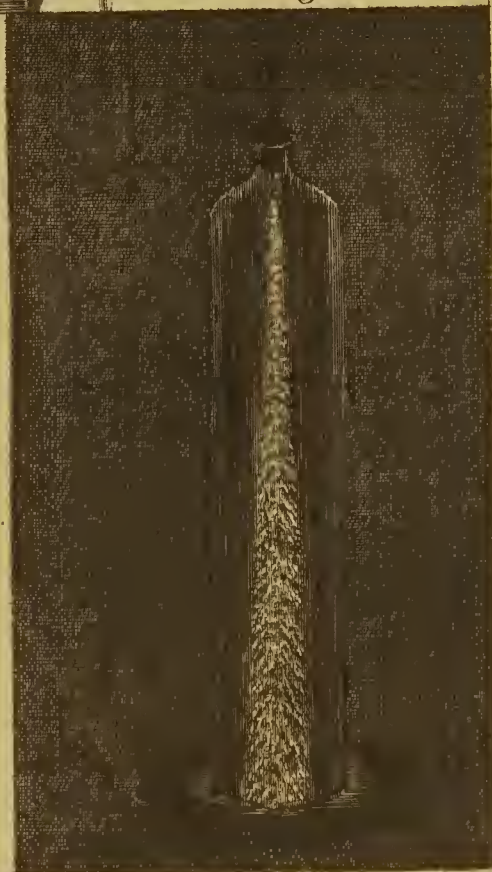
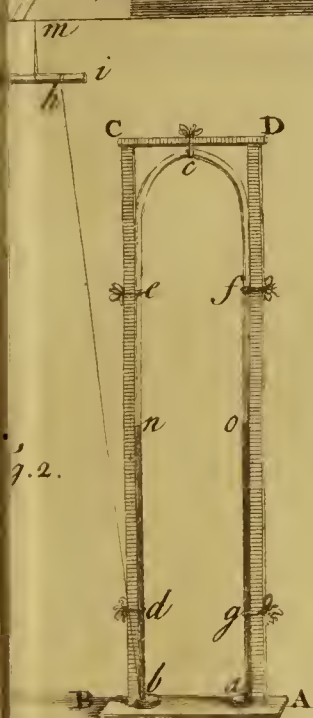
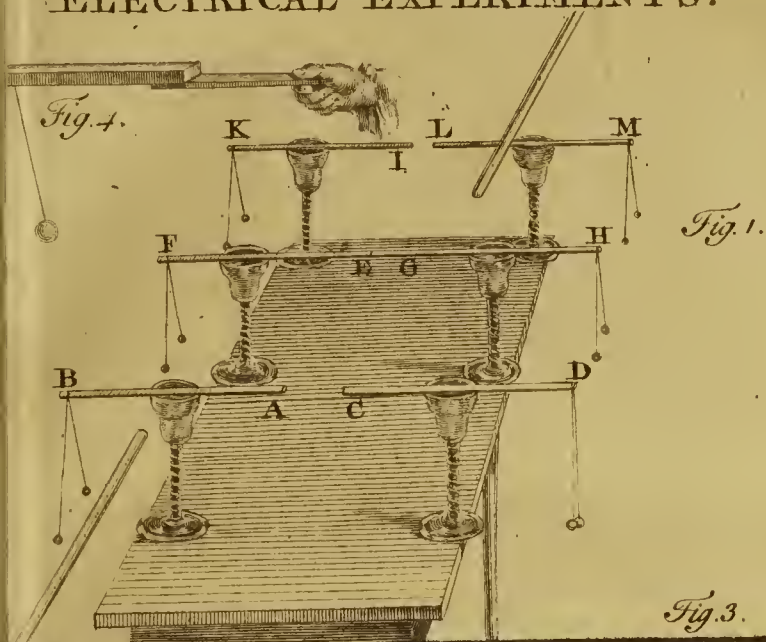
the Balls hanging from CD remain in Contact, placed at a Distance AC from the other; but if, while the Balls at B are electrified, the Glass be moved so that the Piece of Wood AB come in Contact with the Piece CD, then the Balls at B will lose half the electric Fluid, which will be communicated to the Balls at D, and they will now repel each other; and the Distance between the Balls at B and D will be but half what it was at first between the two Balls at B. This Case is represented by the two Pieces of Wood at EF and GH.

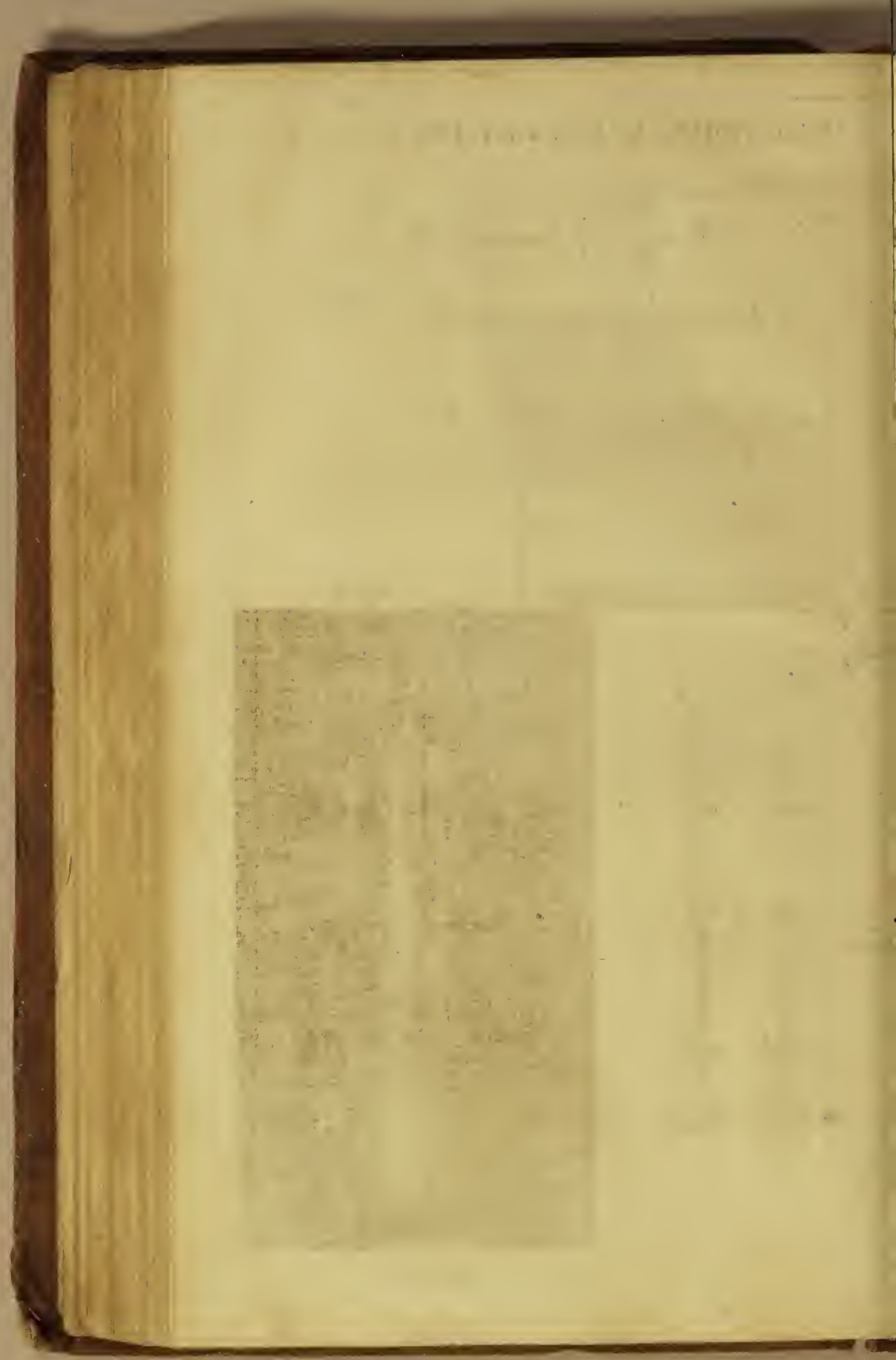
THE Pieces of Wood at IK and LM represent the Case of the sixth Experiment, where the Balls of LM are electrified negatively, and those of IK positively.

THIS Method of supporting the Pieces of Wood with the Balls on Glasses, may be applied to a great Variety of curious Experiments, with regard to positive and negative Electrification, under all the different Circumstances which have been mentioned in this Paper, and in those that follow in the next by the same Author.

THE Torricellian *Vacuum*, mentioned in the eighth Experiment, is here represented in *Figure the 2d*, where *ac b* represents

ELECTRICAL EXPERIMENTS.





sents a hollow incurved Tube of Glass, whose Height *ac* is about three Feet. This Tube is first filled with pure Mercury, and then the Orifice of each Leg is inverted in two small Basons of Mercury *ab*. The Mercury in the Tube will subside from the upper Part *c* to the equal Heights *no* in each Leg; then will there be a *Vacuum*, as in the common Barometer, in all the upper Part of the Tube. This Tube is then fixed in the Frame *A B C D*, either hapsed or tied on with Strings, as represented at *d g e f c*, which Frame is fastened to the Side of the Room. If now, from the prime Conductor *i k*, a Wire or Flaxen String *b* carry the electric Matter to the Bason *b*, it will run through the Quick-Silver into the vacuous Part of the Tube, and produce the surprizing Appearance of a Torrent of electrical Fire in the darkened Room, which will continue as long as the electrical Machine is in Motion.

THE 3d *Figure* represents a tall Glass Receiver, exhausted by the Air-Pump; and a pointed Wire passing through a Cork in the Top conducts the electrical Fluid from the Machine to this *Vacuum*. The Machine being put in Motion, and the Room darkened, the Spectator will observe the

Electricity descending from the Point of the Iron, in the Appearance of a liquid Stream of Fire, of a pale whitish hue, like *Phosphorus*, and of a thick Consistence, like Cream; which plainly shew that the electrical Matter, as it is produced from common Bodies, is of a very different Nature from the Matter of common Light or Fire, as it hath a very different Colour, Smell, and Consistence; and that it is not the pure Matter of Light, or indeed any Thing similar to it, is evident from hence, that it has no different Refrangibility in its Rays, which is the well known Property of Light; and therefore upon the Whole we may conclude, that if Lightning, the *Aurora Borealis*, &c. be the Effects of Electricity, the Matter is still supplied to the Clouds and the upper Regions of the Air from the Earth below, in the manner as Sir *Isaac Newton* has taught in his Book of Optics, and which we have already taken Notice of in the first Part of this Lecture.

ANOTHER of Mr. Canton's Papers is contained in Number XCIII. Part II. of the same Vol. under the following Title:

*A Letter to the Right Honourable the Earl of
Macclesfield, President of the Royal Society,
concerning some new electrical Experiments,
by John Canton, M. A. and F. R. S.*

My Lord,

AS Electricity, since the Discovery of it in the Clouds and Atmosphere, is become an interesting Subject to Mankind; your Lordship will not be displeased with any new Experiments or Observations that lead to a farther Acquaintance with its Nature and Properties.

THE resinous and vitreous Electricity of Mr. Du Fay, which arose from his observing Bodies of the one Class to attract, what those of the other would repel, when each were excited by Attrition, received no Light till the Publication of the second Part of Mr. *Franklin's* Experiments; wherein it appears, that the one Kind of Bodies electrify positively, and the other negatively; that excited Glass throws out the electric Fire, and excited Sulphur drinks it in. But no Reason has yet been assigned, why vitreous Bodies should receive, and resinous Bodies part with this Fire, by rubbing them. Some Persons indeed, of considerable

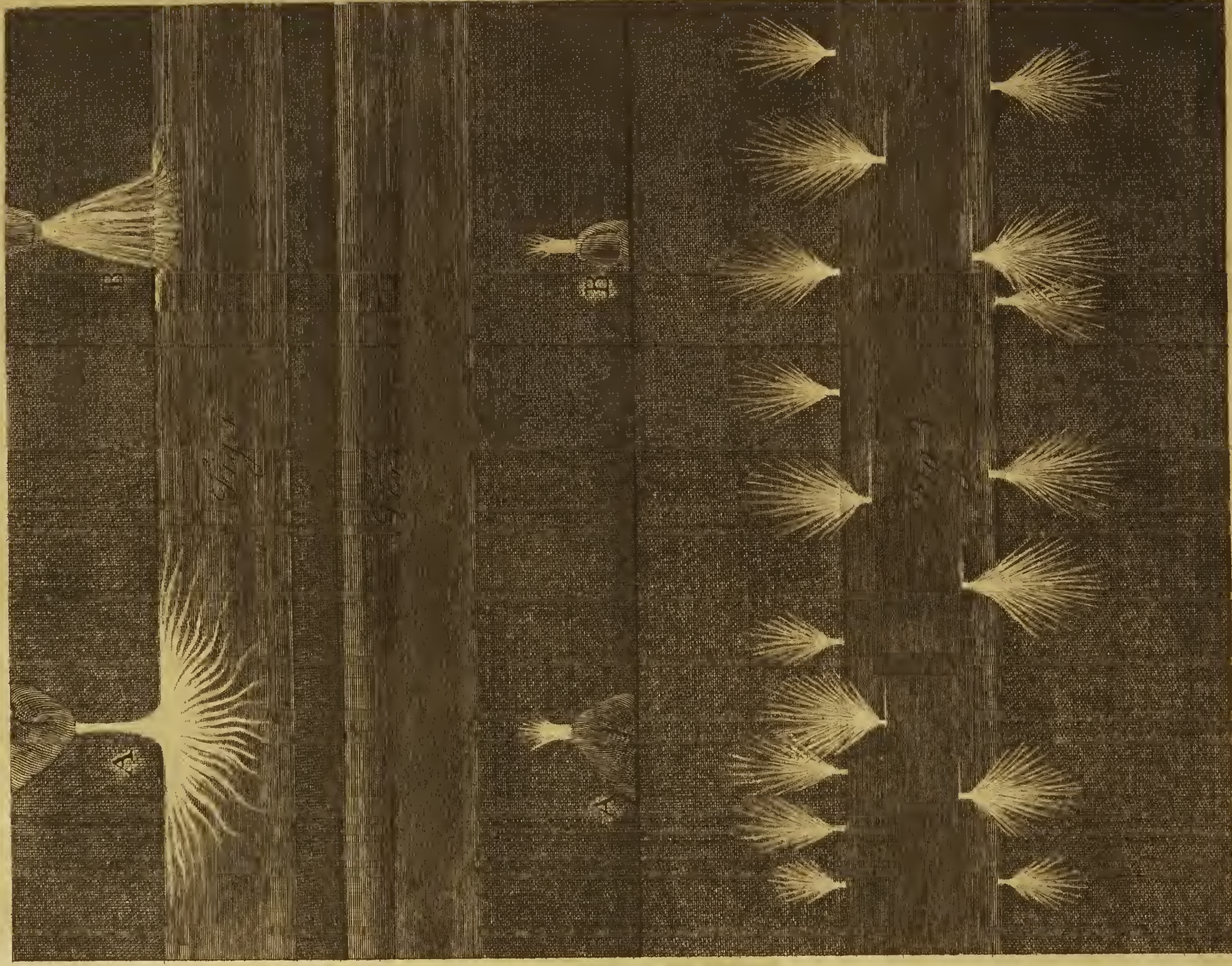
rable Knowledge in these Matters, have supposed the Expansion of Glass, when heated by Friction, to be the Cause of its receiving more of the electric Fluid than its natural Share; but this Supposition cannot be made with regard to Bodies of the other Sort, such as Sulphur, Sealing-Wax, &c. which part with it when treated in the same Manner. The following Experiments, first made at the latter End of *December* 1753, and often repeated since, may perhaps cast new Light on this difficult Subject.

HAVING rubbed a Glass Tube with a Piece of thin Sheet-lead and Flower of Emery mixt with Water, till its Transparency was entirely destroyed; after making it perfectly clean and dry, I excited it with new Flannel, and found it act in all Respects like excited Sulphur or Sealing-Wax. The electric Fire seems to issue from the Knuckle, or End of the Finger, and to spread itself on the Surface of this Tube, in the beautiful Manner represented at *A*

Plate II. and *B* in *Fig. 1.*

If this rough or unpolished Tube be excited by a Piece of dry oiled Silk (especially when rubbed over with a little Chalk or Whiting), it will act like a Glass Tube
with

ELECTRICAL EXPERIMENTS *in a Darkend Room*



with its natural Polish. And in this Case, the Fire appears only at the Knuckle, or End of the Finger; where it is very much condensed before it enters; as at *A* and *B* ^{Plate II.} in *Fig. 2.*

BUT if the rough Tube be greased all over with Tallow from a Candle, and as much as possible of it wiped off with a Napkin, then the oiled Silk will receive a Kind of Polish by rubbing it, and after a few Strokes, will make the Tube act in the same Manner as when excited at first by Flannel.

THE oiled Silk, when covered with Chalk or Whiting, will make the greased rough Tube act again like a polished one: But if the Friction be continued till the Rubber is become very smooth, the electric Power will be changed to that of Sulphur, Sealing-Wax, &c.

THUS may the positive and negative Powers of Electricity be produced at Pleasure, by altering the Surfaces of the Tube and Rubber; according as the one or other is most affected by the Friction between them: For if the Polish be taken off one Half of a Tube, the different Powers may be excited with the same Rubber at a single Stroke. And the Rubber is

found to move much easier over the rough, than over the polished Part of it.

THAT polished Glass electrifies positively, and rough Glass rubbed with Flannel negatively, seems plain, from the Appearance of the Light between the Knuckle, or End of the Finger, and the respective Tubes; but yet may be farther confirmed by observing, that a polished Glass Tube, when excited by smooth oiled Silk, if the Hand be kept at least three Inches from the Top of the Rubber, will at every Stroke appear to throw out a great Number of diverging Pencils of electric Fire, as

Plate II. in *Fig. 3*; but not one was ever seen to accompany the rubbing of Sulphur, Sealing-Wax, &c. nor was I ever able to make any sensible Alteration in the Air of a Room, merely by the Friction of those Bodies: Whereas the Glass Tube, when excited so as to emit Pencils, will, in a few Minutes, electrify the Air to such a Degree, that (after the Tube is carried away) a Pair of Balls, about the Size of the smallest Peas, turned out of Cork, or the Pith of Elder, and hung to a Wire by Linen Threads of six Inches long, will repel each other to the Distance of an Inch and an Half, when held at Arm's-length in the Middle

Middle of the Room. But their Repulsion will decrease as they are moved toward the Floor, Wainscot, or any of the Furniture; and they will touch each other when brought within a small Distance of any Conductor. Some Degree of this electric Power I have known to continue in the Air above an Hour after the Rubbing of the Tube, when the Weather has been very dry.

THE Electricity from the Clouds, in the open Air, may be discovered in the same Manner, if the Balls are held at a sufficient Distance from Buildings, Trees, &c. as I have several Times experienced, by a Pair which I carry in a small narrow Box with a sliding Cover, (*Fig. 4.*) so contrived as to ^{Plate I.} keep their Threads straight, and that they may be properly suspended, when let fall out of it: and these Balls determine whether the Electricity of the Clouds or Air be positive, by the Decrease; or negative, by the Increase of their Repulsion, at the Approach of excited Amber or Sealing-Wax.

To electrify the Air, or Moisture contained in it, negatively; I support by Silk, between two Chairs placed Back to Back at the Distance of about three Feet, a Tin

F 4

Tube

Tube with a fine Sewing-needle at one End of it; and rub Sulphur, Sealing-Wax, or the rough Glass Tube, as near as I can to the other End, for three or four Minutes. Then will the Air be found to be negatively electrical; and will continue so a considerable Time after the Apparatus is removed into another Room.

THE Air without-doors I have sometimes known to be electrical in clear Weather; but never at Night, except when there has appeared an *Aurora Borealis*, and then but to a small Degree, which I have had several Opportunities of observing this Year. How far positive and negative Electricity in the Air, with a proper Quantity of Moisture between, to serve as a Conductor, will account for this, and other Meteors sometimes seen in a serene Sky, I shall leave to the Curious in this Part of Natural Philosophy to determine. That dry Air at a great Distance from the Earth, if in an electric State, will continue so till it meets with a Conductor, seems probable from this Experiment: An excited Glass Tube with its natural Polish, being placed upright in the Middle of a Room, by putting one End of it in a Hole made for that Purpose in a Block of Wood, will generally

rally lose its Electricity in less than five Minutes, by attracting to it a sufficient Quantity of Moisture, to conduct the electric Fluid from all Parts of its Surface to the Floor. But if, immediately after it is excited, it be placed in the same Manner before a good Fire, at the Distance of about two Feet, where no Moisture will adhere to its Surface, it will continue electrical a whole Day; and how much longer I know not. It may not be improper to mention here, that if a solid Cylinder of Glass be set before the Fire till quite dry, it may as easily be excited as a Glass Tube, and will act like one in every Respect: The first Stroke will make it strongly Electrical.

IN a Paper I laid before the *Royal Society*, on the 6th of *December* last, I conjectured, that the Electricity of the Atmosphere might be observed even in the Winter; which I have since found to be true: For in the succeeding Months of *January*, *February*, and *March*, my Apparatus was electrified no less than twenty-five Times, both positively and negatively, by Snow, as well as by Hail and Rain; and to almost as great a Degree when *Fahrenheit's* Thermometer was between 28 and 34, as I ever knew it in the Summer, except in a Thunder-Storm.

I SHALL

I SHALL be glad, if these Observations and Experiments may engage Persons of more Leisure and superior Abilities to pursue this Inquiry; as it is highly probable their Researches would be rewarded by many useful Discoveries. I have the Honour to be,

My Lord, &c.

PART II.

The Method of making ARTIFICIAL MAGNETS.

WE have already intimated, that Magnetism may be communicated by many different Ways to Bars of Iron, without the Touch or Use of real Magnetism; and this has been done in an extraordinary Manner by Mr. Savory, Mr. Lovelace, Dr. Knight, Mr. Mitchell, Mr. Canton, and others at home; also the Messrs. Du Hamel and Antheaume at France. But though the Methods taken by those Gentlemen are different, yet the Force communicated to Steel Bars, properly tempered, is nearly the same in all; and the Method
of

of succeeding best in these Cases is generally deduced from a great Number of Experiments. For we are yet too little acquainted with Nature, and the Laws of Magnetism, to reason directly, or *a priori*, concerning its Properties; and as the strongest artificial Magnets I have yet seen are those made by Mr. *Canton*, I shall give his Paper directing the Method of making them, as it is contained in N° VI. Vol. XLVII. of the *Transactions*, under the following Title :

A Method of making artificial Magnets without the Use of natural ones ; communicated to the Royal Society by John Canton, M. A. and F. R. S.

PROcure a Dozen Bars ; six of soft Steel, each three Inches Long, one Quarter of an Inch broad, and one-twentieth of an Inch thick, with two Pieces of Iron, each Half the Length of one of the Bars, but of the same Breadth and Thickness ; and six of hard Steel, each five Inches and an Half long, Half an Inch broad, and three-twentieths of an Inch thick, with two Pieces of Iron of half the Length, but the whole Breadth and Thickness of one of the

the hard Bars: And let all the Bars be marked with a Line quite round them at one End.

Plate II. (Fig. 1.) THEN take an Iron Poker and Tongs * the larger they are, and the longer they have been used, the better; and fixing the Poker upright between the Knees, hold to it near the Top one of the soft Bars, having its marked End downward, by a Piece of sewing Silk, which must be pulled tight with the left Hand, that the Bar may not slide: Then grasping the Tongs with the right Hand a little below the Middle, and holding them nearly in a vertical Position, let the Bar be stroked by the lower End, from the Bottom to the Top, about ten Times on each Side, which will give it a magnetic Power sufficient to lift a small Key at the marked End: Which End, if the Bar was suspended on a Point, would turn toward the North, and is therefore called the North Pole, and the unmarked End is, for the same Reason, called the South Pole of the Bar.

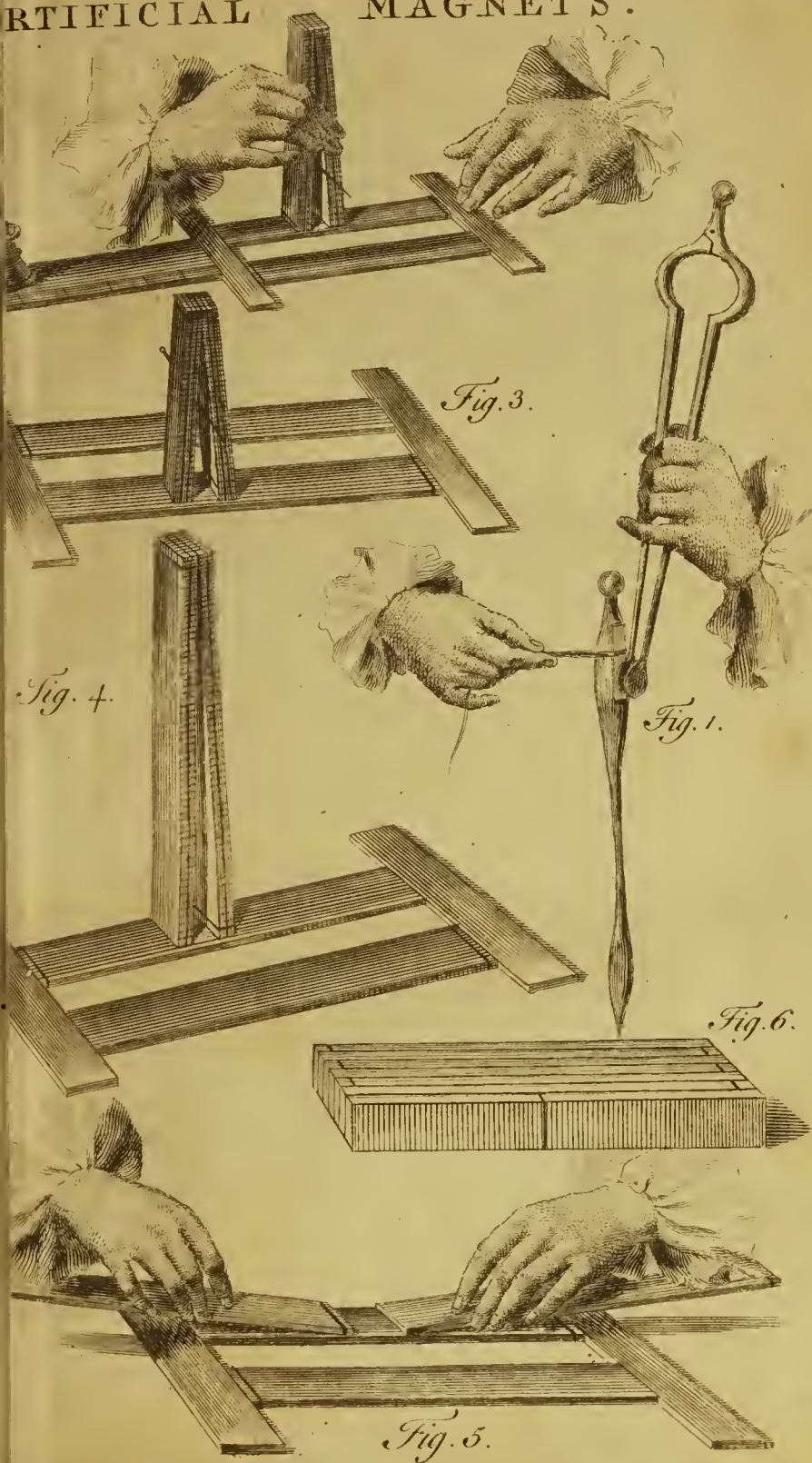
FOUR of the soft Bars being impregnated after this Manner, lay the other two (Fig. 2.) parallel to each other, at the Distance of about one-fourth of an Inch, between

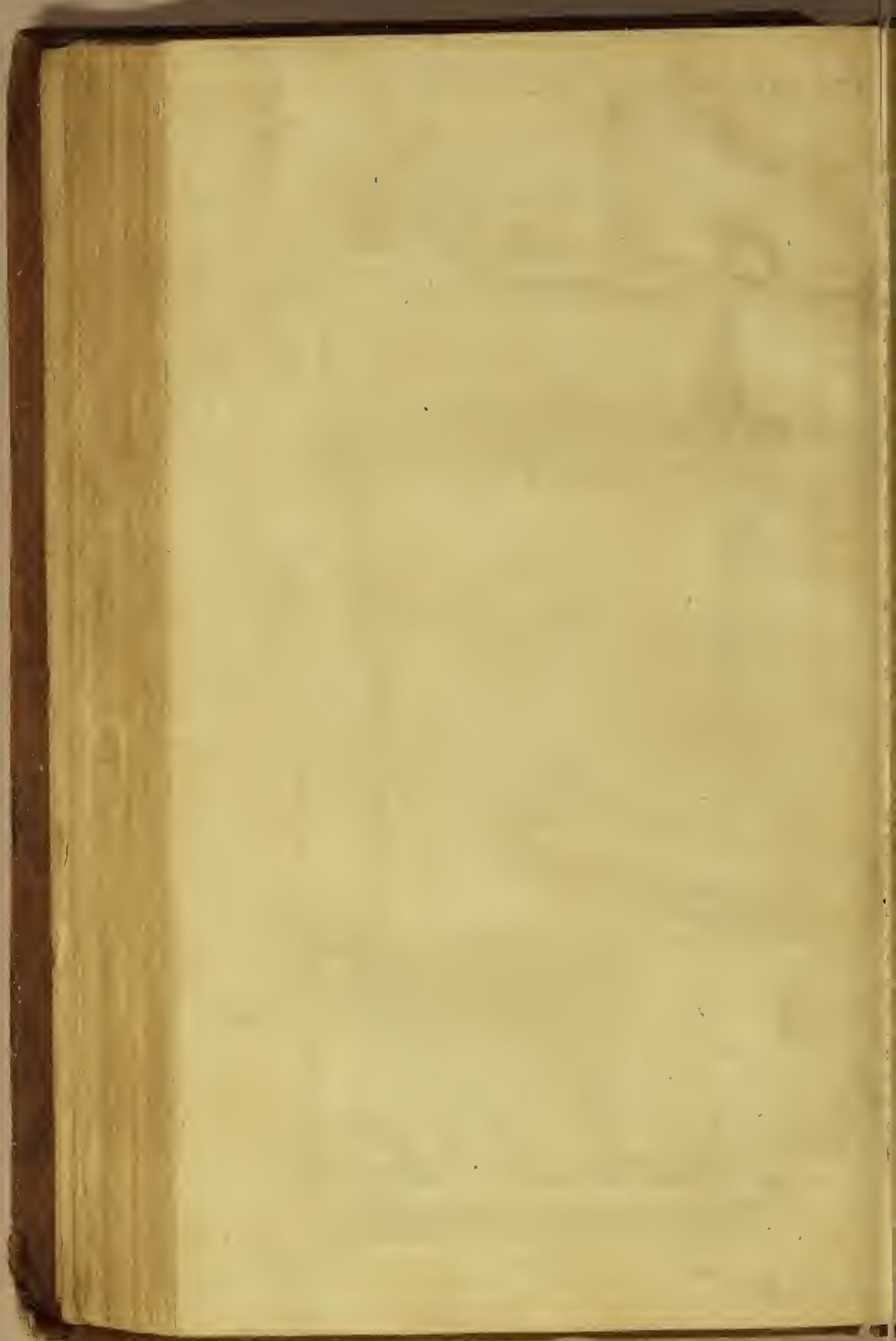
* Or two Bars of Iron.

the two Pieces of Iron belonging to them, and a North and South Pole against each Piece of Iron : Then take two of the four Bars already made magnetical, and place them together, so as to make a double Bar in Thickness, the North Pole of one, even with the South Pole of the other ; and the remaining two being put to these, one on each Side, so as to have two North and two South Poles together, separate the North from the South Poles at one End by a large Pin, and place them perpendicularly with that End downward, on the Middle of one of the parallel Bars, the two North Poles towards its South, and the two South Poles towards its North End : Slide them backward and forward three or four Times the whole Length of the Bar, and removing them from the Middle of this, place them on the Middle of the other Bar, as before directed, and go over that in the same Manner ; then turn both the Bars the other Side upward, and repeat the former Operation : This being done, take the two from between the Pieces of Iron, and placing the two outermost of the touching Bars in their Room, let the other two be the outermost of the four to touch these with : And this Process being

being repeated till each Pair of Bars have been touched three or four Times over, which will give them a considerable magnetic Power, put the half dozen together after the Manner of the four (*Fig. 3.*) and touch with them two Pair of the hard Bars, placed between their Irons, at the Distance of about half an Inch from each other: Then lay the soft Bars aside; and with the four hard ones let the other two be impregnated (*Fig. 4.*) holding the touching Bars apart at the lower End near two tenths of an Inch, to which Distance let them be separated after they are set on the parallel Bar, and brought together again before they are taken off: This being observed, proceed according to the Method described above, till each Pair have been touched two or three Times over. But as this vertical Way of touching a Bar will not give it quite so much of the magnetic Virtue as it will receive, let each Pair be now touched once or twice over in their parallel Position between the Irons (*Fig. 5.*) with two of the Bars held horizontally, or nearly so, by drawing at the same Time the North of one from the Middle over the South End, and the South of the other from the Middle over the North End of a parallel

ARTIFICIAL MAGNETS.





parallel Bar; then bringing them to the Middle again without touching the parallel Bar, give three or four of these horizontal Strokes to each Side. The horizontal Touch, after the vertical, will make the Bars as strong as they can possibly be made: As appears by their not receiving any additional Strength, when the vertical Touch is given by a greater Number of Bars, and the horizontal by those of a superior magnetic Power. This whole Process may be gone through in about half an Hour, and each of the larger Bars, if well-hardened *, may be made to lift twenty-eight Troy Ounces, and sometimes more. And when these Bars are thus impregnated, they will give to an hard Bar of the same Size, its full Virtue in less than

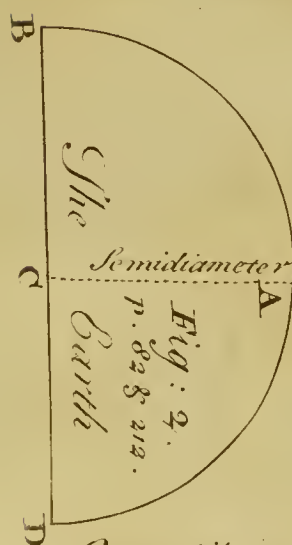
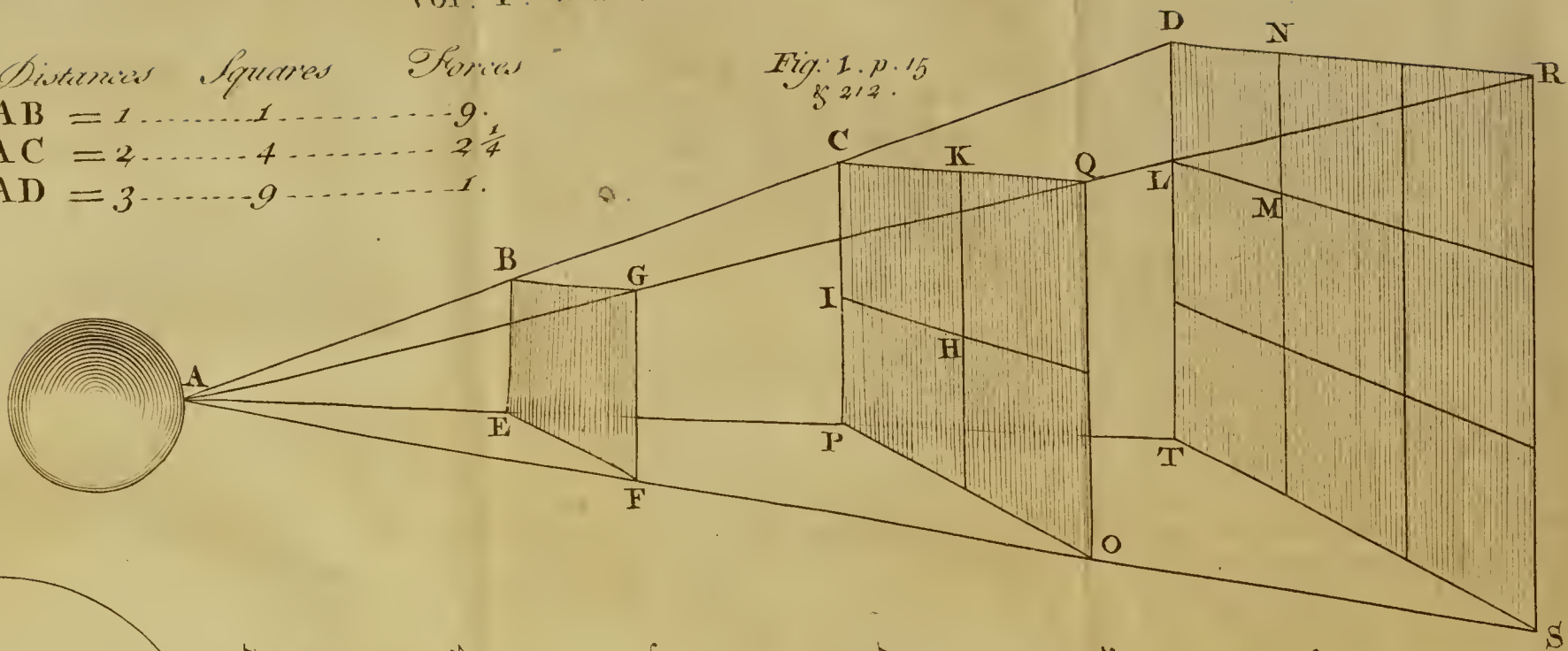
* The Smith's Manner of hardening Steel, whom I have chiefly employed, and whose Bars have constantly proved better than any I could meet with beside, is as follows: Having cut a sufficient Quantity of the Leather of old Shoes into very small Pieces, he provides an Iron Pan, a little exceeding the Length of a Bar, wide enough to lay two Side by Side without touching each other or the Pan, and at least an Inch deep. This Pan he nearly half-fills with the Bits of Leather, upon which he lays the two Bars, having fastened to the End of each a small Wire to take them out by: He then quite fills the Pan with the Leather, and places it on a gentle flat Fire, covering and surrounding it with Charcoal. The Pan being brought to somewhat more than a red heat, he keeps it so about half an Hour, and then suddenly quenches the Bars in a large Quantity of cold Water.

two

two Minutes: And therefore will answer all the Purposes of Magnetism in Navigation and experimental Philosophy, much better than the Loadstone, which is well known not to have sufficient power to impregnate hard Bars. The half dozen being put into a Case (*Fig. 6.*) in such a Manner, as that two Poles of the same Denomination may not be together, and their Irons with them as one Bar, they will retain the Virtue they have received: But if their Power should, by making Experiments, be ever so far impaired, it may be restored without any foreign Assistance in few Minutes. And if, out of Curiosity, a much larger Set of Bars should be required, these will communicate to them a sufficient Power to proceed with, and they may in a short Time, by the same Method, be brought to their full Strength.

<i>Distances</i>	<i>Squares</i>	<i>Forces</i>
AB = 1	1	9.
AC = 2	4	2 $\frac{1}{4}$
AD = 3	9	1.

Fig: 1. p. 15
§ 212.



1

4

9

16

25

36

1

2

3

4

5

6

3600

900

400

225

145

100

Quantity of Matter: Q = 3
Celerity: C = $\frac{9}{27}$
Momentum: M =

Quantity of Matter: Q = 5
Celerity: C = $\frac{7}{35}$
Momentum: M =

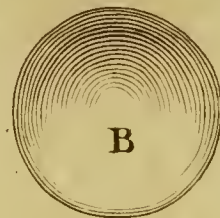


Fig: 3. p. 93.

X

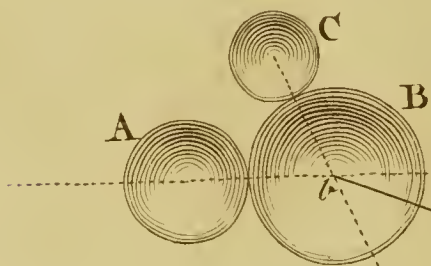


Fig: 4. p. 86
§ 102.

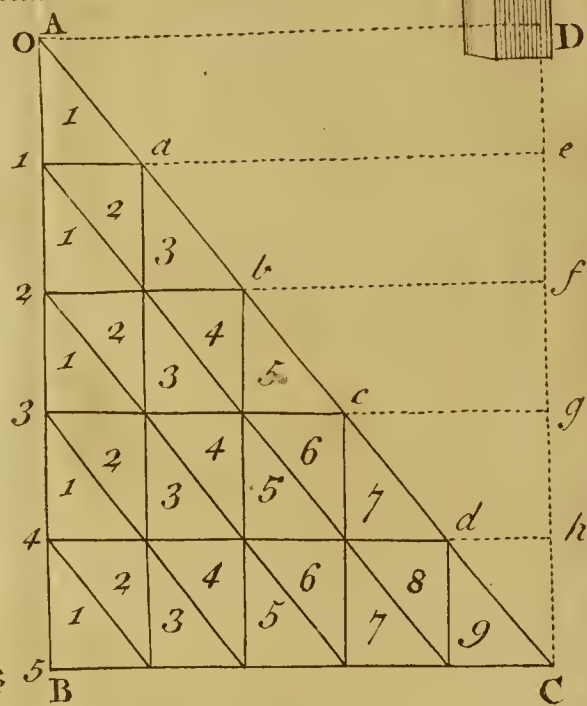


Fig: 5. p. 92
§ 109.

L E C T U R E II.

Of the Attraction of GRAVITATION, and its Laws. Of MOTION and REST, absolute and relative; equable, accelerated, and retarded MOTION. Of the Spaces, Times, and Velocities of BODIES in MOTION. Of the COLLISION of BODIES Elastic and Non-Elastic. Of the MOMENTUM or FORCE of STRIKING BODIES. Of the LAWS of MOTION. Of the COMPOSITION and RESOLUTION of MOTION. Of ACTION and RE-ACTION. Of the DESCENT of heavy Bodies. Of the DESCENT of Bodies on the inclined Planes. Of the DOCTRINE of PENDULUMS. Of the CYCLOID, its PROPERTIES and USE; the Curve of quickest Descent. Of the Centre of OSCILLATION and PERCUSSION. The various Uses of the PENDULUM, as a CHRONOMETER, &c. Of a new constructed PYROMETER, and the EXPANSION of METALINE BODIES. Of the DOCTRINE of PROJECTILES, or ART of GUNNERY. Of CENTRAL FORCES, and the various Laws and Properties of that Sort of Motion.

G

tion. Of the General Law of the PLANETARY MOTIONS. Of the FIGURE of the EARTH, and how determined.

THE *Fourth and last Species of Attraction* is that of GRAVITATION, which is evident only between large Bodies, as the Earth and Moon, the Sun and its Planetary Attendants. The Laws of this Attraction are as follows, viz. (1.) It is common to all Bodies, and mutual between them. (2.) It is proportional to the Quantity of Matter in Bodies. (3.) It is exerted every way from the Centre of the attracting Body in Right-lined Directions. (4.) It decreases as the

Plate III.
Fig 2.

Squares of the Distances increase: That is, if a Body at A, on the Earth's Surface, distant *one Semidiameter* from the Centre C, weighs 3600 Pounds, it will at the Distance of 2, 3, 4, 5, 6, Semidiameters weigh 900, 400, 225, 145, 100 Pounds; which Numbers decrease as the Squares of the Distances 4, 9, 16, 25, 36, increase (XVI).

(XVI.) 1. Though we reckon this Species of Attraction different from that of Cohesion, yet when well consider'd, it may be found perhaps to differ no otherwise than as the *Whole from the Parts*; for the Gravity of large Bodies may be only the Result or Aggregate of the particular Powers of the constituent Particles, which singly act only upon Contact, and in small Distances; but

HENCE

Of the Attraction of GRAVITATION.

83

HENCE we learn that all Bodies have WEIGHT, or are *heavy*; and that there is no such Thing as *absolute* LEVITY in Nature; and by the second Law, the Weight of all Bodies is proportional to the Quantity of Matter they contain; and hence, since Bodies of equal Bulks are found to have unequal Quantities of Matter, it evidently follows, that a VACUUM, or Space void of Matter, must necessarily exist, and that an absolute PLENUM is a

with their joint Forces, in vast Quantities, produce a mighty Power, whose Efficacy extends to very great Distances, proportional to the Magnitude of Bodies.

2. This Force of Gravity is, to Sense, the same for any Distances near the Earth's Surface; since a small Distance from the Surface of the Earth does not sensibly alter the Distance from the Centre, which is near 4000 Miles: At the Height therefore of one Mile, for Instance, the Distances from the Centre will be as the Numbers 4000 to 4001, and the Powers of Gravity inversely as their Squares, *viz.* as 16008001 to 16000000, or as 16008 to 16000, which are so near an Equality as not to be sensibly different from each other.

3. But when the Distance is so great as to cause a Disproportion between that and the Semidiameter of the Earth, then will the Power of Gravity decrease very sensibly, according to the Law above laid down: Thus at the Distance of the Moon, which is, at a Mean, about 60 Semidiameters of the Earth, the Power of Gravity is to that on the Earth's Surface as 1 to 3600, as will be shewn under the Article of *Central Forces* with sufficient Demonstration.

Of the Attraction of GRAVITATION.
 Doctrine unphilosophical, and equally false
 and absurd (XVII).

(XVII.) 1. Since there is no Sort of Matter that we have any Knowledge of but what will gravitate, to suppose there is any Sort that will not, is arbitrary, and contrary to the Rules of Philosophizing, and all sound Reasoning; by which we are confined to the Phenomena of real, and not imaginary Existence.

2. The Weight or Quantity of Matter in Bodies may be express'd by their Bulks multiplied by their Densities. By the DENSITY of a Body is meant the greater or lesser Quantity of Matter under the same Bulk; and therefore the Density (D) of any Body A , is to the Density (D) of any other Body B of equal Bulk, as the Quantity of Matter (M) in the former is to the Quantity of Matter (m) in the latter. That is, $D : D :: M : m$; $\therefore Dm = DM$.

3. But if the Bulks are unequal, and the Quantities of Matter the same, or equal in any two Bodies B and C ; then will the Density (D) of the Body B be to the Density (d) of the Body C , as the Bulk (b) of the latter, to the Bulk (B) of the former; viz. $D : d :: b : B$; $\therefore DB = db$.

4. Hence $D = B = \frac{db}{M}$; and so $Mdb = mBD$; and consequently $M : m :: BD : bd$; that is, *The Quantity of Matter in any two Bodies A and C , which differ in Bulk and Weight, are to each other as the Products of the Densities by their Bulks, as was above asserted.*

5. Hence also in such Bodies, $D : d :: Mb : mB$; that is, *the Densities of the two Bodies A and C are directly as the Quantities of Matter, and inversely as their Bulks.*

6. Lastly, *The Bulks or Magnitudes of two Bodies A and C are directly as their Quantities of Matter or Weights, and inversely as their Densities*; for we have $B : b :: M : mD$.

7. What has been hitherto said relates to the absolute Weight of Bodies; but when the Weight or Gravitation

ALSO

Also, from the third Law it follows, that all Bodies descending freely by their Gravity tend towards the Earth in Right Lines perpendicular to its Surface; and with equal Velocities, (abating for the Resistance of the Air) as is evident by the second Law above, and what will immediately follow in *Mechanics* (XVIII.)

of Bodies of equal Bulk are considered and compared together, they are then called the Relative Weights or Specific Gravities of those Bodies, and since they are likewise proportional to the Quantities of Matter, 'tis evident, that the *Densities and Specific Gravities are proportional to each other*, in all Bodies; and therefore what has been said of the one is equally applicable to the other.

8. As a Corollary to this Article, we may observe with Sir *Isaac Newton*, that all Space cannot be fill'd with Matter, as the *Plenists* assert, but that there must necessarily be Vacuities or Interstices void of Matter in the Composition of natural Bodies; for if there were not, then Bodies of equal Bulk would ever be of the same Weight and Density, or, in other Words, the Density and Specific Gravity of *Cork* or *Air* would be the same with that of *Quicksilver* or *Gold*. And 'tis surprizing to think, that such Absurdities are not gross enough to be heeded by those who hesitate about a *Vacuum*

(XVIII.) The Reason why all Bodies descend with an equal Velocity towards the Centre of the Earth (*in Vacuo*) is because the Force of Gravity in each is proportional to the Quantity of Matter, or Number of Particles only, independent of any other Circumstance or Consideration whatsoever. Thus suppose the Body A has only one Particle of Matter, and the Body B consists of one hundred, then will the Force of Gravity in A be to that in B as 1 to 100; and therefore the Force acting on each single Particle in B is but $\frac{1}{100}$ of the whole Force, and conse-

AGAIN : Since the Attraction is always directly as the Quantity of Matter, and inversely as the Square of the Distance, it follows, that were the internal Parts of the Earth a perfect Void, or hollow Concavity, a Body placed any where therein would be absolutely light, or void of Gravity; but supposing the Earth a solid Body throughout, the Gravity from the Surface to the Centre will decrease with the Distance, or it will be directly proportional to the Distance from the Centre (XIX).

quently is but equal to the Force acting on the single Particle A ; since then each single Particle in B is acted upon with the same Force as the Particle A, the Velocities in each must be equal ; that is, each Particle in the Body B, (and therefore the whole Body) must descend with an equal Velocity with that of A. This would be easy to conceive, if the Particles in the Body B were to fall separate ; and I think 'tis as easy to understand that their Connection or Cohesion can no ways contribute to the Acceleration of their Motion, which is entirely the Effect of the attracting Force of the Earth.

(XIX.) 1. For Illustration, suppose ABC be the outward Shell or Crust, and all the internal Parts of the Earth an hollow Space or Concavity, in which suppose any Body as R, to be placed in any Part whatsoever ; thro' the Centre of the said Body let there be drawn the Lines *ad* and *bc*, which by their Revolution would describe Cones, whose Bases *ab*, and *cd*, in the spherical Superficies, are in proportion to each other as the Squares of the Distances from R respectively. Now the Particle R is attracted by the Particles in those two Bases with an equal Force ; for the Force is as the Square of the Distance of the Bases inversely, and as the Number of Particles

HAVING

HAVING premised the necessary *Præcognita* to the Science of MECHANICS, which in each Base directly, which Number of Particles is as the Square of the Distance from the Particle directly. But this *direct* and *inverse Ratio* of the Forces makes a Ratio of Equality; that is, the Forces on each S de are equal, and being in contrary Directions they destroy each other; and this being the Case in any other Part of the Concave Superficies, 'tis evident the Body R cannot be impell'd towards any Part, but will retain its Position as if not affected by Gravity at all.

2. The Forces with which a Body will be attracted on the Surface of two Spheres of the same Density, but unequal Bulks, *will be directly as the Diameters of the Spheres.* For the Force (f) of the lesser Sphere ABC will be to the Force (F) of the greater DEF inversely as the Squares of the Diameter (d^2) of the lesser to the Square of the Diameter (D^2) of the greater; that is,

Plate II
Fig. 5, 6

$f : F :: \frac{1}{dd} : \frac{1}{DD}.$ Note, *This is so because the Forces are inversely as the Squares of the Distances from the Centres, which are equal to half the Diameters.*

3. Again: The Forces are as the Number of attracting Particles, which are as the Bulks or Magnitudes of the Spheres, which are as the Cubes of the Diameter; therefore also $f : F :: d^3 : D^3$; whence both these Ratio's compounded give $f : F :: \frac{d^3}{d^2} : \frac{D^3}{D^2} :: d : D$; or, *the Forces of Attraction are in proportion to the Diameters of the Spheres directly.*

4. Hence it follows, that were there a Perforation made, and a Body to descend therein from the Surface of the Earth to the Centre, its Gravity would always decrease with the Distance from the Centre; because in every Place it might be consider'd as on that Surface of a Sphere whose Diameter is equal to twice the Distance from the Centre, and which is proportional to the Sphere's Attraction; the Parts of the Earth above the Body being only Part of a Spherical Shell, which has no Effect on the Body, as was shewn above.

G 4 entirely

entirely depends on the Principle of Gravitation, we come immediately to consider the Object thereof, *viz.* the *Nature, Kinds,* and various *Affections* of *Motion*, and moving Bodies; and the Structure and Mechanism of all Kinds of Machines, commonly call'd *Mechanical Powers*, whether Simple or Compound.

MOTION is the continual and successive Change of Space, and is either *Absolute* or *Relative*. *Absolute Motion* is the Change of *Absolute Space* or Place of Bodies, as the Flight of a Bird, the Motion of a Projectile, &c. But *Relative Motion* is the Change of *Relative Space*, or that which has Reference to some other Bodies: As of two Ships under Sail, the Difference of their Velocities is the *Relative Motion* of the Ship sailing fastest; and is that alone which is discernible by us. The same is to be understood of *Absolute and Relative Rest* (XX).

(XX) To make this Matter yet plainer, we are to consider that SPACE is nothing else *but an absolute and infinite Void*; and that the PLACE of a Body is that Part of the immense Void which it takes up or possesses; and that Place may be consider'd either absolutely, or in itself alone, and then it is call'd the *Absolute Place* of the Body; or else with Regard to the Place of some other Body, and then it is call'd the *Relative or Apparent Place* of Bodies.

AGAIN :

AGAIN : Motion is either *Equable* or *Accelerated*. *Equable Motion* is that by which

2. Now as Motion is only the Change of Place in Bodies, 'tis evident that will come under the same Distinction of *Absolute*, and *Relative* or *Apparent*. All Motion is in itself *Absolute*, or the Change of Absolute Space; but when the Motions of Bodies are consider'd and compared with each other, then are they *Relative* and *Apparent* only; they are *Relative*, as they are compared with each other; and they are *Apparent* only, inasmuch as not their *true* or *absolute Motion*, but the *Sum* or *Difference* of the *Motions* only is perceivable by us.

3. In comparing the Motions of Bodies, we may consider them as moving both the same way, or towards contrary Parts; in the first Case the *Difference* of the *Motions* is only perceived by us, in the latter the *Sum* of the *Motions*. Thus for Example; suppose two Ships, A and B, set sail from the same Port upon the same Rhumb, and that A sails at the Rate of five Miles *per* Hour, and B at the Rate of three; here the Difference of the Velocities (*viz.* two Miles *per* Hour) is that by which the Ship A will appear to go from the Ship B forwards, or the Ship B will appear at A to go with the same Velocity backwards, to a Spectator in either respectively.

4. If the two Ships A and B move with the same Degree of Velocity, then will the Difference be nothing, and so neither Ship will appear to the other to move at all. Hence it is, that tho' the Earth is continually revolving about its Axis, yet as all Objects on its Surface partake of the same common Motion, they appear not to move at all, but are relatively at Rest.

5. If the two Ships A and B, with the Degrees of Velocity as above, meet each other; the one will appear to a Spectator in the other to move with the Sum of both Velocities, *viz.* at the Rate of eight Miles *per* Hour; so that in this case the *Apparent Motion* exceeds the *True*, as in the other it fell short of it. Hence the Reason why a Person riding *against* the Wind finds the Force of
a Body

a Body passes over equal Spaces in equal Times. *Accelerated Motion* is that which is continually augmented or increased; as *Retarded Motion* is that which continually decreases: And if the Increase or Decrease of Motion be equal in equal Times, the Motion is then said to be *equally accelerated or retarded* (XXI).

it much greater than it really is, whereas if he rides *with it*, he finds it less.

SCHOLIUM.

6. The Reason of all these different Phænomena of Motion will be evident if we only consider, that we must be absolutely at Rest, if we would discern the true or real Motion of Bodies about us. Thus a Person on the Strand will observe the Ships sailing with their real Velocity; a Person standing still will experience the true Strength and Velocity of the Wind; and a Person placed in the Regions between the Planets will view all their true Motions; which he cannot otherwise do, because in all other Cases the Spectator's own Motion must be added to, or subducted from, that of the moving Body; and the Sum or Difference is therefore the *Apparent or Relative Motion*, and not the *True*.

(XXI.) 1. *Equable Motion* is generated by a single *Impetus* or Stroke; thus the Motion of a Ball from a Cannon is produced by the single Action of the Powder in the first Moment, and therefore the Velocity it first sets out with would always continue the same, were it void of Gravity, and to move in an unresisting Medium, which therefore would be always equable, or such as would carry it through the same Length of Space in every equal Part of Time.

2. *Accelerated Motion* is produced by a constant Impulse, or Power which keeps continually acting upon the Body, as that of Gravity, which produces the Motion of Falling Bodies, which Sort of Motion is constantly acce-

THE

THE *Celerity* or *Velocity* of Motion is that Affection by which a Body passes over a *given Space in a given Time*, or what we

lerated, because Gravity every Moment adds a new Impulse, which generates a new Degree of Velocity; and the Velocity thus increasing the Motion must be quicken'd each Moment, or fall faster and faster the longer it falls.

3. In like Manner, a Body thrown perpendicularly upward, as a Ball from a Cannon, will have its Motion *continually retarded*, because Gravity acts constantly upon it in a Direction contrary to that given it by the Powder, so that its Velocity upwards must be continually diminished, and so its Motion is continually retarded, till at last it be all destroy'd. The Body has then attain'd its utmost Height, and is for a Moment motionless, after which it begins to descend with a Velocity in the same Manner accelerated, till it comes to the Earth's Surface.

4. And because at a small Distance from the Earth's Surface, the Power of Gravity is every where the same, it follows, that in equal Moments of Time the Impulses on the Body will be equal, which will therefore generate an equal Increase of Velocity each Moment, and of Consequence the Motion resulting from thence will be equally quicken'd if downwards, or retarded if upwards, in Mediums without Resistance.

5. In this Case we may observe in Bodies that are projected upwards, (1.) That the Time of the Ascent is equal to that of the Descent. (2.) That at equal Heights above the Earth, the Velocity in the Ascent and Descent is equal, or the same; and therefore, (3.) That the Velocity acquired (by falling) at the Earth's Surface, is equal to that which is generated by the Powder, or other *Impetus* (which threw it up) in the first Moment of its Motion. What Difference will arise from a resisting Medium (as to Projectiles thrown up in the Air, &c.) will be hereafter more fully consider'd and specified.

com-

commonly call the *Swiftness* or *Slowness* of Motion (XXII).

THE MOMENTUM or *Quantity of Motion* is all that Power or Force which a moving Body has to affect or strike any Obstacle or Impediment which opposes its Motion, and is equal to that impressed

(XXII). 1. From the above Definitions we may determine the Theorems, for the Expressions of the Time (T,) the Velocity (V), and the Space (S) passed over in Equable or Uniform Motion, very easily thus :

2. If the Time be given, or the same ; the *Space passed over will be as the Velocity*, viz. $S : V$; that is, with twice the Velocity, twice the Space ; with three Times the Velocity, three times the Space, will be pass'd over in the same Time ; and so on.

3. If the Velocity be given, or remain the same ; *then the Space pass'd over will be as the Time* (viz. $S : T$) that is, it will be the greater or lesser as the Time is so.

4. But if neither the Time nor Velocity be given, or known ; *then will the Space be in the compound Ratio of both*, viz. $S : TV$.

5. Hence, in general, since $S : TV$, we have $V : \frac{S}{T}$;

that is, *the Velocity is always directly as the Space, and inversely as the Time*.

6. And also $T : \frac{S}{V}$; that is, *the Time is as the Space*

directly, and as the Velocity inversely ; or, in other Words, it increases with the Space, and decreases with the Velocity.

7. If therefore in any Rectangle ABCD one Side AB represent the Time, and the other Side BC the Velocity, 'tis evident the Area of the said Rectangle will represent the Space pass'd over by an uniform Motion in the Time AB and with the Velocity BC.

Plate III.
Fig. 5.

Force

Force by which the Body is compell'd to change its Place.

SINCE the Force of a Body is greater or lesser as the Quantity of Matter is so, when the Velocity of its Motion is the same ; also since the said Force in the same Body is proportional to its Velocity : Therefore it follows, that the *Momentum* or Force of *Bodies* in Motion is in the compound Ratio of their Quantities of Matter and their Velocities conjointly. Thus if one Body A strike an Obstacle X, with 3 Parts of Matter and 9 Degrees of Celerity ; and another Body B strike it with 5 Parts of Matter, and 7 Degrees of Celerity ; the *Momenta* of the respective Strokes will be as 27 to 35 (XXIII).

Plate III.
Fig. 3.

(XXIII.) Since the *Momentum* (M) of a Body is compounded of the Quantity of Matter (Q), and the Velocity (V), we have this general Expression $M=QV$, for the Force of any Body A ; and suppose the Force of another Body B be represented by the same Letters in *Italicks*, viz. $M=QV$.

2. Let the two Bodies A and B in Motion impinge on each other directly ; if they tend both the same Way, the Sum of their Motions towards the same Part will be $QV+QV$. But if they tend towards contrary Parts, or meet, then the Sum of their Motions toward the same Part will be $QV-QV$; for since the Motion of one of the Bodies is contrary to what it was before, it must be connected by a contrary Sign. Or thus ; because when the Motion of B conspires with that of A, it is *added to it* ; so when it is contrary, it is *subducted from it*, and the Sum or Difference of the *absolute Motions* is

THE general Laws of Motion which all Bodies observe, are the three following, *viz.*

is the whole *relative Motion*, or that which is made towards the same Part.

3. Again, this total Motion towards the same Parts is the same both before and after the Stroke, in case the two Bodies A and B impinge on each other; because whatever Change of Motion is made in one of these Bodies by the Stroke, the same is produced in the other Body towards the same Part; that is, as much as the Motion of B is increased or decreased towards the same Part by the Action of A, just so much is the Motion of A diminished or augmented towards the same Part by the equal Re-action of B, by the *third Law* of Motion.

4. In Bodies *not elastic*, let x be the Velocity of the Bodies after the Stroke (for since we suppose them not elastic, there can be nothing to separate them after Collision, they must therefore both go on together, or with the same Celerity). Then the Sum of the Motions after Collision will be $Qx + 2x$; whence if the Bodies tend the same Way, we have $QV + 2V = Qx + 2x$, or if they meet, $QV - 2V = 2x + 2x$; and accordingly

$$\frac{QV + 2V}{Q + 2} = x, \text{ or } \frac{QV - 2V}{Q + 2} = x.$$

Plate IV. 5. If the Body (B) be at rest, then $V = 0$, and the
Fig. 1.

Velocity of the Bodies after the Stroke will be $\frac{QV}{Q + 2} = x$. Thus if the Bodies be equal (*viz.* $Q = 2$) and A with 10 Degrees of Velocity impinge on B at rest;

Plate IV. then $\frac{QV}{Q + 2} = \frac{10}{2} = 5 = x$. If $Q = 2$, and $V : V :: 10$
Fig. 2.

: 6; we have $\frac{QV + 2V}{Q + 2} = \frac{16}{2} = 8 = x$, the Velocity after the Stroke.

L A W

LAW I. *Every Body will continue in its State of Rest, or moving uniformly in a Right*

6. If the Bodies are both in Motion, and tend to- Plate IV. wards opposite Parts, or meet ; then when $Q = \mathcal{Q}$, and Fig. 3.

$V = V$, 'tis plain $\frac{QV - \mathcal{Q}V}{Q + \mathcal{Q}} = 0 = x$; that is, *the Bodies*

which meet with equal Bulks and Velocities will destroy each other's Motion after the Stroke, and remain at rest. If

$Q = \mathcal{Q}$, but $V : V :: 6 : 14$, then $\frac{QV - \mathcal{Q}V}{Q + \mathcal{Q}} = -\frac{8}{2}$ Fig. 4.

$= -4 = -x$; which shews that *equal Bodies meeting with unequal Velocities, they will after the Stroke both go on the same Way which the most prevalent Body moved before.*

7. If the Velocity $\frac{QV \pm \mathcal{Q}V}{Q + \mathcal{Q}}$ be multiplied by the Quantities of Matter Q and $\mathcal{Q}$, we shall have $\frac{Q^2V \pm Q\mathcal{Q}V}{Q + \mathcal{Q}}$
 $=$ the *Momentum* of A after the Stroke ; and $\frac{QV\mathcal{Q} \pm \mathcal{Q}^2V}{Q + \mathcal{Q}}$
 $=$ the *Momentum* of B. Therefore $QV - \frac{Q^2V \pm Q\mathcal{Q}V}{Q + \mathcal{Q}}$
 $= \frac{Q\mathcal{Q}V \pm \mathcal{Q}QV}{Q + \mathcal{Q}} = \frac{Q\mathcal{Q}}{Q + \mathcal{Q}} \times V \pm V =$ the Quantity

of Motion lost in A after the Stroke, and consequently is equal to what is gain'd in B, as may be shewn in the same Manner.

8. But since a Part of this Expression (*viz.* $\frac{Q\mathcal{Q}}{Q + \mathcal{Q}}$) is constant, the Loss of Motion will be ever proportional to the other Part $V \pm V$. But this Loss or Change of Motion in either Body is the whole Effect, and so measures the Magnitude or Energy of the Stroke. Where-

Line,

Line, except so far as it is compell'd to change that State by Forces impress'd.

fore any two Bodies, not elastic, strike each other with a Stroke always proportional to the Sum of their Velocities ($V+V$) if they meet, or to the Difference of their Velocities ($V-V$) if they tend the same Way.

9. Hence if one Body (B) be at rest before the Stroke, then $V=0$; and the Magnitude of the Stroke will be as V , that is, as the Velocity of the moving Body A; and not as the *Square of its Velocity*, as many Philosophers (*viz.* the Dutch and Italians) maintain.

10. In Bodies perfectly elastic, the restituent Power or Spring, by which the Parts displaced by the Stroke restore themselves to their first Situation, is equal to the Force impress'd, because it produces an equal Effect; therefore in this Sort of Bodies, there is a Power of Action twice as great as in the former Non-elastic Bodies, for these Bodies not only strike each other by *Impulse*, but likewise by *Repulse*, they always repelling each other after the Stroke.

11. But we have shewn that the Force with which Non-elastic Bodies strike each other is as $V=V$; therefore the Re-action of Elastic Bodies is the same; that is, the Velocity with which Elastic Bodies recede from each other after the Stroke is equal to the Velocity with which they approach'd each other before the Stroke. Whence if x and y be the Velocities of two Bodies A and B, tending the same Way, after the Stroke, since $V-V=y-x$, we have $x+V-V=y$; whence the Motion of A after the Stroke will be Qx , and that of B will be $Qx+QV-QV$; and the Sum of these Motions will be equal to the Sum of the Motions before the Stroke, *viz.* $Qx+Qx+QV-QV=QV+QV$. Whence by reducing the Equation, it will be $Qx+Qx=QV-QV+2QV$; and $x=\frac{QV-QV+2QV}{Q+Q}$ the Velocity of the Body A.

LAW

LAW II. *The Change of Motion is always proportional to the moving Force im-*

12. Again, the Velocity of B is $x + V - V =$

$$\frac{QV - 2V + 2QV}{Q + 2} + V - V = \frac{2QV - QV + 2V}{Q + 2}$$

Here we have supposed the Bodies tend the same Way before the Stroke ; and it is evident from the Equation above, that so long as $QV + 2QV$ is greater than $2V$, the Velocity (x) of A after the Stroke will be affirmative, or the Body A will move the same Way after the Stroke as before ; but when $2V$ is greater than $QV + 2VQ$, the Velocity (x) will be negative, or the Body A will be reflected back.

13. If the Body B be at rest, then $V = 0$; and $x =$

$$\frac{QV - 2V}{Q + 2}$$
 ; which shews the Body A will go forwards

or backwards, as QV is greater or lesser than $2V$, or A greater or lesser than B.

14. If $Q = 3$, $2 = 2$, and $V = 10$, and $V = 0$; then Plate IV.
 after the Stroke the Velocity of A will be $\frac{QV - 2V}{Q + 2} =$ Fig. 5.

$$\frac{30 - 20}{5} = \frac{10}{5} = 2$$
, and the Velocity of B will be y

$$= \frac{2QV}{Q + 2} = \frac{60}{5} = 12$$
. If the Bodies are both in Mo-

tion, and $V = 5$, the rest the same as before ; then

$$\frac{QV - 2V + 2QV}{Q + 2} = 6 = \text{Velocity of A after the}$$

Stroke, and $\frac{2QV - QV + 2V}{Q + 2} = 11 = \text{Velocity of B after the Stroke.}$ Fig. 6.

15. If the Bodies A and B move towards contrary Parts, or meet each other, then will the Relative Velocity, to which the Force of the Stroke is proportional, be $V + V$, and so the Velocities of A and B after the

H *press'd,*

press'd, and is always made according to the Right Line in which that Force is impress'd.

Stroke will be x and $x + V + V$; and so the Motion of A will be Qx and $2x + 2V + 2V$; the Sum of these Motions is $Qx = 2x = 2V = 2V = QV - 2V =$ the Motion towards the same Part before the Stroke. Whence we have $x = \frac{QV - 2V - 22V}{Q + 2}$; and the Velocity of B will be $\frac{QV - 2V - 22V}{Q + 2} + V + V = \frac{2QV + QV - 2V}{Q + 2}$.

16. If $QV + 22V$ be greater than QV , the Motion of the Body A will be backwards; otherwise it will go on forwards as before. If $Q = 3$, $2 = 2$, $V = 10$, and $V = 5$; then will the Velocity of A be $\frac{QV - 2V - 22V}{Q + 2} = \frac{-10}{5} = -2$, and so the Body A will go back with two Degrees of Velocity. The Velocity of B, after the Stroke, will be $\frac{2QV + QV - 2V}{Q + 2} = 13$.

17. If the Bodies are equal, that is, if $Q = 2$, then $x = \frac{-22V}{22} = -V$; which shews, that when equal Bodies meet each other, they are reflected back with interchanged Velocities; for in that Case also the Velocity of B becomes $\frac{2QV}{2Q} = V$. An Example of this you have in Fig. 8 of Plate IV.

18. If the Bodies are equal, and one of them at rest, as B; then since $Q = 2$, and $V = 0$ we have the Velocity of A after the Stroke $x = 0$; or the Body A will abide at rest, and the Velocity of B will be $= V$, the Velocity of A before the Impulse, as appears by the Example of Fig. 9.

19. If several Bodies B, C, D, E, F, are contiguous in a Right Line, if another equal Body A strike B with

LAW

LAW III. *Re-action is always equal and contrary to Action; or the Actions of two*

any given Velocity, it shall lose all its Motion, or be quiescent after the Stroke; the Body B which receives it will communicate it to C, and C to D, and D to E, and E to F; and because Action and Re-action between the Bodies B, C, D, E, are equal, as they were quiescent before, they must continue so; but the Body F having no other Body to re-act upon it, has nothing to obstruct its Motion, it will therefore move on with the same Velocity which A had at first, because it has all the Motion of A, and the same Quantity of Matter by Hypothesis.

20. Let there be three Bodies A, B, C; and let A strike B at rest; the Velocity generated in B by the Stroke

will be $y = \frac{2 Q V}{Q + Q}$, and so the Momentum of B will be

$\frac{2 Q V Q}{Q + Q} = 2 y$. With this Momentum B will strike C

at Rest and contiguous to it; the Velocity generated in

C will be $\frac{2 Q y}{Q + C}$; and its Momentum will be $\frac{2 Q y C}{Q + C} =$

$\frac{2 Q C}{Q + C} \times \frac{2 Q V}{Q + Q} = \frac{4 Q V Q C}{Q Q + Q C + Q^2 + Q C}$.

21. If now we suppose B a variable Quantity, while A and C remain the same, we shall find what Proportion it must have to each of them in order that the Momentum of C may be a Maximum, or the greatest possible, by putting the Fluxion thereof equal to Nothing;

that is, $\frac{4 Q^2 C^2 V \dot{Q} - 4 Q C Q^2 \dot{Q}}{Q C + Q Q + Q C + Q^2} = 0$; whence we

get $Q C - Q Q = 0$, and so $Q C = Q Q$; Consequently $Q : Q :: Q : C$, or $A : B :: B : C$; that is, the Body B is a Geometrical Mean between A and C.

22. Hence if there be any Number (n) of Bodies in a Geometrical Ratio (r) to each other; and the first be

H 2

Bodies

Bodies upon each other are always equal, and in contrary Directions: That is, by Action and

A, the second will be $r A$, the third $r^2 A$, and so on to the last, which will be $r^{n-1} A$.

23. Also, the Velocity of the first being V , that of the second will be $\frac{2 V}{1+r}$, (for $\frac{2 Q V}{Q+Q}$ is here $= \frac{2 A V}{A+r A}$
 $= \frac{2 V}{1+r}$) that of the third $\frac{4 V}{1+r^2}$, that of the fourth $\frac{8 V}{1+r^3}$, and so on to the last, which will be $\frac{2}{1+r} \Big|^{n-1} V$.

24. The *Momentum* of the first will be $A V$, that of the second $\frac{2 r A V}{1+r}$, that of the third $\frac{4 r^2 A V}{1+r^2}$, that of the fourth $\frac{8 r^3 A V}{1+r^3}$, and so on to the last, which will be $\frac{2 r}{1+r} \Big|^{n-1} A V$.

25. To give an Example; if $n = 100$, and $r = 2$; then will the first Body A be to the last $r^{n-1} A$, as 1 to 633825300000000000000000000000, nearly; and its Velocity to that of the last nearly as 271022000000000000 to 1: Lastly, the *Momentum* of the first to the last will be nearly as 1 to 2338480000000000.

26. If the Number (n) of Bodies be required, and the Ratio of the *Momenta* of the first and last be given as 1 to M , and the Ratio of the Series r given also; then putting $\frac{2 r}{1+r} = R$, we have the *Momentum* of the

last Body express'd by $\frac{2 r}{1+r} \Big|^{n-1} = M = R^{n-1}$; therefore the Logarithm of M is equal to the Logarithm of R

Re-

Re-action equal changes of Motion are produced in Bodies acting upon each other; and these Changes are impress'd towards contrary Parts.

THE first of these Laws is founded on the *Vis Inertiæ* of Matter, whereby it is indifferently disposed to persevere in its State of Motion or Rest. 'Tis not more evident that Matter at Rest requires an extrinsic Power to give it Motion, than that, when in Motion, the Force of some other Body resisting it is necessary to bring it to a State of Rest. For want of such Resistance we see the Planets and Comets long conserve their Motions undiminis'd; while moving Bowls, and Wheels, are gradually reduced to a State of Rest, by the Friction or Rubbing of the Parts on which they move, against contiguous resisting Bodies; as is evident by the experiment of the *Axis in Peritrochio*, moving first on fix'd Parts, and afterwards on *Friction-Wheels*.

FROM this Law, and what will be farther demonstrated hereafter, it follows,

multiplied by the Power $n-1$; that is, $L.M = n-1$
 $< L.R$; consequently $\frac{L.M}{L.R} + 1 = n$, the Number of
 Bodies required.

H 3

that

that no *perpetual Motion* can be effected, at least by any human Power, with Bodies in a resisting Medium.

By the *second Law* we are instructed how to estimate the Sum of the Motions of Bodies moving the same or contrary Ways, when they directly strike, or impinge on each other. Also we hence learn the *Composition and Resolution of Motion* arising from Forces impress'd in oblique Directions; a Doctrine of the utmost Use in Philosophy, and the Foundation of all *Mechanics*. To illustrate this: Let the Body

Plate III. B at Rest be impell'd by the Body A in the
Fig. 4. Direction bc , with a Force that would, in a given Time, cause it to move from b to c ; at the same Instant, let another Body C strike it in the Direction bd , with a Force that will carry it from b to d in the same Time; then compleat the parallelogram $bced$, and draw the Diagonal be , that will represent the Direction and Distance through which the Body will move in the same Time by both the Forces conjointly (XXIV).

(XXIV.) This is evident if we consider that the Force impress'd by the Body C does no ways diminish the Velocity of a Body approaching to the line ce , at the End of the given Time, and therefore it will then be found

THE *third general Law* is founded on Reason and Experience: We know from

somewhere in the said Line *ce*. For the same Reason it will at the End of the said Time be carried to a Distance from *bc* equal to *bd*, and therefore it must also at the same Moment be found somewhere in the Line *de*; but it cannot be in the Line *ce* and *de* at the same Time, unless in that Point *e*, where they intersect each other, as the Proposition asserts.

2. We may now conceive the Body B moving by the single Impulse of some Power in the Direction *be*, such as will carry it thro' the Space *be* in a given Time; then this may be resolved into any other two Forces acting in the Directions *be* or *de*, and *bd* or *ce*, which Lines will also represent the Efficacy of the said Forces in the same Time.

3. This Doctrine of the *Composition and Resolution of Forces* will be found of very frequent Use; and will be farther illustrated in an Application to the Doctrine of *Oblique Percussion*, as follows. Let the Body A im-
 pinge on B at rest in the Direction A C, which be-
 cause it does not go thro' the Center of the Body B,
 the Stroke will not be *Direct*, but *Oblique*; and let the
 Force of the Stroke be represented by the said Line
 A C. Plate IV.
Fig. 12.

4. This Force A C may be resolved into the two Forces A I, and I C; the former of which being parallel to the horizontal Line G F, cannot affect the Body B at all; for the Motion in that Line would only pass by the Superficies of B, and touch the uppermost Point without any Stroke or Force to remove it.

5. But that Part of the Stroke represented by I C, passes thro' the Center of B, and therefore expresses the whole Force with which the Body A strikes it in the Direction A C. *Therefore the Force of an Oblique Stroke is to the Force of a Direct Stroke, as I C to A C, passing thro' the Center of the Body B.*

the Nature of Attraction or Gravity, that if a Stone fall towards the Earth, the

6. Now since $IC=AG$, we have this Analogy, *The direct Stroke is to the oblique Stroke, as AC to AG, that is, as Radius to the Sine of the Angle of Inclination ACG.* Whence, 'tis evident, the less this Angle is, the more oblique the Stroke, and the less its Force, because the less must $AG (=IC)$ be in respect of AC.

7. If the Bodies A, B, are *Non-elastic*, in the Line GF, take $CF=CG=AI$, and this will be the horizontal Velocity of A after the Stroke, which will be the same as before, and since the whole Force IC of the Striking Body A is known, we can find (by *Annot. XXIII. 7.*) what Part thereof remains after the Stroke, which let be express'd by FD, (taken from F to the Right) then drawing CD it shall represent the Motion and Direction of A after the Stroke, while B will go on towards E with the Motion BE generated by the Stroke, (which also may be found as before) and will be equal to FD, if the Bodies A and B are equal.

8. If the Bodies are *Elastic*, and B less than A, the Direction of A after the Stroke, (*viz.* CD) will make an *Acute Angle* with BE the Direction of B, or lie on the Right of CF. If B be greater than A, it will be reflected so that CD will make an *Obtuse Angle* with CE, or lie on the Left of it. But if both the Bodies are equal, the whole Motion of A in the Direction CE will be destroy'd (by *Annot. XXIII. 19.*) and it will proceed with only the horizontal Velocity in the Direction CF, making a Right Angle with CE, the Direction of B.

9. If two Bodies A, B, in Motion, impinge on each other in the Point C in the oblique Directions AC, BC, these are each resolvable into two, *viz.* AE and EC, BD and DC. Now since EC and DC are parallel, they cannot obstruct each other, so will be the same after the Stroke as before it. But AE and BD, being opposite, will express the Forces with which the Bodies strike each other directly, and may be found by the Rules above deliver'd.

Plate IV.
Fig. 13.

Quantity

Quantity of Motion both in the Earth and Stone is the same. That the Iron attracts the Loadstone with an equal Power of Magnetism, is evident by Experiment. That *Action* and *Re-action* are equal between impinging Bodies, or that the same Quantity of Motion that is generated in one Body is destroy'd in the other by the Stroke, whether the Bodies be *elastic* or *non-elastic*, will also be made apparent to the Senses by Experiment. Whence also it will appear, that the Action or Effect of *elastic Bodies* is twice as great, as that of Bodies void of Elasticity (XXV).

10. If then we make $CE=CE$, and $CD=CD$: Also CG =Motion of A after the Stroke, and CF =Motion of B, and compleat the Parallelograms EG , and DF , their Diagonals CA and CB will be the Directions, and express the Velocities A and B after the Stroke. This Construction is general, and may be accommodated to particular Cases, varying with the Magnitudes and Velocities, &c. of the impinging Bodies.

(XXV.) Let Q =Quantity of Matter in the Earth, and V =the Velocity with which it moves by Attraction; and let q and v denote the same Things in the falling Stone; then since the Earth and Stone act mutually on each other by Attraction, the Velocity of each, and consequently the Spaces they describe in the same Time, will be as the Power acting on each Body, which therefore will be inversely as the Quantities of Matter in each: Consequently $Q : q :: v : V$; and so $QV=qv$, or the *Momentum* of the Earth is equal to that of the falling Body.

FROM

FROM this Law we have a Solution of divers *Phænomena* otherwise not to be ac-

2. In like Manner, when a Horse draws a Stone, the Cord being equally stretch'd between both, acts equally upon both, and that which has least Resistance yields, and is drawn along. This is usually the Case of the Stone; but if its Weight be increased, and therewith its Resistance, till it be equal or greater than that of the Horse, then neither Horse nor Stone will move; unless the Stone be laid on a Descent, and then it will move the contrary Way, and draw the Horse after it.

3. In the same Manner we may understand how Rowing, Swimming, Flying, &c. is performed; for the Boat, the Fish, and the Bird, are Bodies easily moveable with the least *Impetus*, but the Water and Air in which they move, tho' fluid Bodies, yet give great Resistance to others which strike them, or re-act with an equal Force, in a contrary Direction; and by this means impel the Boat, the Fish, and Bird in a Direction nearly contrary to that in which they strike it, and with an equal Force.

4. *Mersennus* tells us a Cannon 12 Feet in Length, weighing 6400 lb. gives a Ball of 24 lb. an uniform Velocity at the Rate of 640 Feet per Second. Put $w=6400$, $w=24$, $v=640$, v =Velocity, with which the Cannon recoils. Now since the *Momentum* of the Cannon and

Ball are equal, we have $wv=uv$, and so $v=\frac{wv}{w}=$

$\frac{640 \times 24}{6400}=2, 4$, the Velocity of the Cannon; that is,

it would recoil at the Rate of $2\frac{4}{10}$ Feet per Second, if free to move.

5. But if the Cannon be fix'd, it will receive a Shock or Pressure from the expansive Force of the Powder, equal to the Pressure of a certain Weight, which Weight may thus be found. As the Powder constantly acts on the Ball while in the Cannon, it will drive it along

counted

Fig. 1.



Fig. 2.



Fig. 3.

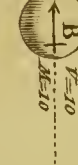


Fig. 4.



Fig. 5.

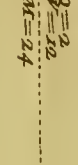
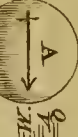


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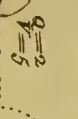
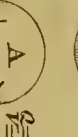


Fig. 7.

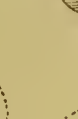


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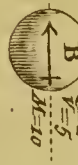
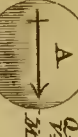


Fig. 8.

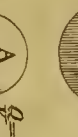


Fig. 9.

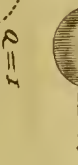
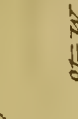
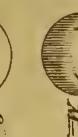


Fig. 9.



Fig. 10.

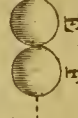


Fig. 11.



Fig. 12.

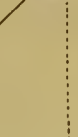
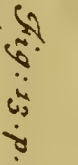
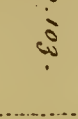
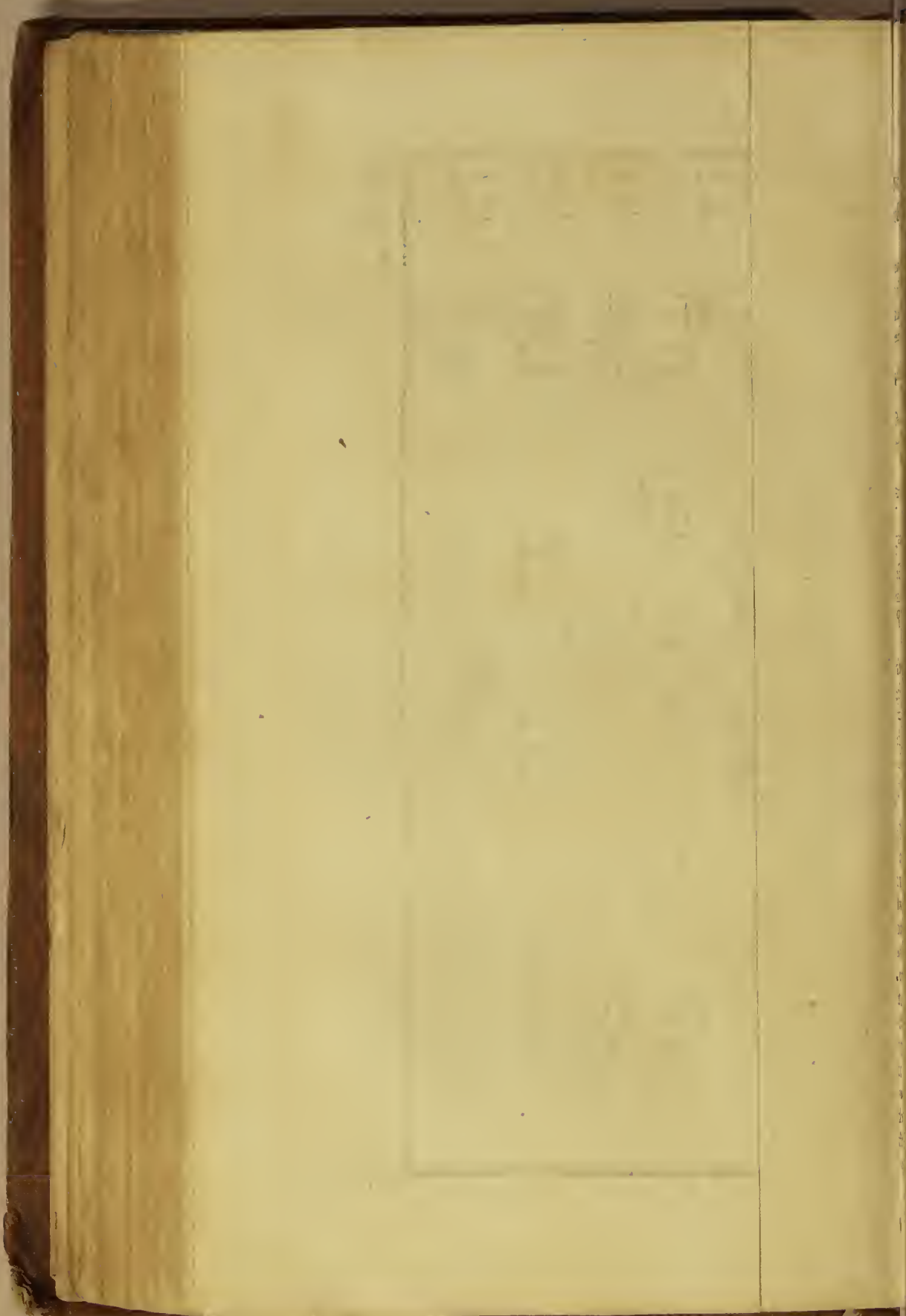


Fig. 13.





counted for : As, why soft and unelastic Bodies *retain the Impressions* of others,

with an accelerated Velocity, which accelerated Velocity will produce an uniform Velocity of 24 Feet in the same Time (as we shall shew hereafter) when free from the Cannon. And since the Ball goes at the Rate of 640 Feet *per* Second, it will pass over 24 Feet in $\frac{1}{26\frac{2}{3}}$

of a Second, which therefore is the Time of its passing thro' the Cannon.

6. Now since in accelerated Velocities the Spaces passed over are as the Squares of the Times, therefore as $\frac{1}{26\frac{2}{3}} : 1''^2$, or as $1''$ is to $26\frac{2}{3}''^2$ ($=711\frac{1}{4}$) so is 12 Feet to $8533\frac{1}{3}$ Feet, which the Ball would be carried through in one Second by the accelerating Force of the Powder.

7. But since the accelerating Force of Gravity gives the Ball a Weight of 24 Pounds, the greater accelerating Force of the Powder will give it the Force of Weight $=12800$ Pounds. For the Weights of Bodies are as the accelerating Forces, and these are the Spaces passed through in the same Time; therefore $16 \text{ F.} : 24 \text{ lb.} :: 8533\frac{1}{3} \text{ F.} : 12800 \text{ lb.}$ as required.

8. But if the Cannon be free to recoil, Part of this Force will be spent in giving it a Velocity of 2,4 Feet in a Second, and to find what Part of the whole Weight 12800 Pounds; that is, we are to consider that as the whole Force produces a Velocity of 640 Feet in a Second one Way, so when the Piece recoils, this Velocity is divided into two Parts in contrary Directions, *viz.* into $640-2,4$ and $2,4$; the whole Force therefore of 12800 Pound must be divided into Parts of the same Proportion; thus, as $640-2,4 : 2,4 :: 637,6 : 2,4 :: 797 : 3 :: 12800-x : x :: 797+3 (=800) : 3 :: 12800 : x=48$ Pounds. Wherefore of the 12800 Pounds only 48 are spent for giving a Recoil to the Cannon, the other 12772 Pounds spend themselves in Pressure on the Gun and Ball, giving it a Velocity of 637,6 Feet *per* Second.

while

while hard and elastic Bodies *rebound with equal Force*, and under *equal Angles*. Hence, the Reason of *Rowing with Oars*, and the *Swimming of Fishes* in Water; also the *Flying of Birds* in Air; the *Recoiling of Guns*; the retro-vertiginous Motion of the *Æolipile*; the *Hydrostatic Paradox*; and many other Things hereafter occasionally mentioned, are accountable for on this and no other Principle.

WE proceed next to consider the *Nature of Motion belonging to Bodies which descend freely by the Force of Gravity in Vacuo*, or an unresisting Medium: And this kind of Motion we shall find affected with the following Properties, *viz.* (1.) That it is equally accelerated. (2.) That its Velocity is always proportional to the Time of the Fall. (3.) That the Spaces passed through are as the Squares of the Times or Velocities. And therefore, (4.) That the Velocity, and consequently the *Momentum*, which is proportional thereto, is as the Square Root of the Space descended through. (5.) That the Space passed through the first Second is very nearly $16\frac{1}{16}\frac{3}{6}$ Feet. (6.) That a Body will pass over twice the Space in an horizontal Direction, with the last acquired Velocity of the

the descending Body, in the same Time (XXVI).

(XXVI.) 1. It has been shewn (*Annot. XXI.*) that the Motion of a Body falling freely by its Gravity is *equably accelerated*; and that its *Velocity is at all Distances proportional to the Time of the Fall*, is evident from hence, that in every equal Particle of Time, the Body receives an equal Impulse from Gravity, which generates an equal Increment of Velocity; which, therefore, as it increases with, must also be proportional to the Times.

2. *That the Spaces passed thro' are as the Squares of the Times or Velocities* is hence evident, that if the Time of the Fall thro' a certain Space be represented by $A1$, and the Velocity acquired in the End of that Time be $1a$; then drawing Aa , the Triangle $A1a$ will represent the Space pass'd thro' in that Time. For if we conceive the Line $A1$ to be divided into an infinite Number of equal Parts. and thro' each of these Divisions, Lines drawn parallel to $1a$, these will represent the Velocities in the several Particles of Time. Now the Space described in each Moment will be as the Velocity (because for a Moment the Velocity may be considered as uniform) consequently the Sum of all the Spaces described in the Moments in the Time $A1$ will be as the Sum of all the Velocities, or Lines parallel to $1a$, which represent them: But the Sum of all these Lines make up the Area of the Triangle $A1a$, therefore the whole Space pass'd thro' in the Time $A1$ will be as the Area of the Triangle $A1a$.

Plate III.
Fig. 5.

3. Therefore the Triangle $A2b$ represents the Space pass'd thro' in the Time $A2$, and the Triangle $A3c$ the Space pass'd thro' in the Time $A3$, and so on. But the Triangles $A1a$, $A2b$, &c. are similar, and therefore are to each other as the Squares of their Sides $A1$, $A2$, or $1a$, $2b$, &c. That is, *The Spaces are as the Squares of the Times or Velocities.*

4. Consequently the *Velocities and Times of the Fall are as the Square Roots of the Spaces pass'd thro'*; and since

HENCE it follows, that if one Leg AB of a right-angled Triangle represent the

the *Momentum* of a Body is always $M=QV$, and in this Case Q being a given Quantity, we have $M : V$, or the *Momentum of the Body as the Velocity, or as the Square Root of the Height thro' which the Body falls.*

5. It has been found by very accurate Experiments made by letting Bodies of various Sorts fall from the Height of the Dome of St. Pauls, to the Pavement, that Gravity accelerates Bodies in the beginning of their Fall, at the Rate of 16,13 Feet in the first Second of Time. This may also be otherways shewn *a priori*, because we demonstrate that the Time (T) of the Vibration of a Pendulum (in the Arch of a Cycloid) is to the Time (t) of a Body's falling thro' Half the Length of the Pendulum ($\frac{1}{2}L$) as the Circumference of a Circle (P) to the Diameter (D). (*Annotat. XXVIII. 9.*)

6. Again, it is found that the Length of a Pendulum vibrating Seconds is 39,2 Inches. Now since $T : t :: P : D$; 3,14159 : 1, we have $T^2 : t^2 :: 314159^2 : 1^2 :: x : \frac{1}{2}L$; whence we have $x = \frac{1}{2}L \times 3,14159^2 = 16$ Feet = the Space descended in the Time of one Vibration, or 1 Second.

Plate III. 7. Since the Space pass'd thro' by an uniform Motion
Fig. 5. in the Time AB with the Velocity BC is $AB \times BC = ABCD = 2ACB$; it is evident that a Body moving uniformly with the last acquired Velocity BC of a descending Body, will move thro' twice the Space in the Time AB of the Fall.

8. It is farther evident, that since the Spaces descended thro' in each Second are as the odd Numbers, 1, 3, 5, 7, 9, 11, 13, 15, &c. and these Numbers constantly approach nearer and nearer to an Equality, so the accelerated Motion of the Descent does by Degrees approach nearer and nearer to an uniform Motion; thus the Spaces 101, 103, 105, differ but little; and the Spaces 1001, 1003, 1005, differ less; but when we come to the Spaces 1000000001, 1000000003, 1000000005,

Time

Time of the Fall, and the other Leg BC the *Velocity* acquired at the End of the Fall; then will the Area ABC of the Triangle represent the Space pass'd through. And hence the Spaces descended through at the End of every Second will be as the Square Numbers 1. 4. 9. 16. 25. 36. &c. and therefore the Spaces pass'd through in each Second separately will be as the odd Numbers 1. 3. 5. 7. 9. 11. 13. 15. &c. as in the *Figure*.

THE next Sort of Motion is that of Bodies descending on *inclined Planes*, and *curved Surfaces*, which we find distinguished with the following Properties. (1.) The Motion on the inclined Planes is equally accelerated, as arising from Gravity. (2.) The Force of Gravity compelling a Body, as A, to descend on an inclined Plane BD, is to the absolute Force of Gravity as the Height of the Plane BC to its Length BD. (3.) The Spaces descended

Plate V.
Fig. 1.

these are so very near equal, that the Motion may now be esteem'd uniform.

9. If the Descent be in a *resisting Medium*, the Motion will actually become uniform at a certain Distance, and the sooner as the Medium is denser; thus a Body falling in Air will be longer in acquiring an uniform

are

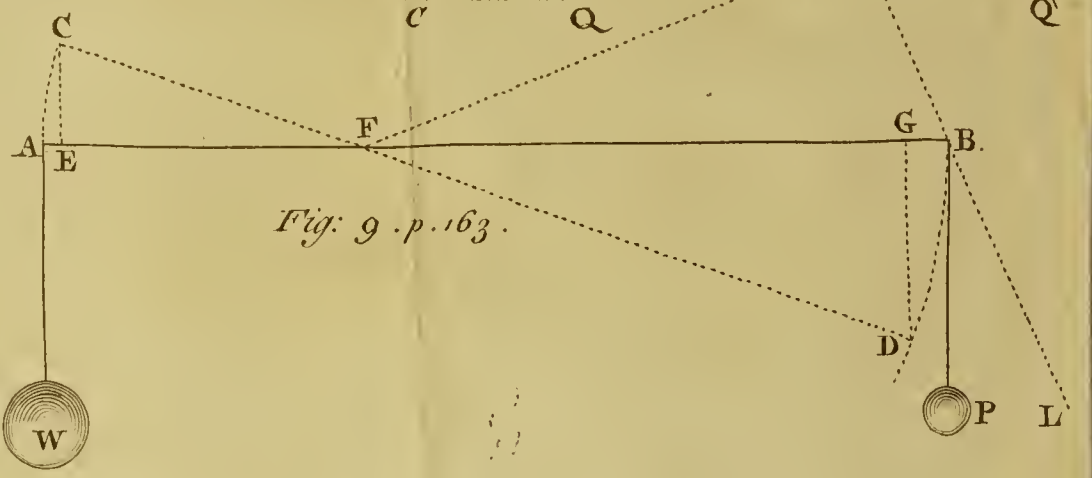
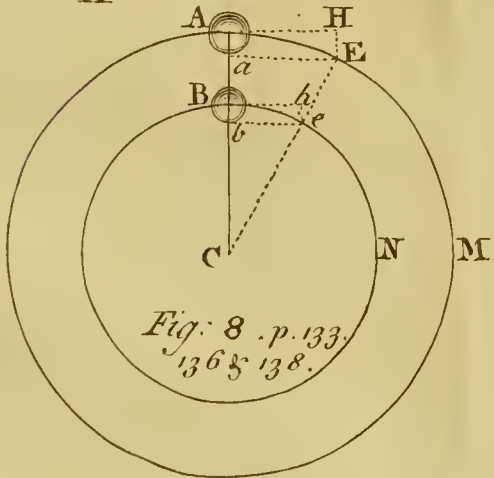
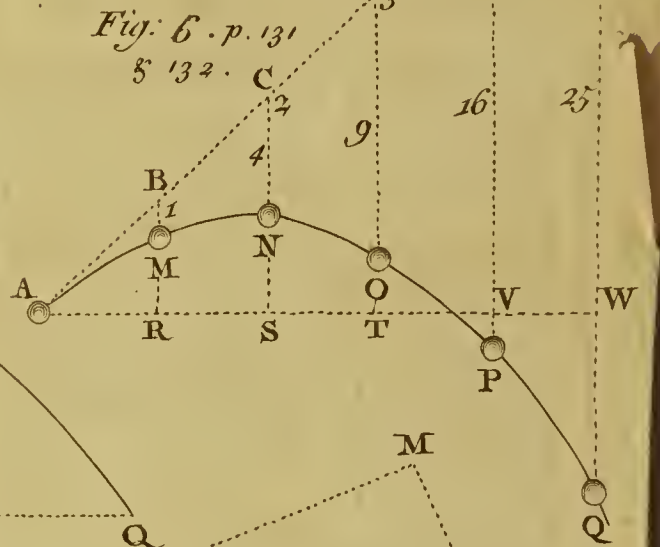
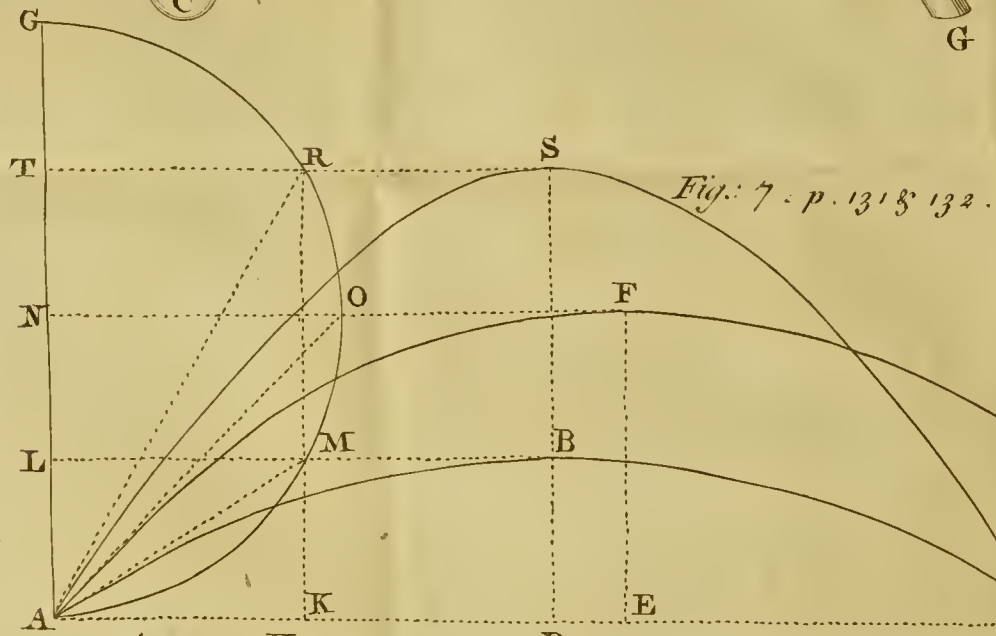
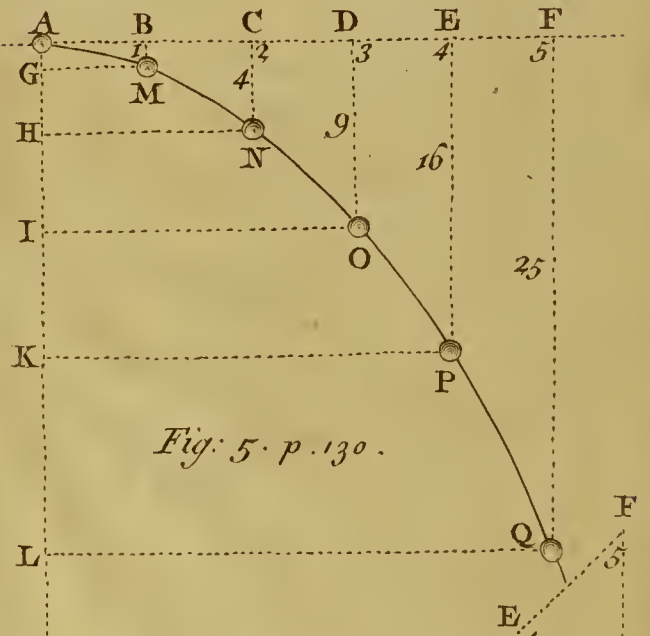
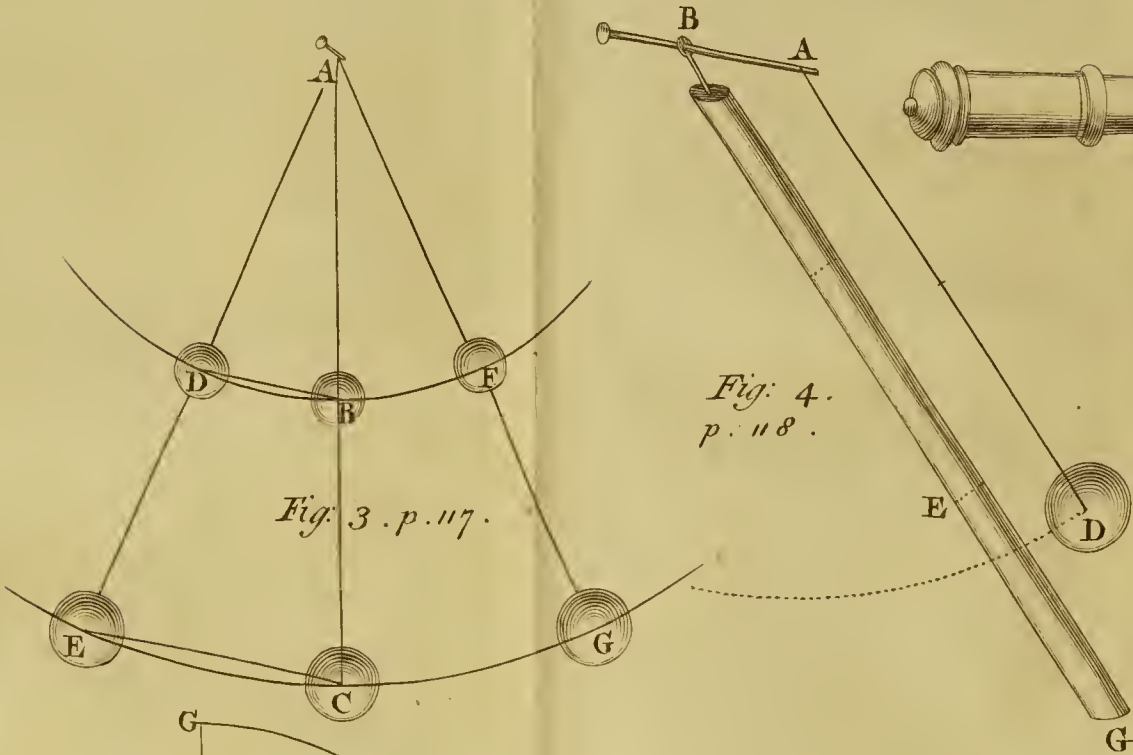
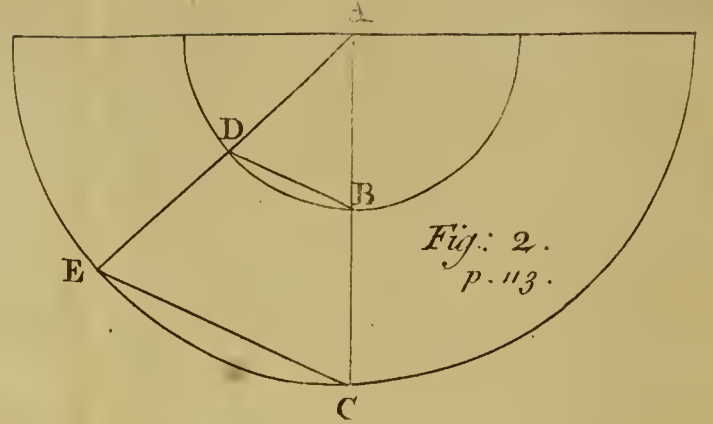
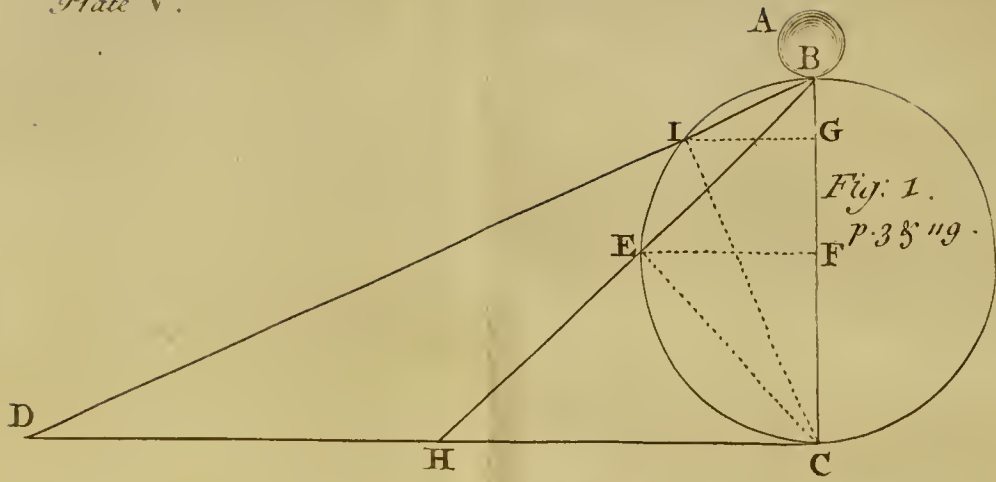
are as the Squares of the Times. (4.) The Times in which different Planes, BD, BH, of the same Altitude BC, are passed over, are as their Lengths respectively. (5.) The Velocities acquired by descending through such Planes at the lowest Points, DH, are all equal. (6.) Therefore if a

Motion than in Water, and in Quicksilver it will soon obtain it; because as the Density increases, so does the Resistance, and consequently the Increments of Velocity are annihilated in the same Ratio, and the Motion reduced to Uniformity.

10. Bodies of the same Matter and Figures will sooner come to an uniform Motion as the Magnitudes are lesser; for 'tis shewn (in Geometry) that the Quantities of Matter decrease in Bodies with the Cubes of the Diameters, but the Surfaces decrease only with the Squares of the Diameters. And since the Resistance is proportional to the Surfaces of Bodies, moving in the same Medium with the same Velocity, it will follow that smaller Bodies will be more resisted than larger ones, and so come sooner to an uniform Motion. Hence it is, that a Body reduced to Powder descends very slowly and with nearly an uniform Motion in Water, tho' in the Solid much heavier than Water. Also a Bullet shot from a Gun will go much farther and with a greater Velocity than a Charge of small Shot of the same Weight, or a Ball of Cork of the same Size, which *in Vacuo* would go much farther than the Bullet, as admitting a greater Velocity with a less Quantity of Matter, from the same *Momentum* of the Powder.

11. If Bodies (equal and homogeneous) move in the same Medium with different Velocities, those which move fastest will soonest acquire an uniform Motion, since (as will be shewn) the Resistance in such a Case increases with the Squares of the Velocity. If the Velocity with which a Body is projected downwards be
Body

RPJCB



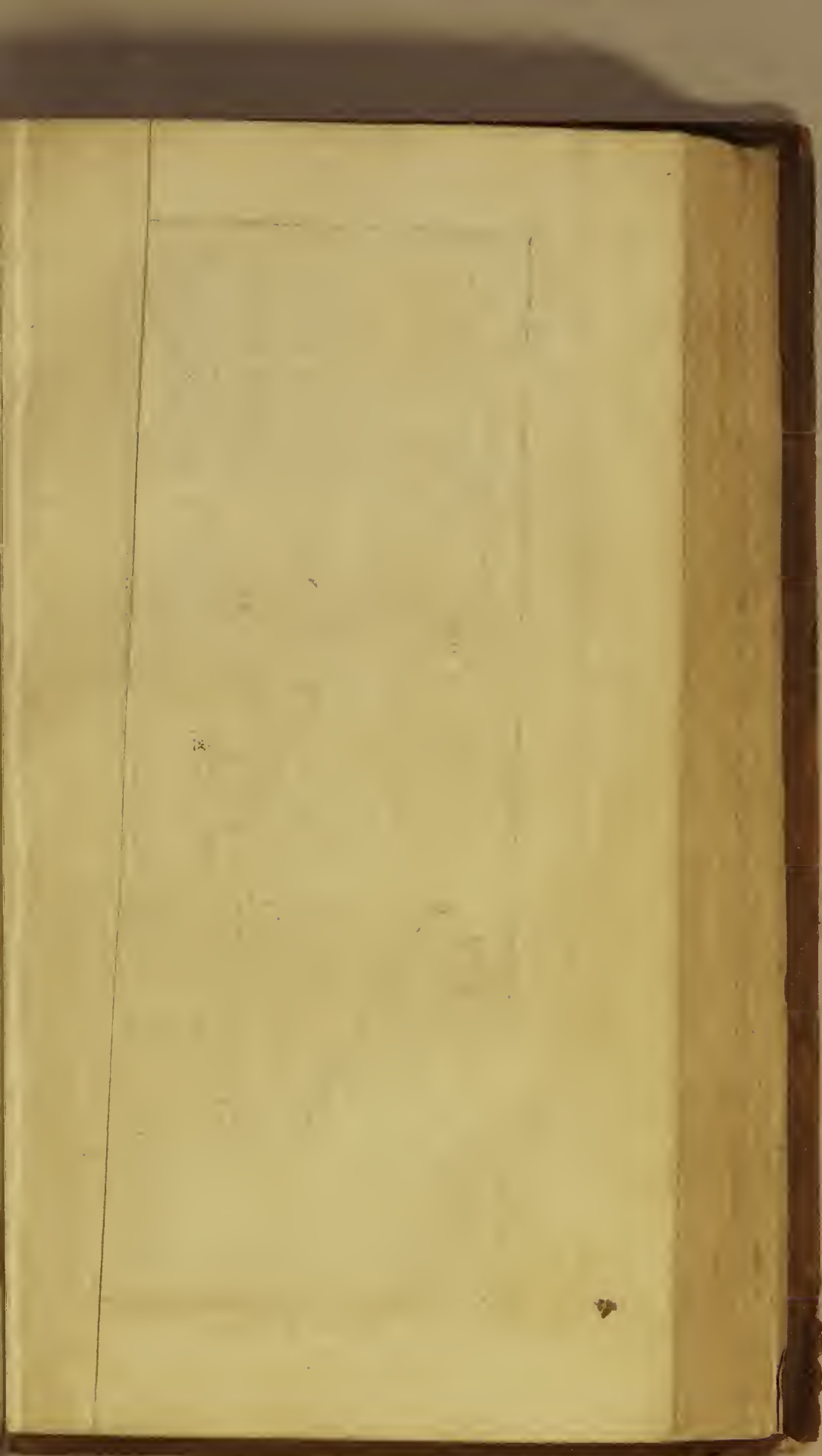


Fig. 1. p. 113.

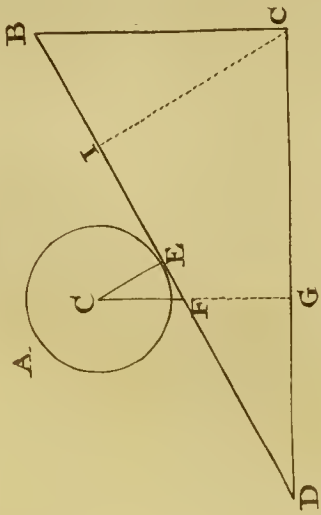


Fig. 3. p. 116.

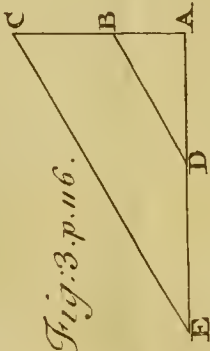


Fig. 6.
p. 122.

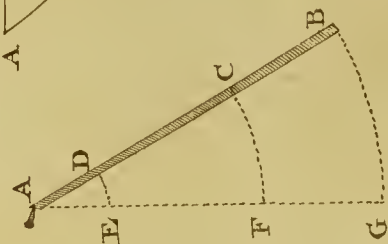


Fig. 2. p. 115.

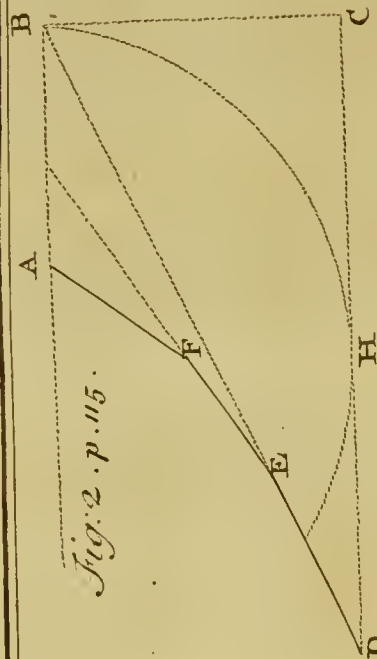


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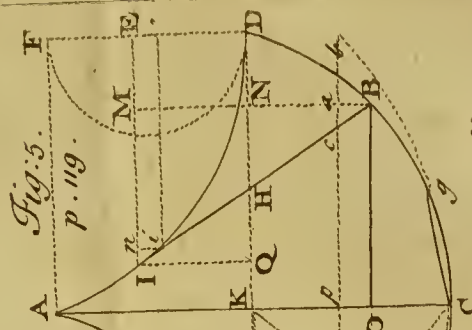


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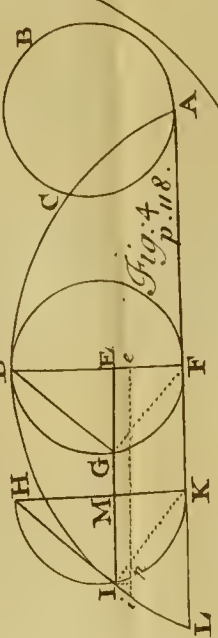


Fig. 7. p. 128.

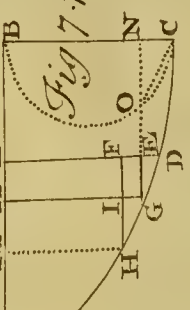


Fig. 9. p. 134.

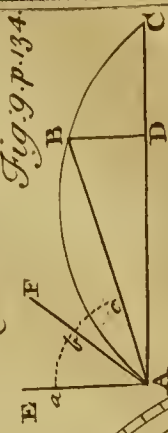


Fig. 8. p. 125.

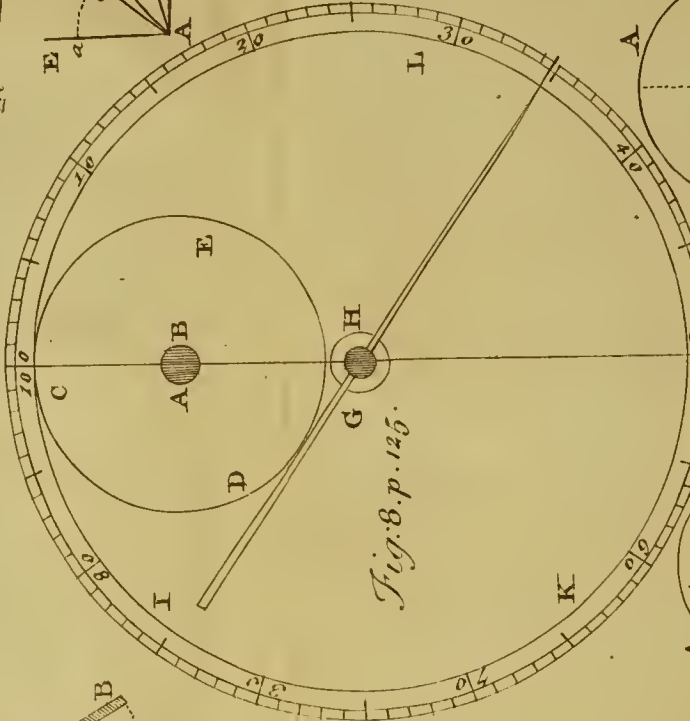


Fig. 10. p. 134.

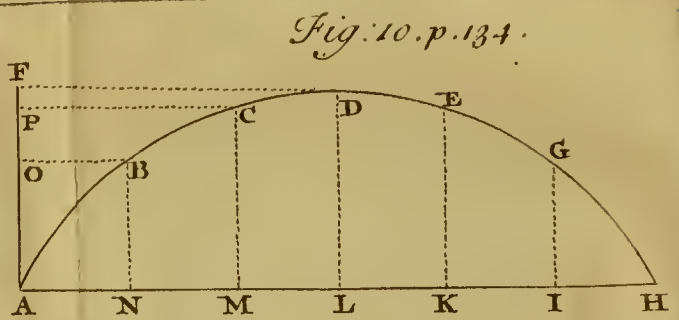


Fig. 13. p. 150.

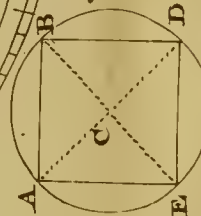


Fig. 14

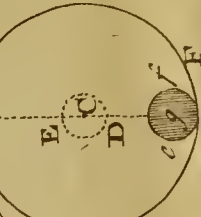


Fig. 12.
p. 142.

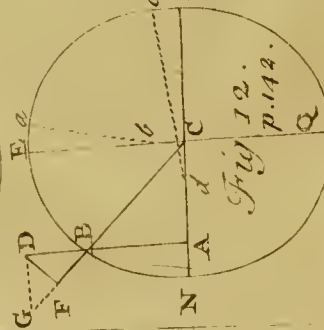
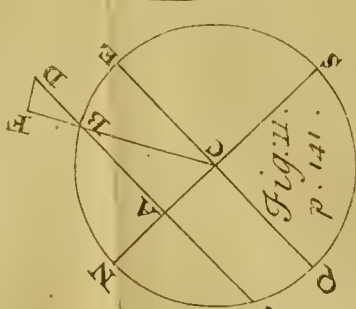


Fig. 11.
p. 141.



Body descends from the same Height thro' several contiguous Planes, ever so different in Number or Situation, it will always acquire the same Velocity in the lowest Point. (7.) The Times or Velocities of Bodies descending through Planes similarly situated, or alike inclined to the Horizon, as DB, EC, will be as the Square Roots of their Lengths DB, EC (XXVII.) Fig. 2.

equal to the uniform Motion of a free Descent, the Motion will be every where uniform, because the Resistance of the Medium is equal to Gravity. But if a Body (as a Ball from a Cannon) be projected with a Velocity greater than that, the Motion will be retarded, inasmuch as the Resistance is now greater than Gravity, and so will diminish the uniform Motion, arising from the Equilibrium just mentioned.

12. Hence light Bodies (as an Arrow) thrown directly upwards, spend less Time in rising than falling, because the Motion upwards is altogether retarded, whereas that of the Descent is in some Part uniform. Also the greater the Velocity with which a Body (as a Bullet) enters a dense Medium (as Water) the less the Effect at a given Distance therein; and with the greater Force they are reflected or thrown out again, when projected obliquely, as we see in a Stone thus thrown into the Water, &c.

(XXVII.) Let BD be an inclined Plane, A a Body descending thereon; from the Centre C draw CE perpendicular to the Plane, and CG perpendicular to the Horizon or Base CD, and meeting the Plane in F. Now since the Body gravitates in the Direction CF, let CF represent the Weight of the Body or absolute Force of Gravity; this may be resolv'd into two Forces, acting in the Direction CE and EF, of which the first is perpendicular to the Plane, and is that by which it

Plate VI.

Fig. 1.

FROM these Properties of Bodies descending on Inclined Planes, we deduce the following Corollaries, *viz.* (1.) That the Times in which a Body descends through

presses or acts upon the Plane; but since the Plane equally acts upon the Body in a contrary Direction, that Force CE is wholly destroy'd. The Remainder therefore, EF, is that which carries the Body down the Plane, as acting in a Direction parallel thereto; and this is call'd the relative Gravity.

2. Now because of the similar Triangles CFE, DFG, DBC, we have $CF : FE :: DF : FG :: BD : BC$; that is, *the absolute Gravity CF is to the Residual or Relative Gravity FE, as the Length of the Plane BD to the Height thereof BC.*

3. Also since the Triangles BDC and BCI are similar (CI being drawn perpendicular to BD); and since the Spaces describ'd in the same Time in the Perpendicular and on the Plane will be as the Powers of Gravity, or accelerating Forces in those Directions, that is, as BD to BC, or as BC to BI (for $BD : BC :: BC : BI$). Therefore BI will be the Distance descended on the Plane in the same Time as a Body would descend freely in the Perpendicular from B to C.

4. Since the Motion on the Plane is accelerated, the Time of describing BI will be to the Time of describing BD, as $\sqrt{BI} : \sqrt{BD} :: t : T$; and so $t^2 : T^2 :: BI : BD$; but, (because $BI : BC :: BC : BD$) we have $BI : BD :: \overline{BI}^2 : \overline{BC}^2$; therefore $t^2 : T^2 :: \overline{BI}^2 : \overline{BC}^2$; and so $t : T :: BI : BC :: BC : BD$; consequently the Time of describing BI or BC will be to the Time of descending through the Plane BD, as the Height of the Plane BC to its Length BD.

5. Hence the Time (T) of descending thro' any other Plane BH of the same Height (*Fig. 6.*) is to the Time (t) of the Descent thro' the Perpendicular BC, as BH to BC; that is, $T : t :: BH : BC$; but it is also $T : t :: BD : BC$; therefore *ex æquo*, we have $T : T ::$
the

the Diameter BC, or any Chord BE of a Circle, are equal. Hence, (2.) All the Chords of a Circle are described in equal Times. (3.) The Velocities acquired in

BD: BH; or the Times of descending thro' several Planes BD, BH, of the same Height, are as the Length of the Planes.

6. The Velocity acquired in falling to C, is to the Velocity at I as BC to BI, (for the Velocities generated in the same Time, must be as the Powers which produce them). Also the Velocity at D is to that at I as $\sqrt{BD}$ to the $\sqrt{BI}$; or $V:V::\sqrt{BD}:\sqrt{BI}$; and so $V^2:V^2::BD:BI::\overline{BC}^2:\overline{BI}^2$; therefore $V:V::BC:BI$; since then the Velocity at C and D have both the same Ratio to the Velocity at I, they must be equal to each other.

7. In the same Manner it is shewn that the Velocity at H is equal to the Velocity at C; and consequently the Velocities acquired in descending thro' Inclined Planes of the same Height are always equal.

8. Hence a Body descending thro' several Inclined Plate VI. Planes AF, FE, ED, contiguous to each other, will Fig. 2. have the same Velocity at the Point D, as it would have acquired in falling freely through the same perpendicular Height BC, for the Velocity at F is the same it would have had by descending thro' GF, which has the same Altitude; and the Velocity at E is the same it would have gained by descending thro' BE; consequently the Velocity of D is the same it would acquire by descending thro' BD, and therefore the same it would have by falling freely thro' the Perpendicular BC.

9. If now we suppose the Number of those contiguous Planes infinite, and their Lengths infinitely small, they will then constitute a *Curve Line*; whence it follows, That a Body descending thro' the Arch of any Curve BH will acquire the same Velocity in the lowest Point H, as it would have at C, by falling thro' the same perpendicular Height BC.

descending through any Arch of a Circle EC, in the lowest Point C, is equal to that which would be acquired in falling through the same perpendicular Height FC. (4.) The Velocities acquired in descending through the Chords IC, EC, of a Circle,

Plate VI.
Fig. 3.

10. If there are two Planes, BD and CE, similarly situated, or alike inclined to the Horizon AE; the Time of the Descent on DB, will be to the Time of the Descent on CE, as Square Roots of their Lengths; for the Time on BD is to the Time thro' BA, as DB to BA (by Art. 4.) and the Time on CE is to the Time thro' CA, as CE to CA; but the Times thro' BA and CA are as the Square Roots of BA and CA; and since by similar Triangles we have $\sqrt{BA} : \sqrt{CA} :: \sqrt{BD} : \sqrt{CE}$, it follows, *That the Times of Descent on two similar Planes DB and CE are as the Square Roots of their Lengths, or as $\sqrt{BD}$ to $\sqrt{CE}$.*

Plate V.
Fig. 1.

11. From what has been demonstrated in the preceding Articles, it follows, (1.) That the Times of descending from B to I, and to E, are equal, being each equal to the Time of Descent thro' the Perpendicular BC. (2.) For the same Reason the Descent thro' IC and EC, and consequently thro' all the Chords of a Circle, are equal. (3.) The Velocities at C acquired by the said Descents are as the Lengths of the Chords IC and EC. For since $BC : EC :: EC : CF$, we have $BC = \frac{EC^2}{FC}$; and for the same Reason $BC = \frac{IC^2}{GC}$, whence $\frac{EC^2}{FC} = \frac{IC^2}{GC}$; and therefore $IC : EC :: \sqrt{GC} : \sqrt{FC}$; that is, as the Velocities in the Descents thro' IC and EC. (4.) Lastly, since the Times of Descent through the Planes DB and EC are as $\sqrt{DB}$ to $\sqrt{EC}$, they will also be as $\sqrt{BA}$ to $\sqrt{CA}$; for $DB : EC :: BA : CA$.

are,

are, at the lowest Point C, as the Lengths of those Chords. (5.) The Times of Descent through Chords of similar Arches DB, EC, are as the Square Roots of the Semidiameters AB, AC, of the respective Circles.

FROM these Properties, and their Corollaries, the Doctrine of PENDULUMS is derived. Plate V.
Fig. 3.

A PENDULUM is any Body B, suspended upon, and moveable about a Point A, as a Centre. The Nature of a Pendulum consists in the following Particulars. (1.) The Times of the Vibrations of a Pendulum in very small Arches are all equal. (2.) The Velocity of the Bob in the lowest Point will be nearly as the Length of the Chord of the Arch which it describes in the Descent. (3.) The Times of Vibration in different Pendulums, AB, AC, are as the Square Roots of their Lengths. (4.) Hence the Lengths of Pendulums AB, AC, are as the Squares of the Times of their Vibrations. (5.) The Time of one Vibration is to the Time of Descent through half the Length of the Pendulum, as the Circumference of a Circle to its Diameter. (6.) Whence the Length of a Pendulum vibrating Seconds will be found 39 Inches nearly; and of an *Half-*

I 3

Second

Second Pendulum 9,8 Inches. (7.) An uniform homogeneous Body BG, as a Rod, Staff, &c. which is *one third Part* longer than a Pendulum AD, will vibrate in the same Time with it (XXVIII.) (8.) This

(XXVIII.) 1. As the Doctrine of Pendulums and Time-keeping Instruments depends in a great Measure upon the *Cycloid*, I think it necessary here to shew the Nature and Use of that Curve, with regard thereto. If a Circle ABC, inscribing on a right Line AL, begin to revolve in the Manner of a Wheel, from A towards L, the Point A will by its twofold Motion describe the Curve ACDIL, while the Circle makes one Revolution from A to L.

Plate VI.
Fig. 4.

2. This Curve is called the CYCLOID, and from the Definition 'tis evident (1.) That the *Base* of the Cycloid AL is equal to the Periphery of the *generating Circle* ABC. (2.) The Axis of the Cycloid FD is equal to the Diameter of the said Circle. (3.) That the Part of the Base KL is equal to the Arch of the Circle IK. (4.) Therefore KF (=ME=IG) is equal to the remaining Arch IH, or GD. (5.) That the Chord of the Circle KI is perpendicular to the Cycloid in the Point I; and (6.) Therefore the Chord HI (being at right Angles with IK) is a Tangent to the Curve in the Point I. (7.) The said Tangent HI is parallel to the Chord DG.

3. Parallel to EI draw *ei* indefinitely near, and *In* perpendicular thereto; then will the Triangles DGE, DGF, *Ini*, be similar, and so we have DE : DG :: DG : DF :: In : Ii; that is, (putting DF=*a*, and DE=*x*,

$$DI=z,) x : \sqrt{ax} :: \sqrt{ax} : a :: \dot{x} : \dot{z} = Ii = \frac{a \dot{x}}{\sqrt{ax}} =$$

Fluxion of the Arch DI, whose Fluent is $2\sqrt{ax} = 2 DG =$ the Arch DI; and consequently, the Semicycloid DIL = 2 DF, the Diameter of the generating Circle.

4. Let

4. Let AID be a Semicycloid inverted; and suppose a flexible String fasten'd at one end in A, and stretch'd along the said Curve AID, so that the other End of the String should be made to coincide with the Point D; whence the Length of the String will be equal to that of the Curve: If now the End of the String be taken from D, and with a tight hand be drawn from the Cycloid, it will, in its Evolution, describe the Curve DBC, the Nature of which we are now to investigate.

5. Let the String, in evolving, be in any Situation AIHB, then is IB (called the *Radius of Evolution*) equal to the Part of the Cycloid ID. Put $FD=AK=a$, $DE=x$, $CO=y$, $CB=z$, then is $ni=\dot{x}$, $Ba=Op=\dot{y}$, $Bb=\dot{z}$; and (by Art. 3.) $DI=(IB=) 2\sqrt{a-x}$, whose

Fluxion is $\dot{x} \sqrt{\frac{a}{x}} = Ii$; therefore $\dot{x} \times \sqrt{\frac{a}{x}} - \dot{x}^2 =$

$$(\overline{Ii^2} - \overline{i n^2}) = \frac{\dot{x}^2 a - x \dot{x}^2}{x} = \overline{In^2}, \text{ whence } In =$$

$$\dot{x} \sqrt{\frac{a-x}{x}}; \text{ but } In : Ii :: ca : cB :: aB : Bb = \dot{z} =$$

$$\frac{\sqrt{a} \times \dot{y}}{\sqrt{a-x}}; \text{ and } Ii : ni :: IB : MB = 2x; \text{ therefore}$$

$BM - BN = MN = IQ = ED = x$. Consequently $BH = HI$; and also $CK = KA$, because all the Radii of Evolution (of which AC is the last) are bisected by the base LD; therefore $CK - OK =$

$$a-x = OC = y; \text{ whence } \sqrt{y} = \sqrt{a-x}, \text{ and so } \dot{z} =$$

$$\frac{\sqrt{a} \times \dot{y}}{\sqrt{y}}; \text{ therefore } z = 2\sqrt{ay}; \text{ consequently the}$$

Curve CBD, having the same Property with AID, is every way equal and similar thereto.

6. Whence it appears that if AC be the Length of a Pendulum so disposed, as to vibrate between the two Semicycloids AD and AL, the Bob will describe in its Motion the Cycloid DCL. The Properties of which Motion will now easily appear: For with respect to the

Velocity acquired by descending thro' any Arch RC or BC , it is always as $\sqrt{OC}$ which is as the Chord $SC = \frac{1}{2}RC$; consequently the Velocity at C is every where as the Space passed through, or as the Arch of the Cycloid described in the Descent.

7. But since it is always $S = TV$, in all kind of Motions; if in any Case S be as V , 'tis evident T must be a given Quantity, or always the same. That is, When the Spaces (S, s ,) are as the Velocities (V, v ,) the Times (T, t ,) will be always equal; and therefore all the Arches of a Cycloid, great or small, are described in equal Times.

8. If we put $CK = a$, $KO = x$, then will $2SC = RC = 2\sqrt{aa - ax}$; if the Descent be from L to R , the Velocity at R will be $\sqrt{x} = \sqrt{OK}$. Now $CO :$

$$CS (:: CS : CK) :: Rq : Rr = \frac{a\dot{x}}{\sqrt{aa - ax}} = \dot{z}, \text{ the}$$

Fluxion of the Arch LR , which divided by the Velo-

$$\text{city } \sqrt{x} \text{ gives the Fluxion of the Time} = \frac{a\dot{x}}{\sqrt{aux - axx}}$$

$$= \frac{a\dot{x}}{\sqrt{a} \times \sqrt{ax - xx}}; \text{ but } \frac{a\dot{x}}{2\sqrt{ax - xx}} \text{ is the Flu-}$$

xion of the circular Arch KS ; therefore the Fluent of

$$\text{the Time of Descent thro' } LR \text{ is the Arch } \frac{2KS}{\sqrt{a}};$$

wherefore when S coincides with C , LR will become LC ; and so the Time of Descent thro' the Semicyc-

$$\text{loid will be } \frac{2KSC}{\sqrt{a}}.$$

9. Therefore the Time of Vibration thro' the whole Cycloid LCD is $\frac{4KSC}{\sqrt{a}}$; and the Time of Descent

thro' the Perpendicular $KC = a$, being as $2\sqrt{a}$, we

$$\text{have } \frac{4KSC}{\sqrt{a}} : 2\sqrt{a} :: 2KSC : a. \text{ The Time of a}$$

Vibration

Vibration in the Cycloid is to the Time of Descent thro' half its Length, as the Circumference of a Circle to its Diameter, or as 3,14159 to 1.

10. If A C be the Length of a Pendulum vibrating Seconds, it may be easily found, since it is known by most accurate Experiments, that a Body descends freely by Gravity thro' $193\frac{1}{2}$ Inches in the first Second of Time; therefore by making this Analogy, as $3,14159^2 : 1^2 :: 193\frac{1}{2} : 19,6 = AK$, and so A C = 39,2 Inches, *the Length of a second Pendulum, as requir'd.* And since the Lengths are as the Squares of the Times of Vibrations, we have $4 : 1 :: 39,2 : 9,8$ Inches, the Length of an half-second Pendulum.

11. Since the Length of a Pendulum A C, for vibrating Seconds, is so considerable, the Bob, without the Cycloidal Checks A L and A D, will indeed describe the Arch of a Circle, as $f C e$, when it vibrates, but then this Circular Arch will, for some Distance on each Side the lowest Point C, coincide with the Cycloid very nearly; as for Example, to g and h ; and hence it follows that a common Pendulum, vibrating through very small Arches $g h$, will perform all its Vibrations in equal Times. And hence the above-mentioned Cycloidal Checks came into Disuse.

12. Hence the Time of Descent through the Chord Cg is to the Time of Descent through the Arch of the Circle or Cycloid belonging to it as $\sqrt{4a}$ (supposing A C = $2a$) to $\frac{\frac{1}{2}P}{\sqrt{a}}$, or as $2a$ to $\frac{1}{2}P$, that is, as a to $\frac{1}{4}P = \frac{3,14159}{4} = 0,7854$, or as 1 to 0,7854, *which is the Proportion of a Square to its inscribed Circle.*

13. In all that has been hitherto said, the Power of Gravity has been supposed constantly the same. But if the said Power varies, the Lengths of Pendulums must vary in the same Proportion, in order that they may vibrate in equal Times; for we have shewn that the Ratio of the Times of Vibration and Descent through half the Lengths is given, and consequently the Times of Vibration and Descent through the whole Length is given: But the Times of Vibration are supposed equal,

Centre of Oscillation E in the Rod, is also the *Centre of Percussion*, or that Point in which the Force of the Stroke is the greatest possible (XXIX).

therefore the Times of Descent thro' the Lengths of the Pendulums are equal. But Bodies descending thro' unequal Spaces in equal Times are impell'd by Powers that are as the Spaces described, that is, *the Powers of Gravity are as the Lengths of the Pendulums*. See Aunot. XXVII. Art. 2. and 3.

Plate VI. (XXIX.) 1. If A B be an uniform Rod vibrating about the Point A, the Velocities of every Point D, C, B, will be as the Arches D E, C F, B G described in the same Time; but those Arches are as A D, A C, A B; consequently *the Velocity of every Point is proportional to its Distance from the Center of Motion A*.
 2. Therefore if x represent the Length of the Line, it will represent also the Sum of all the Velocities, of which $\dot{x}$ will be the Fluxion; therefore $x \dot{x}$ will be the Fluxion of the *Momenta*, and so the Fluent $\frac{x^2}{2}$ will be the Sum of all the *Momenta*, which divided by the Quantity of Matter x , will give $\frac{x}{2}$ for an Expression of all the Velocities on each Side the common Center of Gravity of all the Points, which therefore is in the middle Point of the Line A B.
 3. In regard of striking an Object, the *Momentum* of each Particle will be increas'd in Proportion to its Velocity and Distance from the Center A, and this is call'd the *Force* of the Particle; therefore if the Fluxion of the *Momenta* $x \dot{x}$ be multiplied by the Velocity x , the Product $x \dot{x} x$ will be the Fluxion of the Forces of all the Particles: The Fluent of which $\frac{x^3}{3}$ will be the Sum of all the Forces, which divided by the Sum of all the *Momenta* $\frac{x^2}{2}$ will give $\frac{2}{3} x =$ Distance of the Center of Forces from the Point A. In this Center C, therefore, the

FROM

FROM these Properties of the Pendulum we may discern its Use as an *universal* CHRONOMETER, or Regulator of Time, as it is used in Clocks and such-like Machines. By this Instrument also we can measure the Distance of a Ship, by measuring the Interval of Time between the Fire and the Sound of the Gun; also the Distance of a Cloud, by numbering the Seconds or Half-Seconds between the Lightning and Thunder. Thus, suppose between the Lightning and Thunder we number 10 Seconds; then, because Sound passes through 1142 Feet in one Second, we have the Distance of the Cloud equal to 11420 Feet. Again; the Height of any Room, or other Object, may be measured by a Pendulum vibrating from the Top hereof. Thus, suppose a Pendulum from the Height of a Room vibrates once in three Seconds; then say, As 1 is to the

force of all the Particles will be united, and consequently this Point, and no other, will strike with the force of the whole Rod, which is therefore called *the Center of PERCUSSION*.

4. This Point is also the *Center of Oscillation or Vibration*; for since it strikes with the Force of all the particles in the Line A B, it will move in the same Manner as if all the Particles were in that Point collected together, that is, it will oscillate or vibrate in the same time with a single Pendulum, whose Length is equal to that Distance, *viz.* $\frac{2}{3}$ of A B.

Square

Square of 3, *viz.* 9, so is 39,2 to 352,8 Feet, the Height required. Lastly, by the Pendulum we discover the different Force of Gravity on diverse Parts of the Earth's Surface, and thence the *true Figure of the Earth* (XXX).

(XXX.) 1. Since it is found by Experiment, that a Pendulum under the Equator vibrating Seconds is shorter by $\frac{1}{10}$ of an Inch than such an one in our Latitude; it follows, that the Gravity under the Equator is to the Gravity with us, as 391 to 392; as being proportional to the Lengths of the Pendulums (*Annotat. XXIII. Art. 13.*) Whence it is evident the Surface of the Earth is farther distant from the Centre under the Equator than in our Latitude, and therefore is not Spherical. What the true Figure is, and whence it proceeds, will be shewn hereafter.

2. To the Uses of a Pendulum above-mention'd may be added another most considerable, *viz.* *That a Pendulum is an universal Standard of Measure*, because the Length of any Pendulum is known by the Time of its Vibration; and therefore may be compared with the Length of a Second Pendulum as a Standard. Thus suppose a *Japonese* was enquiring the Length of the *English* Foot, and was told it made 100 Vibrations in the Time a Second Pendulum made 55, he would immediately know the Time of a single Vibration in each must be inversely as those Numbers, *viz.* as 55 to 100; and then, that the Lengths of those Pendulums were as the Squares of the Times, *viz.* as $100^2 : 55^2 :: 39,2 : 12 =$ the Length of one Foot.

3. Hence, if the Length of a Second Pendulum were made to consist of 1000 equal Parts, the Lengths of the Measure made use of by all Nations might be compared therewith, and express'd in those Parts; thus the *English* Foot would be found equal to 306,9 nearly for as $39,2 : 12 :: 1000 : 306,9 \text{ fere.}$ And thus for any other. If this Method of ascertaining the Mea-

THE

THE greatest Inconvenience attending this most useful Instrument is, that it is constantly liable to an Alteration of its Length from the Effects of *Heat* and *Cold*, which very sensibly expand and contract all Metalline Bodies, as will be very evident by the *Pyrometer* (XXXI).

asures were used by all Nations, it would entirely preclude all Doubts and Obscurity in this Affair to Posterity.

(XXXI.) 1. The PYROMETER which I have contrived to shew the Extension of heated Metals, &c. is perhaps the most simple, exact, and easy, that the Nature of such a Machine will admit; and the Manner of computing its Effect is as follows:

Plate VI.
Fig. 8.

2. A B is the Diameter of the Axis or Spindle on which the heated metalline Bar is laid, and is moved by its Extension; on the other End is fixed a small Wheel C D E, which is connected by a filken Thread to a small Pinion Wheel G H, which on its Extremity carries a fine Index over the Divisions of a large graduated Circle I K L.

3. Now admitting the metalline Rod extends $\frac{1}{1000}$ Part of an Inch, 'tis plain the Circumference of the Axis A B, on which it lies, must be moved through the same Space; and supposing the Wheel D E to be in Diameter 10 Times as large as the said Axle A B, the Motion at D will be 10 Times as great, or the Point D will move through $\frac{1}{100}$ Part of an Inch. But just so much as D moves, will the Periphery of the Pinion G H move, they both moving with the same Thread; if therefore the Circle I K L be in Diameter 10 Times as large as that of the Pinion G H, the Motion of the Index (which it carries) on its Circumference will be 10 Times as great, viz. $\frac{1}{10}$ Part of an Inch, which will be visible to the Eye: Consequently every Inch on the graduated Circle will shew the extension of $(100 \times \frac{1}{1000} =) \frac{1}{10}$ Part of an Inch in the

WHEN

WHEN Pendulums were first applied to Clocks, they were made very short; and

Metal. And so if the whole Circumference of the Circle I K L be 10 Inches, one Revolution of the Index will shew the Expansion of $\frac{1}{10}$ of an Inch in the Metal.

4. Hence it is easy to observe, that if the Wheel C D E, and Circle I K L, be enlarged in any Proportion to the Axles A B and G H greater than what I have assign'd, the Power or Effect of the Machine will be increased in the same Degree; and hence this Instrument may be constructed to shew very great Expansions of the Metals with the utmost Degree of Accuracy.

5. The Reader may be curious to know in what Proportion the several Sorts of Metals are expansible, which I shall here shew in a Table made from the Experiments of Professer *Muschenbroek*, with 1, 2, 3, 4, 5, Flames of Spirits of Wine.

		<i>Iron.</i>	<i>Steel.</i>	<i>Copper.</i>	<i>Brass.</i>	<i>Tin.</i>	<i>Lead.</i>
Flame	1	80	85	89	110	153	155
	2	117	123	115	220	*	274
	3	142	168	193	275	*	*
	4	211	270	270	361	*	*
	5	230	310	310	377	*	*

6. Here we observe the Expansion of *Iron* to be the least of all Metals, and that of *Tin* and *Lead* the greatest, and nearly equal with one Flame. Hence the Rods of Pendulums ought to be made of *Iron*, and also all Measures of Length, as Yards, &c. and whatever else we would not have alter'd by Heat and Cold.

7. The Method of remedying the Inconvenience arising from the Extension and Contraction of the Metalline Rod of Pendulums, is by applying the Bob with a Screw, by which means the Pendulum is at any Time made longer or shorter, as the Bob is screw'd downwards or upwards, and so the Time of its Vibration is continually the same.

8. If a Glafs or Metalline Tube, uniform throughout, fill'd with Quicksilver, and the Length of 58,8

the Arches of the Circle described being large, the Times of Vibration through different Arches could not, in that Case, be equal; to effect which, the Pendulum was contrived to vibrate in the Arch of a *Cycloid*, the Property of which Curve is, *that a Body will descend from any Part thereof to the lowest Point in the same Time, and sooner than by any other Way* (XXXII).

inches, were applied to a Clock, it would *vibrate Seconds* (for $39,2 = \frac{2}{3}$ of 58,8) and such a Pendulum admits of a twofold Expansion and Contraction, *viz.* one of the Metal and the other of the Mercury, and these will be at the same Time contrary, and therefore will correct each other.

9. For by what we have shewn, the Metal will extend in Length with Heat, and so the Pendulum will vibrate slower on that Account. The Mercury also will expand with Heat, and since by this Expansion it must extend the Length of the Column upward, and consequently raise the *Center of Oscillation*, so that by this means its Distance from the Point of Suspension will be shorten'd, and therefore the Pendulum on this account will vibrate quicker: Wherefore if the Circumstances of the Tube and Mercury are skilfully adjusted, the Time of the Clock might by this Means, for a long Course of Time, continue the same, without any sensible Gain or Loss.

10. This is the Invention of Mr. *Graham* in the Year 1721, who made a Clock of this Sort, and compared it with one of the best of the common Sort for three Years together, and found the Errors of the former but about $\frac{1}{8}$ Part of the latter; of which the Reader may see a farther Account in *Phil. Transf.* N^o 392.

(XXXII.) 1. How far the *Cycloid* is concern'd in the Pendulum, has been shewn; I shall now shew that this

THE

THE Motion of PROJECTILES comes next to be consider'd. A PROJECTILE is

is the *Curve of quickest Descent*; for let A H C be the
 Plate VI. Curve of quickest Descent, then will a Body descending
 Fig. 7. from A pass from H to D sooner in an Arch of this
 Curve than in any other, between H and D; let the
 Point G be taken such, that, drawing H F, G E parallel
 to the Axis A B, they may cut off I G = E D, in the
 Perpendicular K G and L D. Since the Point D is
 given, L D is constant, as also I G = D E, because
 the Point H is invariable, and therefore also K G is
 constant: but the variable Quantities are H I, H G,
 and G E, G D.

2. Now put K G = b , L D = c , and I G = E D = d ; also H I = v , and G E = z ; then will the Velocity of the Body at G be as $\sqrt{K G}$, and at D as $\sqrt{L D}$. And since the Times are as the Spaces pass'd over directly, and the Velocities inversely, the Time of passing thro' H G will be as $\frac{H G}{\sqrt{K G}} = \frac{\sqrt{d^2 + v^2}}{b}$ and the Time of describing G D, as $\frac{G D}{\sqrt{L D}} = \frac{\sqrt{d^2 + z^2}}{c}$.

3. But the Sum of those Times is the least possible viz. $\sqrt{\frac{d^2 + v^2}{b}} + \sqrt{\frac{d^2 + z^2}{c}}$, a *Minimum*. Consequently

its Fluxion $\frac{v \dot{v}}{b^{\frac{1}{2}} \sqrt{d^2 + v^2}} + \frac{z \dot{z}}{c^{\frac{1}{2}} \sqrt{d^2 + z^2}} = 0$, or $\frac{v \dot{v}}{b^{\frac{1}{2}} \sqrt{d^2 + v^2}} = -\frac{z \dot{z}}{c^{\frac{1}{2}} \sqrt{d^2 + z^2}}$; but since H F = v - z = a constant Quantity, we have $\dot{v} + \dot{z} = 0$, and $\dot{v} = -\dot{z}$; and therefore $\frac{v}{b^{\frac{1}{2}} \sqrt{d^2 + v^2}} = \frac{z}{c^{\frac{1}{2}} \sqrt{d^2 + z^2}}$, therefore $v c^{\frac{1}{2}} \sqrt{d^2 + v^2} = z b^{\frac{1}{2}} \sqrt{d^2 + z^2}$, when an

any Body A, thrown or projected in an *upright, oblique, or horizontal* Direction; as a *Stone* from the Hand, an *Arrow* from the Bow, or a *Ball* or *Bomb* from a Cannon or Piece of Ordnance. The Force with which the Body is projected is call'd the IMPETUS, and the Distance to which it is thrown is call'd the HORIZONTAL RANDOM OR AMPLITUDE of the Projection.

EVERY Projectile is acted upon by two Forces or Powers, *viz.* the *Impetus* of the

$$\sqrt{d^2 + v^2} : \sqrt{d^2 + z^2} :: v c^{\frac{1}{2}} : z b^{\frac{1}{2}}; \text{ that is, } GD : GH :: HI \times LD^{\frac{1}{2}} : GE \times KG^{\frac{1}{2}}.$$

4. But GD, HG are the Fluxions of the Curve; HI, GE, the Fluxions of the *Abfciffæ* AM, AK; and KG, LD are the Ordinates to the Points G and D. *Therefore the Fluxions of the Curve of swiftest Descent are every where as the Fluxions of the Abfciffæ directly, and the Square Roots of the Ordinates inverfely.*

5. But this is the Property of the Cycloid; for putting AK = x, KG = BN = y, and BC = a; the Triangles ONC and GED are fimilar by the Nature of the Curve; and therefore ON : OC :: GE : GD;

$$\text{that is, } \frac{ay - yy^{\frac{1}{2}}}{aa - yy^{\frac{1}{2}}} :: \dot{x} : \frac{\dot{x} \times a^2 - y^{\frac{1}{2}}}{ay - yy^{\frac{1}{2}}} =$$

$$\frac{\dot{x} \times a^{\frac{1}{2}} \times a - y^{\frac{1}{2}}}{y^{\frac{1}{2}} \times a - y^{\frac{1}{2}}} = \frac{\dot{x} \times a^{\frac{1}{2}}}{y^{\frac{1}{2}}} = GD; \text{ but } a^{\frac{1}{2}} \text{ is a given}$$

Quantity, therefore GD is every where a $\frac{\dot{x}}{y^{\frac{1}{2}}}$, or di-

rectly as the Fluxion of the Abfcifs AK, and inverfely as the Square Root of the Ordinate RG, and confequently the Cycloid is the Curve of swiftest Descent. See *Ann:at.* XXVIII. 3.

projectile Force, and that of *Gravity*. By the first, the Body passes over *equal Spaces*, *AB, BC, CD, &c. in equal Times*; and by the second, it descends through Spaces *AG, AH, AI, &c.* which are as the *Squares of the Times*; and therefore by these two Forces compounded, the Body will describe, not a *Right Line*, but a *Curve AQ*; and of that Sort which we call a *Parabola*; and this will be the Case in all Directions but that in the Perpendicular, when the Path of the Projectile will be (to Appearance) a *Right Line*. The greater the Angle of Elevation *KAM* of the Cannon is, the greater will be the Height *DB* to which the projected Body will rise. Also, the greater will be the Distance or Amplitude of the Projection, till the said Angle becomes equal to 45 Degrees *KAO*; upon which Elevation the Random *AC* will be the greatest possible, and equal to twice the Altitude *AG* of the perpendicular Projection. On any Elevation *AM* or *AM*, equally above or below 45 Degrees, as on 40 and 50, 30 and 60, 20 and 70 Degrees, the Random *AC* will be the same; which Case an Engineer frequently finds of very great Use.

Plate V.
Fig. 7.

IF the Object be situated above the Horizon, then, in order to strike it with the least *Impetus*, let a Piece of Looking-glass

be fix'd to the Cannon perpendicular to its Axis; and holding a Plumb-Line over the Glas directly under the Eye, the Cannon is to be elevated till the Object appears exactly under the Plummet, and there fix'd; if then it be discharged, it will strike the Object as required.

FROM what has been said, we may easily understand how a Body projected upright from the Earth's Surface does really describe a Parabola, though to Appearance it ascends and descends in a Right Line. For it is urged by two Forces, *viz.* the Projectile upwards, and the Force arising from the Motion of the Earth about its Axis from *West* to *East*; in which Case it must necessarily describe a Parabola (XXXIII).

(XXXIII.) 1. In the *Parabola*, it is demonstrated Plate V. that the several *Abscissæ* AG, AH, AI, &c. are as Fig. 5, 6. the Squares of the Ordinates GM, HN, IO, &c. and since this is the Property of the Curve which the Projectile describes, 'tis evident the Projectile describes a *Parabola*.

2. Let AM represent the Force with which a Ball is Fig. 7. projected from the Cannon in the Direction AM; this may be resolved into two others AL and LM, of which the first is perpendicular, and the latter parallel to the Horizon. The perpendicular Force or Velocity AL is that by which the Ball rises, and the Heights to which it will rise with the Velocities AL and AM are as the Squares of the Velocities, *viz.* as $\overline{AL}^2$ to $\overline{AM}^2$; but (because $AL : AM :: AM : AG$) $\overline{AL}^2 : \overline{AM}^2 :: AL : AG$; therefore since AG represents the

AM

K 2

I SHALL

I SHALL in the last Place consider the Nature of CIRCULAR MOTION and CEN-

Height to which the Ball will rise with the Velocity AM , AL will be the Height to which it will rise with the Velocity AL , or in the Projection on the Elevation AM ; thus $BD = AL$.

Fig. 6, 7. 3. The Velocity of the Projection LM in the horizontal Direction is every where uniform, or carries the Ball through equal Spaces AR , RS , ST , TV , VW , in equal Times. For by the single *Impetus* of the Powder, the Ball describes equal Spaces AB , BC , CD , &c. in equal Times in the Direction AF , as has been shewn; therefore by the Similarity of Triangles AR , RS , ST , &c. will be as AB , BC , CD , &c. that is, *the horizontal Spaces pass'd over in the Projection, will, in equal Times, be equal.*

Fig. 7. 4. Now since the perpendicular Velocity is to the horizontal Velocity as AL to LM , and AL is the Space pass'd through by the former in half the Time of the Projection, LM would be the Space pass'd through in the horizontal Direction in the same Time, if the Velocity LM were of the same Sort with AL , viz. *uniformly retarded and accelerated*; but since LM is not retarded, but uniform, it will carry the Body through twice the Space LM in the same Time (*Annot. XXVI. 7.*) that is, through $AD = LB = 2LM$.

5. And since the Time of the Ascent from A to B is equal to the Time of Descent, the Ball will pass over $DC = AD$ in the Time of the Descent from B to C ; therefore the horizontal Space AC , which the Ball passes over during the whole Time of the Projection, is equal to 4 times LM , wherever the Point M be taken in the Circumference AOG .

6. Hence when AO is the Direction of the Cannon, (O being the middle Point of the Semicircle) the Line NO , which is the horizontal Velocity, will be the greatest possible (from the Nature of the Circle) and consequently $4NO = AQ$, *the greatest possible horizontal Range*; which therefore is made on an Angle

TRAL

TRAL FORCES. If a Body A be suspended ^{Plate V.}
at the End of a String AC, moveable ^{Fig. 8.}

$\angle OAK = 45$ Degrees. For $\angle OAK = \angle AON = \angle OAN$, because $NO = AN$; therefore since $\angle ANO = 90^\circ$, $\angle AON = \angle OAN = \angle OAK = 45^\circ$.

7. Hence $AN = FE$ ($= NO = \frac{1}{2} AQ$) will be the Height of the Projection on the Angle $\angle OAK = 45^\circ$. *Whence AQ will be the Parameter, or Latus Rectum, of the Parabola AFQ.*

8. If $MO = OR$, then will $LM = TR$; and consequently the Horizontal Random on the Elevation AM , and AR , will be equal, *viz.* AC ; but the Height of the Projection on AR will be $AT = SD$. *Whence the Height SD is to the Height DBD (as RK to MK) as the Tangent of the Angle of Elevation RAK to the Tangent of the Angle MAK.*

9. From what has been demonstrated 'tis easy to understand, that an Object situated on the Horizon may be hit by a Ball discharged on an Elevation of 45° with a less Charge of Powder, than on any other Angle of Elevation.

10. The Time of the Projection ABC is equal to the Time of the perpendicular Ascent and Descent through AL or DB , (because the horizontal Velocity does no way affect that in the Perpendicular.) But the Time of Ascent and Descent in AL is to that in AG , as $\sqrt{AL}$ to $\sqrt{AG}$; that is, (since $AL : AG :: AM^2 : AG^2$) as AM to AG ; that is, (because $AG : AM :: AM : AL = MK$ as MK to AM , or as the Sine of the Angle of Elevation MAK to the Radius.

11. Therefore the Times of two different Projections ABC , ASC , will be as the Chords AM and AR , or as the Sines of the Angles of Elevation to the Radius AG .

12. Suppose it required to know from what Height a Body ought to fall to acquire such a Velocity, that, being reflected in that Moment in the Direction AF , it should describe the given Parabola AOQ . In order to this we are to consider that (the Equation being $xp = yy$)

about a Point or Pin C as a Centre, and in that Position it receive an Impulse or

the Velocity of a Ball shot from the Cannon commences in the Direction of the Axis AL (or Absciss x) in the Point A; but that in the Direction AF (or of the Ordinate y) is from first to last the same; consequently there must be a certain Point, to which the Ball having descended, acquires a Velocity equal to that in the Direction AF, which is uniform; and this must be when $\dot{x} = \dot{y}$. To determine which, putting the Equation $x \dot{p} = y \dot{y}$ in Fluxions, we have $\dot{x} \dot{p} = 2 y \dot{y}$; but since in this Case we have $\dot{x} = \dot{y}$, therefore $\dot{p} = 2 y$, and $\frac{\dot{p}}{2} = y = \sqrt{x \dot{p}}$, (per Equation;) therefore $\frac{1}{4} \dot{p}^2 = x \dot{p}$, that is, $\frac{1}{4} \dot{p} = x$.

13. Consequently, if an heavy Body fall from the Height of $\frac{1}{4}$ of the Parameter of the given Point A, and in that Point it be reflected (with the Velocity there acquired) in the Direction AF, it will describe the given Parabola AOQ.

14. If it be required to strike the Point B, whose Plate VI. Distance AD and Height BD are given, with the Fig. 9. least Impetus of all that will hit it, then in order to find the Elevation of the Cannon, draw AB, and EA perpendicular to AC; on the Point A describe the Arch abc , which bisect in b , then a Line AF drawn through that Point will be the Elevation of the Cannon as required; and the least Impetus will be equal to $\frac{AB + BD}{2}$. See the Demonstration of this by the

Inventor Dr. Halley in the *Phil. Transactions*, and by Dr. Keill, *Introd. Pag.* 280, 281. From a bare View of the Diagram the Reason of the Use of the *Looking-Glass* and *Plummet* is evident.

15. If a Ball be projected perpendicularly upright in the Direction AF with a Velocity that will carry it to the Height of one Mile, or 5280 Feet; it will be 18 Seconds (nearly) in the Ascent, and as much in the Descent, (for, as 16,2 Feet : 5280 Feet :: $1^2 : x^2$;
Blow

Fig. 10.

Blow in an horizontal Direction, it will be thereby compell'd to describe a Circle about the central Pin: While the circular Motion continues, the Body will have a continual Endeavour to recede or fly off from the Centre, which is call'd its *Centrifugal Force*, and arises from the *Horizontal Impetus*; with this Force it acts upon the fix'd Centre Pin, and that, by its Renitency or Immobility, re-acts with an equal Force on the Body by means of the String, and solicits it towards the Centre of Mo-

whence $x = 18''$, *ferè* the Time of Ascent or Descent.) Now in the Parallel of *London*, the Motion of the Earth's Surface, and of all Bodies upon it, is at the Rate of 10,8 Miles *per* Minute, or 950 Feet *per* Second, from West to East: Therefore in 36 Seconds, the Time of the Ascent and Descent, the Ball will be carried over 34214 Feet or more than six Miles, in the horizontal Direction Eastwards from the Point A to H. Now since the Ball is carried at the same Time with a retarded Force upwards, and an uniform Force forwards, it will describe the Parabola A D H. And because the horizontal Motion in the *Spectator* and in the *Project* is the same, they will be carried through equal Spaces A N, O B; A M, P C; A L, F D, &c. in the same Time; and consequently the projected Ball will always appear perpendicularly over the Spectator in every Point of the Curve; which is the Occasion of the Deception above-mentioned.

16. Hence the Objection usually urged against the Motion of the Earth, from the apparent Ascent and Descent of Projects in a Right Line, is the Result of Ignorance; as indeed are all other Arguments of this Kind.

tion, whence it is call'd the *Centripetal Force*; and when we speak of either, or both of them indefinitely, they are call'd the *Central Forces* of the revolving Body.

IT is of the last Importance to understand the Nature of this Kind of Motion, since by it all the Machinery of the Planetary System is perform'd, as will be easy to understand, if, for the revolving Body, we substitute a Planet; for the Centre, we place the Sun; for the *Centripetal Force*, or String, its Power of Attraction; and for the *Projectile Force*, the Almighty Power of God in the first Creation of Things.

Plate V.
Fig. 8.

THE Theory of this Species of Motion is comprised in the following Propositions, viz. (1.) The projectile Force AH is infinitely greater than the centrifugal Force HE . (2.) The central Force is proportional to the Quantity of Matter in the revolving Body A , all other Things being equal. (3.) If two equal Bodies A, B , describe unequal Circles AM, AN , in equal Times, the central Forces will be as the Distances AC, BC , from the Centre C . (4.) If equal Bodies describe unequal Circles with equal Celerities, the central Forces will be inversely as the Distances. (5.) If equal Bodies describe equal Circles, the

the central Forces will be as the Squares of the Celerities. (6.) If equal Bodies describing unequal Circles have their central Forces equal, their Periodical Times will be as the Square Roots of the Distances. (7.) If equal Bodies describe unequal Circles with equal Celerities, the Periodical Times will be as the Distances directly. (8.) Therefore the Squares of the Periodical Times are proportional to the Cubes of the Distances, when neither the Periodical Times nor the Celerities are given. And in that Case, (9.) The central Forces are as Squares of the Distances inversely.

THESE are the Theorems of circular Motions, the two last of which are found by Astronomers to be strictly observed by every Body of the Planetary and Cometary System. For Example: The Periodical Time of *Venus* is 225 Days, and that of the *Earth* 365; the Squares of which Numbers are 50625 and 133225: Again, the Distance of *Venus* from the *Sun* is to that of the *Earth* as 72 to 100; the Cubes of which Numbers are 373248 and 1000000; but $50625 : 133225 :: 373248 : 1000000$; that is, the Squares of the Periodical Times are as the Cubes of their Distances very nearly. From hence also it will easily appear,

pear, that Bodies under the Equator have the greatest centrifugal Force, which there acts in direct Opposition to Gravity, and diminishes towards the Poles, with the Squares of the Distances from the Earth's Axis. Hence also it is evident, that if ever the Earth was in a fluid State, and at the same Time moving about its Axis, it must necessarily put on the Figure, not of a perfect Sphere or Globe, but of an oblate Spheroid, flattened towards both Poles; as is manifestly shewn by Experiment (XXXIV).

Plate V.
Fig. 8.

(XXXIV.) 1. Let $AD = a$, the Diameter of the Circle AMD ; AH the Space which a Body A moves through in a constant Particle of Time by an uniform Projectile Force; HE the Distance it is drawn from the Tangent AH by the Centripetal Force (which is just sufficient to retain the Body moving in a circular Orbit) in the same Time.

2. Then since AH is indefinitely small, the Arch AE , and its Chord, will nearly coincide and be equal to AH , and will therefore also represent the Projectile Force. But $AD : AE :: AE : Aa = HE = \frac{AE^2}{AD}$ = the Central Force. Now AE is infinitely small in respect of AD , therefore HE or Aa is infinitely small with respect to AE .

3. If the Projectile Force or Velocity $AE = V$, and $AD = a$, the general Expression of the Central Force will be $\frac{V^2}{a}$; which therefore in different Circles will be as the Squares of the Velocities directly, and inversely as the Diameters or Semi-diameters of the Circles.

4. But in the same Circle it will be directly as the Squares of the Velocity; because in that Case a is given Bu

But if the Velocity be given, it will be as the Distance from the Centre inversely, (for then $\frac{V^2}{a}$ becomes $\frac{1}{a}$.)

5. Hence if the Central Forces in two Circles be equal, the Diameters of those Circles, or Distances from the Centre of Force, will be as the Squares of the Velocities directly.

6. As $AE : 1 :: 3,1416 a : P$ = the Periodical Time of describing the whole Circle (for in equable Motions the Spaces are as the Times.) Therefore $P = \frac{3,1416 a}{V}$;

whence $V^2 = \frac{3,1416^2 \times a^2}{P^2}$, which substituted in the

Expression for the Central Force, we have $\frac{3,1416^2 \times a}{P^2}$

= $\frac{a}{P^2}$ (because $3,1416^2$ is given) for the Central Force

in this Case; which therefore is always as the Diameter or Radius of the Circle directly, and the Square of the Periodical Time inversely.

7. Hence if the Central Force be as any Power (n) of the Distance (a) from the Center C, we have $\frac{a}{P^2}$

= a^n ; whence $P = \frac{1-n}{a^{\frac{1}{2}}}$; and therefore when $n = 0$, we have $P = a^{\frac{1}{2}}$, or the Periodical Time will be as the Square Root of the Distance.

8. If $n = 1$, that is, if the Force be directly as the Distance, then will $P = a^{\frac{0}{2}} = a^0 = 1$; that is, the Periodical Time will be given, or the same in every Circle.

9. If $n = 2$, that is, if the Force be directly as the Square of the Distance, then $P = a^{-\frac{1}{2}} = \frac{1}{a^{\frac{1}{2}}}$;

or the Periodical Time will be as the Square Root of the Distance inversely.

10. If $n = -1$, that is, if the Force be inversely as

as the Distance, then $P = a^{\frac{2}{3}} = a$, or the *Periodical Time will be directly as the Distance from the Centre.*

11. If $n = -2$, or the Force be inversely as the Square of the Distance, then $P = a^{\frac{3}{2}}$, or (squaring both Sides) $P^2 = a^3$, that is, *The Square of the Periodical Time will be as the Cube of the Distance from the Center.*

12. When the Central Force is given, $\frac{a}{P^2} = 1$, and $P = a^{\frac{1}{2}}$, that is, *The Periodical Time will be as the Square Root of the Distances.*

13. Since the Central Force is as $\frac{V^2}{a}$, and as $\frac{a}{P^2}$, we have $PV = a$; whence the Distance from the Centre is always in the Compound Ratio of the Periodical Time and Velocity.

14. Hence if the Distance be given, or $a = 1$, then the Periodical Time is inversely as the Velocity, and the Velocity inversely as the Periodical Time.

15. If the Velocity be given, or $V = 1$, the Periodical Time will be directly as the Distance from the Centre.

16. If the Periodical Time be given, the Velocity will be directly as the Distance likewise.

17. If we put $p = 3,1416$; then $pa : P :: AE = \frac{AE \times P}{p a} = \text{Time of describing } AE$. Again, as $1^2 :$

$16,2 :: \frac{AE^2 \times P^2}{p^2 \times a^2} : \frac{16,2 AE^2 \times P^2}{p^2 a^2}$, = the Distance

descended by an heavy Body (in the Time of describing AE) by the Force of Gravity. Lastly, this Force of

Gravity $\left(\frac{16,2 AE^2 \times P^2}{p^2 a^2} \right)$ is to the Central Force

$\left(\frac{AE^2}{a} \right)$ as 1 to $\frac{a p^2}{16,2 P^2}$, that is, as $16,2 P^2$ to $a p^2$

or as P^2 to $0,615 a$.

18. If now we suppose a Body revolving about the Centre of the Earth at the Distance of its Surface with a Centrifugal Force equal to that of Gravity; then $P^2 = 0,615 a$, and so $P^2 = \sqrt{0,615 a}$; and putting $a =$

4200000

42000000 Feet in the Diameter of the Earth, we have $P = 5083$ Seconds, or $84' 43''$; which is the Time of Revolution to acquire a Centrifugal Force equal to that of Gravity at the Earth's Surface. Consequently were the Earth to revolve in $84' 43''$ instead of 24, Hours, the Bodies on its Surface would lose all their Weight, and be as liable to fall off as to abide thereon.

19. From what has been demonstrated we may discover the Law of Gravitation at the Moon; for since the Distance of the Moon from the Earth's Centre is to the Distance of the Earth's Surface as 60 to 1; and since the Periodical Time at the Earth's Surface is $5083''$, therefore the Periodical Time at the Moon may be found (by Art. 11.) thus; $1^3 : 60^3 :: 5083^2 : P^2$; whence $P = 5083 \times 60^{\frac{3}{2}} = 2362000'' = 27,3$ Days, the Periodical Time of the Moon, if moved by a Centripetal Power, as *Gravity decreasing with the Squares of the Distances inversely*: And therefore since the Periodical Time, thus found, is equal very nearly to the real Periodical Time of the Moon, it shews that Law takes Place between the Earth and Moon. This might also be shewn, from the Time of the Moon's Revolution, but *one Proof* is enough. I shall also hereafter shew, that this is the general Law of the whole Planetary System.

20. Since we had $\frac{a}{P^2} =$ the Central Force, when a is given, the Force will be as $\frac{1}{P^2}$; that is, in equal Circles the Centrifugal Forces will be as the Squares of the Velocities inversely. And therefore since in the Time of the Earth's Revolution, viz. 24 Hours, there are $68400''$; if we say, as $5083^2 : 68400^2 :: 1 : 289$ nearly; that is, *the Centrifugal Force under the Equator, arising from the Earth's Rotation, is to the Power of Gravity on its Surface as 1 to 289 nearly*.

21. Since the Time of Revolution of a Body under the Equator $E Q$, and in any parallel of Latitude $B G$, is equal, the Centrifugal Forces are as the Distances $E C$, $B A$, from the Centre of Motion (by Art. 16.) or, as Radius $C B$ to the Co-sine $A B$ of the Latitude. Plate VI.
Fig. 11.

22. But

22. But in any Latitude B, the Centrifugal Force is not (as under the Equator) opposite to the whole Gravity, but only a Part thereof, which is to the Whole as the Co-sine of Latitude to the Radius. For continue out AB (the Direction of the Centrifugal Force) to D; and CB (in which Gravity acts) to F; from D let fall a Perpendicular to F, then will BD represent the whole Centrifugal Force, and BF that Part of it which acts directly against Gravity; but $BF : BD :: AB : BC = EC$. Therefore on both these Accounts, the Centrifugal Force decreases from the Equator towards the Poles N and S in the Proportion of the Squares of the Co-sines of the Latitude.

Plate II.
Fig. 12.

23. We proceed now to demonstrate *the true Figure of the Earth*, which we shall find to be *Spheroidical* and not *Spherical*, if ever its Parts were in a fluid or yielding State, from the Consequence of a Centrifugal Force. In order to this, let NS be the Axis of Rotation, BC a Column of fluid Particles gravitating towards the Centre C, which (because we suppose the Parts of the Fluid every where quiescent) will be of the same Weight with every other Column of Particles CN, or CE. Let $CE = a$, $CN = b$; $CB = x$, $AB = s$ Sine of the Angle BCN; and supposing the gravitating Power to be every where as the Power n of the Distance from the Centre; that is, suppose the Power of Gravity at E (g) to be to that at B as CE^n to CB^n , then will the Gravity at B be $= \frac{g \times CB^n}{CE^n} = \frac{g \times x^n}{a^n}$.

24. Also the Centrifugal Force at E (f) is to that at B as CE to AB (as we shew'd *Art.* 16.) But $AB : CB :: s : 1 = \text{Radius}$; whence $AB = s \times CB = sx$; consequently we have the Centrifugal force at B equal to $\frac{f s x}{a}$.

25. But since this Force acts in the Direction BD, that Part which opposes Gravity is FB (as was shewn above) whence since $BD : FB :: 1 : s :: \frac{f s x}{a} : \frac{f s s x}{a}$ = to Centrifugal force at B, which opposes Gravity.

26. Hence the Power of Gravity on a Particle at B will

will be as $\frac{g \times x^n}{a^n} - \frac{f s^2 x}{a}$ impelling it towards the Center C. Such a Particle will be represented by $\dot{x}$, and therefore its Weight will be $\frac{g \times x^n \dot{x}}{a^n} - \frac{f s s x \dot{x}}{a}$; and as this is the Fluxion of the whole Weight of the Column CB, the Fluent thereof $\frac{g \times x^n + 1}{n + 1 a^n} - \frac{f s s x^2}{2 a}$ will be the Weight of the said Column of Particles CB.

27. If now we consider the Column of Particles CE, which is of equal Weight, since in this Case the Angle NCE is a Right one, we shall have $x = a$, and $s = 1$; wherefore the Weight of the Column CE will be $\frac{g \times a^n + 1}{n + 1 a^n} - \frac{f a a}{2 a}$; therefore the Weight of a

Column every where is equal to $\frac{2g - nf - f}{2n + 1} \times a$

$= \frac{g \times x + 1}{n + 1 a^n} - \frac{f s s x^2}{2 a}$ therefore we have (by proper Reduction) $2g x^n + 1 - \frac{n + 1}{n + 1} \times f s s a^{n-1}$
 $= 2g - nf - f \times a^n + 1$ for a general Equation.

28. Whence, putting $s = 0$, we have $2g x^n + 1 =$
 $2g - nf - f \times a^n + 1$; and therefore $a : x :: 2g - nf - f$

$2g - nf - f \times \frac{1}{n + 1} :: CE : CN$, *so is the Diameter of the Equator to the Axis of the Earth.*

29. Hence, upon the Hypothesis of an uniform Gravity, we have $n = 0$; in that Case $2g : 2g - f :: CE : CN$. But the Gravity (g) is to the Centrifugal Force (f) under the Equator, as 289 to 1 (*per Art. 20.*) Wherefore in this Case $CE : CN :: 578 : 577$.

30. If the Earth were to revolve in $84' 43''$, the Centrifugal Force would be equal to Gravity (*Art. 18.*) or $g = f$; and then $CE : CN :: 2 : 1$. If the Earth were to revolve quicker than that, the Particles would fly off, and the Earth be reduced to a single Atom.

31. If the Gravity be supposed proportional to the Distance from the Center; then $n = 1$, and we have
 CE:

$CE : CN :: g^{\frac{1}{2}} : g - f^{\frac{1}{2}}$ and if in this Hypothesis the Earth were to revolve in $84' 43''$, or $g = f$, then would CE be infinitely greater than CN ; that is, CN would be nothing, or the Spheroid would then become a circular Plane.

32. If Gravity be supposed inversely as the Square of the Distance, we have $n = -2$; and then we shall

have $2g^{-n+1} : 2g + nf - f^{-n+1} :: CE : CN ::$

$2g^{-1} : 2g + f^{-1} :: 2g + f : 2g :: 579 : 578$. And since of all the above Hypotheses, this last is found to be the only true one (see *Art. 19.*) it follows that the Earth in its Chaotic or Fluid State, revolving about its Axis, must necessarily put on a Spheroidal Figure, having the Equatorial Diameter EQ , to the Axis NS , as $579 : 578$.

33. And this would be the Case, were all the Circumstances of the Hypothesis the same in Nature as we have presumed in the Theory; but since they are found to be otherwise, the *Mathematical Theory* (which gives only the true Sort of Figure, but not the true Figure itself) must be adapted to Nature, and the Figure which the Earth really has investigated from other Principles. For Bodies on the Earth's Surface do gravitate in such Directions as pass not thro' the Centre C (as has been supposed) any where but in the Equator or under the Poles, viz. at E and N .

34. In order to find the true Proportion between EQ and NS , Sir *Isaac Newton* makes a Supposition they are as 101 to 100, and then by a Method (too prolix to be here explained) he finds the Gravity at E and N to be as 500 to 501. Now supposing the Matter of the Earth uniform and at Rest, the Weight of the Column of Particles EC will be to that of the Column NC in the Ratio of their Magnitudes 101 to 100, and of their specific Gravities 500 to 501 conjointly, (See *Annot. LV (II. 10.)*) that is, as 505 to 501.

35. Now 'tis plain if the Weight of each Particle in the Column CE were divided into 505 equal Parts, a Centrifugal Force, that should take off four of those equal

equal Parts, would leave the Weight of the Column EC equal to that of the Column NC, whence an Equilibrium must ensue between them.

36. Whence our Author makes this Analogy: If a Centrifugal Force which is $\frac{4}{505}$ of Gravity causes an Excess of $\frac{1}{105}$ Part in the Altitude EC, a Centrifugal Force which is but $\frac{1}{189}$ Part of Gravity can cause an Excess only of $\frac{1}{229}$ Part of EC. Therefore the true Proportion between the Diameter of the Equator EQ, and the Earth's Axis NS, is that of 230 to 229. Whence, since the former is about 8000 Miles, it will exceed the latter by about $34\frac{3}{10}$ English Measure.

37. Whence, since in an Equilibrium of Fluids communicating with each other the Altitudes are as the specific Gravities inversely, it follows that the Gravity at the Equator E is to that at the Pole N as 229 to 230; or a Body which under the Poles weighs 230 lb. will under the Equator weigh but 229 lb.

38. The Gravity increases from the Equator E towards the Pole N *with the Squares of the Sines of the Latitudes*. For continue out CF to G, and draw GD perpendicular to AD; then if BD be the whole Centrifugal Force at the Equator, and BG the whole Diminution of Gravity occasioned there by it; and since in the Latitude B the Centrifugal Force which acts directly against Gravity is reduced to FB, the Line FB will also represent the Diminution of Gravity in the Latitude B, and therefore the Difference GF will be the Increase of Gravity at B. And so (because BG : GD :: GD : GF) we have BG : GF :: $\overline{GB}^2 : \overline{GD}^2$::) $\overline{BC}^2 : \overline{AC}^2$:: *Radius Square : Square of the Sine of the Latitude :: Decrease at the Equator : the Increase at B.*

39. If we put the Diminution of Gravity at the Equator = 10000, then will the Gravities at the Equator, London, and the Pole be as the Numbers 2290000, 2296124, 2300000; therefore the Lengths of Pendulums vibrating Seconds in each of those Places will be in the same Proportion (by Annot. XXVIII. 13.) But 2296124 : 2290000 :: 39,2 : 39,1 = Inches of a Second Pendulum under the Equator, the same nearly as found by Observation.

40. Since in the elliptic Meridian ENQS, the Curvature at E, Q, is much greater than that at N, S; the Radius of Curvature (suppose Eb) in the former will be shorter than that in the latter (suppose Sd;) and therefore if at the Equator the Angle Eba, or the Arch Ea, be equal to *one Degree*, the Angle of one Degree Sdc at the Pole S will contain a greater Arch cS, as is evident from the Diagram. Whence it appears that in this spheroidal Figure of the Earth, the Degrees increase from the Equator to the Pole; so that if in the Equator a Degree consists of 60 Miles, in the several Latitudes the Miles will be as below:

Lat.	0	10°.	20°.	30°.	40°.	50°.	60°.
Miles in a Deg.	} 60, 59,5. 59,57. 59,67. 59,8. 59,93. 60,06.						
Lat.		70°.	80°.	90°.			
Miles in a Deg.	} 60,16. 60,235. 60,26.						

SCHOLIUM.

What has been said hitherto, has been upon Supposition that the Earth is of an uniform Density throughout, but it is reasonable to suppose (from the Earth's having been originally in a fluid State) that the heaviest Matter subsided first, according to the Laws of Gravity; and therefore that the Earth is more dense and compact the nearer we go to the Centre, whence it must follow, that as there is a greater Number of Particles, there will be, on the Whole, a greater Quantity of Centrifugal Force in the Column EC, and consequently a greater Excess in its Altitude above that of NC, than was before stated; that is, EC will exceed NC more than in the Proportion of 230 to 229, which is also found to be still more consonant to Experiments. See farther on this Head my *New Principles of GEOGRAPHY and NAVIGATION*.

L E C T U R E III.

Of the CENTRES of MAGNITUDE, MOTION, and GRAVITY. The METHOD of finding the CENTRE of GRAVITY in all BODIES. The PHÆNOMENON of the DOUBLE CONE and CYLINDER. Of the COMMON CENTRE of GRAVITY between two or more BODIES; consider'd between the EARTH and MOON, the SUN and PLANETS. The FUNDAMENTAL PRINCIPLE of MECHANICS demonstrated. The Impossibility of a PERPETUAL MOTION. The NATURE and KINDS of LEVERS, BALANCES, PULLIES, the AXIS in PERITROCHIO, INCLINED PLANE, WEDGE, and the SCREW. Of FRICTION and FRICTION-WHEELS. Of COMPOUND MACHINES. The THEORY of CLOCK-WORK. A new ORRERY, COMETARIUM, &c. Of the MOVEMENT of WIND-MILLS, WATER-MILLS, SHIPS, &c. Of the MAXIMA and MINIMA in MECHANICS. The THEORY of WHEEL-CARRIAGES at large.

HAVING considered the Nature of every Kind of Motion, with the Properties of each, we should

have come immediately to treat of the MECHANICAL POWERS or MACHINES, but that something still remains to be premised thereto, relating to the *Centres of Magnitude*, of *Motion*, and of *Gravity* in Bodies.

THE CENTRE of MAGNITUDE is that Point which is equally distant from all the external Parts of the Body : And in Bodies that are uniform and homogeneous, it is the same with

THE CENTRE of MOTION, which is that Point which remains at Rest, while all the other Parts of the Body move about it : And this is again the same in uniform Bodies, of the same Matter throughout, as

THE CENTRE of GRAVITY, which is that Point about which all the Parts of a Body do in any Situation exactly balance each other.

THIS Centre of Gravity in Bodies is of the greatest Consequence to be well understood, as being the sole *Principle of all Mechanical Motions*. The particular Properties hereof are as follow. (1.) If a Body be suspended by this Point, as the Centre of Motion, it will remain at Rest in any Position indifferently. (2.) If a Body be suspended in any other Point, it can rest only in two Positions, *viz.* when the said Centre of Gravity is exactly above or below

low the Point of Suspension. (3.) When the Centre of Gravity is supported, the whole Body is kept from falling. (4.) Because this Point has a constant Endeavour to descend to the Centre of the Earth; therefore, (5.) When this Point is at Liberty to descend, the whole Body must also descend or fall, either by sliding, rolling, or tumbling down. (6.) The Centre of Gravity in regular, uniform, and homogeneous Bodies, as *Squares, Circles, Spheres, &c.* is the middle Point in a Line connecting any two opposite Points or Angles. (7.) In a Triangle it is in a Line drawn from any Angle bisecting the opposite Side, one Third of the Length distant from that Side or Base. (8.) It is also *one Third* of the Side distant from the Base of an hollow Cone. (9.) But in a solid Cone it is *one Fourth* of the Side distant from the Base. (10.) In the Human Body, the Centre of Gravity is situated in that Part which is call'd the *Pekvis*, or in the Middle between the Hips.

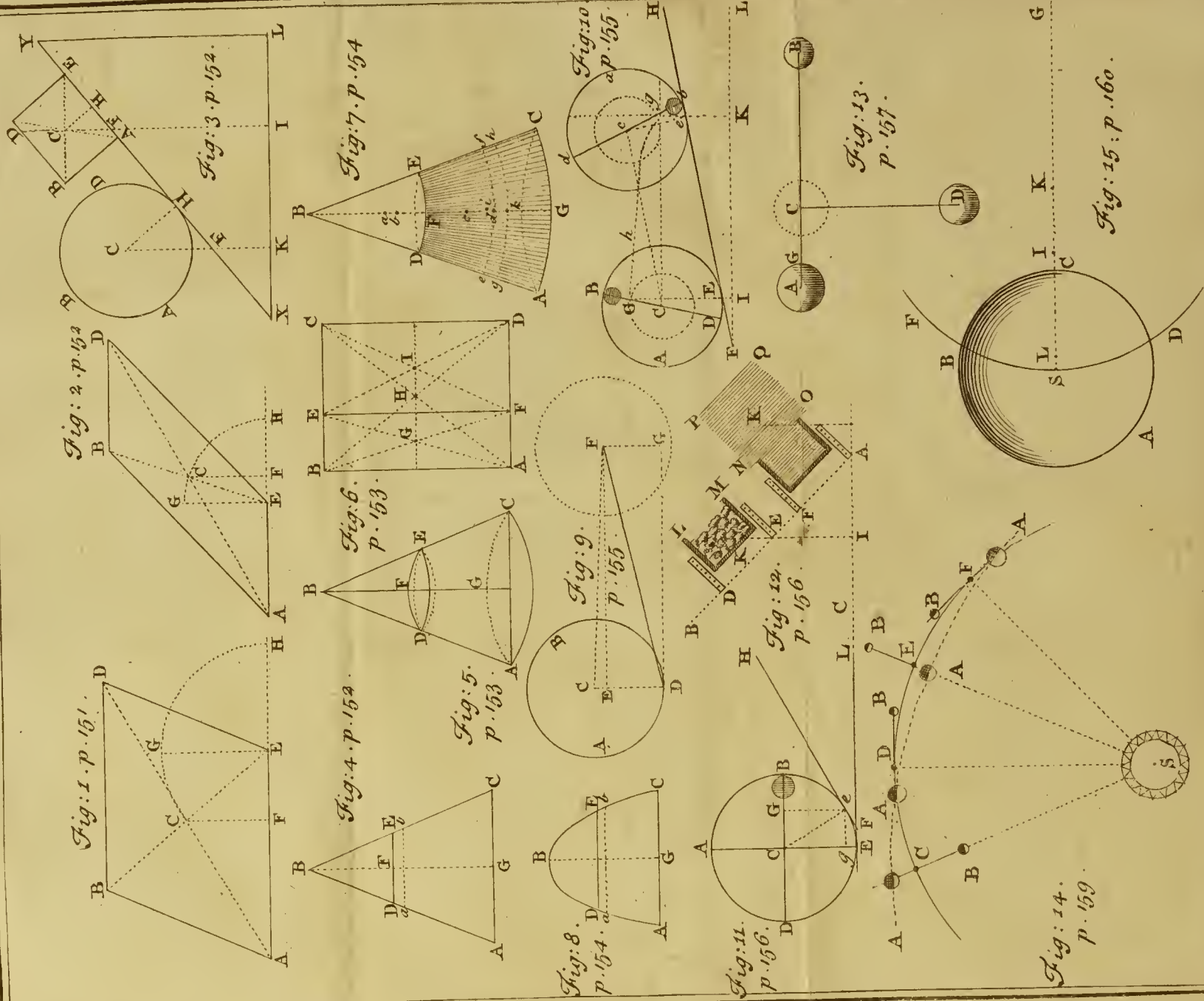
HENCE the Solution of several very curious *Phænomena* will be evident with the least Attention; as why some Bodies stand more firmly on their Bases than others; why some stand more firmly in an inclined Position; why some Bodies fall in one

L 3 manner,

manner, some in another ; why some may seem to rise, while the Centre of Gravity really descends, as the *rolling* CONE, and CYLINDER. Hence the Form of that particular Bucket which descends empty with the Mouth downwards, but is drawn up full with the Mouth upwards. Hence also it appears that a Waggon loaded with heavy Matter, as Iron, Stone, &c. will go safely on the Side of a Hill or rising Ground, where a Load of Hay or Corn would be overturn'd. Again, we hence see the Reason why no Man, standing still, can move or stir, without first moving the Centre of Gravity out of its Place ; also, why we stand firmly, while the Centre of Gravity falls between, or on the Base Line of the Feet ; and how necessarily we fall, when the Centre of Gravity falls on one Side or other of the same : With many other Particulars, which naturally result from this Principle (XXXV).

Plate VI. (XXXV.) 1. That the Centre of Magnitude. of Motion, and of Gravity is the same Point C in regular and homogeneous Bodies, as the circular Area, or Square ABDE is evident ; because any Right Line as AD drawn through the said Point and terminated by the Peripheries of those Bodies, has an equal Number of Particles on each Side ; therefore the said Point is Centre of Magnitude and Gravity, and consequently of uniform Motion in every such Line, and therefore of the whole Area, which is made up of them.

2. But



IF two or more Bodies in Motion be connected together, as *Chain-Shot*, &c. or any

2. But if a Body be not homogeneous but consists of different Matter in different Parts, the Centre of Magnitude C will not be the Centre of Gravity; thus suppose ABF a circular Piece of Wood with a round Piece of Lead *ef* in one Side of it; this Lead will remove the Centre of Gravity from C to some other Point D in the Diameter passing through the Centers C and *g*; and this Centre of Gravity D will of Course be also the Centre of Motion and Rest, for on no other Point can a Body rest but that on which it is equilibrated, since Rest is only the Result of an Equilibrium of the Parts.

Fig. 4.

3. If the Centre of Gravity be supported, the Body will be supported likewise; for since the Power of every Particle to descend is there referr'd, and all unite in one; if that one Point be upheld, all the rest must, as having a mutual Dependence thereon by a necessary Connection of Parts.

4. If a Body be suspended on any other Point C, than the Centre of Gravity D, it will descend in every Position but two, *viz.* when the Centre of Gravity is in E above, or in D below the Point of Suspension C, because in two Cases it gravitates on a fix'd Point, which suspends the Body, and therefore upholds it; but in all other Cases it has Liberty to descend, and therefore will carry the Body down, till it comes to rest in D.

5. If the Centre of Gravity C in the Body ABDE Pl. VII. gravitate on the Base AE in the Perpendicular CF, that Body will stand firm, and so much the more so, as Fig. 1. the Point F falls nearer the middle Point between A and E; for since, were the Body to move, the Point or Angle E would be the Centre of Motion, it follows, that in that Case the Centre of Gravity C must rise in the Arch of a Circle from C to G, which it cannot do without some extraneous Force; and therefore, when left to itself, the Body must stand firm.

6. But in the Body of the next Figure, because the Line CF, in which it gravitates, falls off from the

L 4

how

how depend on each other, they will have a *Common Centre of Gravity*, which will be a

Pl. VII. Base AE, the Centre of Gravity C will be free to descend, as not being supported, and consequently the Body must fall.

Fig. 3. 7. If a Body ABDE, laid on an inclined Plane XY, gravitates in the Direction CI, within the Base AE, that Body will move down the Plane (by *Annot. XXVII. 1.*) but it cannot move over the angular Point A (by 5th of this) it must therefore descend by *sliding down* the Plane. If the Base AE, had been so small that the Point F had fallen off it, it would have turn'd on the Point A in its Motion, and so have *tumbled down* the Plane. But if the Bases AE be supposed infinitely small, or a Point, the falling from one Point to another, will be so momentaneous, that the Body (which then becomes a Circle or Sphere) will descend by *rolling down* the Plane.

Fig. 4. 8. To find the Centre of Gravity of a *Line*, has been shewn (*Annot. XXIX. 2.*) To find that of a *Superficies*, as of a *Triangle*, *Parabola*, &c. this is the Method. Let AG bisect the Base AC of the Triangle ABC, it will also bisect every other Line DE drawn parallel to the Base; consequently the Centre of Gravity of the Triangle will be found somewhere in the Line BG. The Area of the Triangle may be consider'd, as consisting of an infinite Number of indefinitely small Parallelograms DEbaD, each of which is to be consider'd as a small Weight, and also as the Fluxion of the Area of the Triangle, and so may be expressed by $2y\dot{x}$, (putting $BF=x$, and $FE=y$;) if this fluxionary Weight be multiplied by its Velocity x , we shall have $2yx\dot{x}$ for its *Momentum*.

9. Now put $BG=a$, and $AC=b$; then $BG (a) : AC (b) :: BF (x) : DE = \frac{bx}{a} = 2y$; therefore the Fluxion of the Weights $2y\dot{x} = \frac{bx\dot{x}}{a}$; and the Fluxion

Point

Point in the Line joining the Centres, so situated, that its *Distance from the said*

of the *Momenta* $2 y x \dot{x} = \frac{b x x \dot{x}}{a}$; whence the Fluent

of the latter, viz. $\frac{b x^3}{3a}$, divided by the Fluent of the

former, viz. $\frac{b x^2}{2a}$, will give $\frac{2}{3} x$ for the Distance of

the Point from B in the Line B F, which has a Velocity equal to the Mean Velocity of all the Particles in the Triangle D B E, and is therefore its Centre of Gravity.

Consequently the Centre of Gravity of any Triangle A B C, is distant from the Vertex B, $\frac{2}{3}$ B G a right Line drawn from the Angle B bisecting the Base A C.

10. And since the Section of a superficial or hollow Cone is a Triangle, and Circles have the same Ratio as their Diameters, it follows that the Circle whose Plane passes through the Centre of Gravity of the Cone is $\frac{2}{3}$ of the Length of the Side distant from the Vertex of the said Cone.

11. But in a solid Cone, which consists of an infinite Number of circular Areas, which therefore may be consider'd as so many Weights, the Centre of Gravity may be found as before, by putting B E = x , B G = a ; the circular Area D F E = y , and A G C = b ; and from the Nature of the Cone, $a^2 : x^2 :: b : y = \frac{b x^2}{a^2}$, but $\dot{x} y = \frac{b x^2 \dot{x}}{a^2}$ = Fluxion of the Weights; and

$y x \dot{x} = \frac{b x^3 \dot{x}}{a^2}$ = Fluxion of the *Momenta*, whence

the Fluent of the latter, viz. $\frac{b x^4}{4 a^2}$ divided by the Fluent

of the former $\frac{b x^3}{3 a^2}$, will give $\frac{3}{4} x$ of the Centre of

Gravity of the Part D B E F; consequently the Centre of Gravity of the Cone A B C G is distant from the Vertex B, $\frac{3}{4}$ of the Side B G, in a Circle parallel to the Base.

12. If A B C D be a regular Superficies, and A B E F

Centres

Pl. VII.
Fig. 5.

Fig. 6.

Centres will be reciprocally as the Quantity of Matter in each Body.

another similar thereto, their Difference will be $FECD$; and the Centres of Gravity of each will be in the Points H, G, I , as is evident from what has been shewn. Now (from *Euclid's Elements*) we have $FECD$ to $ABEF$ as the Triangle FBD to the Triangle ACF ; but $FBD : ACF :: FD : AF :: GH : HI$; wherefore if any two regular and similar Surfaces or Solids together with their Centres of Gravity G, H , are given, the Centre of Gravity I of their Difference will be given also by the above Analogy. This in many Mechanical Affairs will prove of very considerable Use.

Fig. 7.

13. Thus for Example, suppose $ABCG$ and $BDEF$ were two similar Cones, and their Centres of Gravity when hollow or superficial were c and a ; but when solid, d and b ; if then we suppose the Cone $BDEF$ cut off from the Cone $ABCG$, there will remain the Difference $ADECG$, whose Centres of Gravity, when hollow and when solid, may be found as above; which let be in the Circles eif and gkb , at i and k . If now the truncated Part were made into a Bucket by fixing in a thin Bottom at AC , and suspending it moveable in a Handle by Pins fix'd between the said Circles eif and gkb , on each Side, 'tis evident such a Bucket when empty must hang with its Mouth downwards; and when full of Water, with the Mouth upwards; thus may such a Bucket be applicable to divers Uses, as being capable of filling, and emptying, and moving itself in different Forms, as the Bottom is fix'd in the wide or narrow End.

Pl. VII.
Fig. 8.

14. The Centre of Gravity of a Parabola ABC , is found as in the Triangle and Cone; thus let $BF = x$, $DE = y$; then will $y\dot{x}$ be the Fluxionary Weight, and yxx the Fluxion of the *Momenta*; but from the Nature of the Curve we have $y = x^{\frac{1}{2}}$, whence $y\dot{x} = x^{\frac{1}{2}}\dot{x}$, and $yxx = x^{\frac{1}{2}}\dot{x}x$, whose Fluent $\frac{2}{3}x^{\frac{3}{2}}$ divided by $\frac{2}{3}x^{\frac{3}{2}}$ (the Fluent of $x^{\frac{1}{2}}\dot{x}$) will give $\frac{3}{5}x = BF$ for

WHENCE,

WHENCE, since the Earth and Moon are to each other as about 40 to 1, and

the Distance of the Centre of Gravity from the Vertex B in the Part DBE; and so $\frac{2}{3}$ of BG is the Point in the Axis of the whole Parabola ABC, from the Vertex B.

15. The Phænomenon of the double Cone is easily accounted for, from what we have said of the Centre of Gravity. For let ABD be the common Base of the two Cones, its Centre C will be the Centre of Gravity of the Whole; therefore if DF be the Leg of a Ruler elevated to an Angle FDG, whose Sine FG is less than the Semidiameter of the Cone CD, 'tis plain the Centre of Gravity C at the Position of the Cone in D is more distant from the Centre of the Earth than in its Position between the Legs of the Ruler at F; and therefore it will descend (as on an inclined Plane CFE) from C to F, where it will stop, as being supported on the Ends of the Ruler. Fig. 9.

16. Let ABED represent a Section of a Cylinder of Wood bias'd on one Side with a cylindric Piece of Lead as B, this will bring the Centre of Gravity out of the Centre of Magnitude C to some Point G between C and B; let FH be an inclined Plane, whose Base is FL. It is evident the Cylinder laid upon the Plane will no where rest but there, where a Perpendicular to the Horizon FL passes through the Centre of Gravity G, and that Point of the Plane E in which the Cylinder touches it; and this in all Angles of Inclination of the Plane less than that whose Sine is equal to CG (the Radius being CD) will be in two Situations ABED and *abed*, because when the Cylinder moves, the Centre of Gravity describing a Circle round the Centre of Magnitude C, this Circle will meet the Perpendicular in two Points G and g, in each of which the Centre of Gravity being supported; the Cylinder will rest. Therefore the Cylinder moves from E to *e* by the Descent of the Centre of Gravity from G to g, in the Arch of the Cycloid Gbg. Fig. 10.

the Distance of the Moon 60 Semidiameters of the Earth, the Distance of the

Pl. VII. 17. If the Cylinder A B E D insliding on the horizontal Line E L in the Point E, has the Centre of Gravity G in the horizontal Diameter D B, it will gravitate in the Perpendicular G e; if therefore a Plane F H touch the Cylinder in the Point e, it is plain the Cylinder cannot either ascend or descend on such a Plane; because G in any Situation between e and H, or e and F, it will gravitate to the Left or Right from the Point in which the Cylinder touches the Plane, and so will in either Case bring it back to the Point e.

18. In this Case the Angle of the Plane's Inclination H F L is equal to the Angle E C e or C e G, because E L and F H are Tangents to the Cylinder in the Points E and e, and consequently the Angles E C e + E F e = two Right Angles = E F e + H F L; therefore E C e = H F L. Hence it follows, that the Cylinder cannot ascend on a Plane whose Inclination is greater than the Angle E C e, but on a Plane of less Inclination it may; and there is a certain Inclination of the Plane on which the Ascent will be a *Maximum*, but 'tis not worth while to insist farther on so useless a Subject.

Fig. 12. 19. If D L M E and F P Q O represent two Waggon loaded, the former with heavy Matter, as *Iron, Stone, &c.* which lies low, and the latter with a light Substance that rises high, both on the Side of an Hill A B; 'tis evident the Centre of Gravity of the Load L M, acting in the Direction R I, which falls between the Wheels D E will keep the Waggon firm on the Side of the Hill, (by *Art. 5.*) But in the other Load N P Q O, where the Centre of Gravity K rises so high above the Bottom of the Waggon, it must act in a Direction K H, which falling without the Base Line F G of the Wheels, must necessarily cause the Waggon to overturn.

20. For the same Reason a Man must fall when the Centre of Gravity acts in a Direction falling without the Base-line of the Feet; and 'tis by the artful adjust-

common

common Centre of Gravity of the Earth and Moon will be found about 1854 Miles from the Earth's Surface; and it is this common Centre of Gravity that describes the *Annual Orbit* about the Sun, and not the Earth itself, as is commonly said and thought.

IN like manner there is a common Centre of Gravity of the Sun and all the Planets which circulate about him; and were they all placed in a Right Line on one Side the Sun, then would the common Centre of Gravity of the whole System be distant from the Sun's Surface *eight Tenths* of his Semidiameter: And it is about this common Centre of Gravity, and not about the Sun, that not only all the Planets, but even the Sun itself, do constantly move (XXXVI.)

ing of this Point over the Rope that People can walk or fly thereon; and that Tumblers and Equilibrists perform such Wonders. In fine, 'tis this single Principle that regulates every Kind of Motion, both Animal and Mechanical, and to which we are naturally prone to have a much stricter Regard than we commonly think of; as would be evident were we nicely to consider the Gait, Posture, and Configuration of the Body under the various Modes of Acting, viz. of *Walking, Sitting, Riding, Stooping, Carrying, Lifting, Drawing, Pushing, Reaching, Striking, Throwing, &c.* and the particular Manner in which Artists fix a Wind-Mill, Crane, or any such like Machine.

(XXXVI.) 1. If two Bodies A and B be connected together, by a Wire or Chain A B, they will each of

Pl. VII.
Fig. 13.

THE

THE common Centre of Gravity of any Number of Bodies being supported, none

them affect the other by their gravitating on the Line AB ; and also every Particle of that Line will be affected with those Gravities, or be carried downwards, but with an unequal Force, except *one*, on which the Force of each Body is equal; and since we shew'd the Force of any Body A or B arises from its Quantity of Matter multiplied into its Velocity, 'tis evident, that one Particle must be so situated at C , that $B : A$ ($:: v : V$) $:: AC : BC$. Because then we have $B \times V = A \times v = B \times AC = A \times BC$. Or the Product of each Body into its Velocity is the same at the Point C , but no other.

2. The Point C is therefore call'd the *Common Centre of Gravity* of both the Bodies. If the Distance between the Centres of the Bodies be given, and the Magnitude of each, the Distance of the Point C from either may be thus found; as $B + A : B :: AC + BC$ ($= AB$) $: AC$. Let A represent the Earth, and B the Moon; then will $A : B :: 40 : 1$, nearly; and since $AB = 60$ Semidiameters of the Earth; we have

$$40 + 1 : 1 :: 60 : \frac{60}{41} = AC; \text{ and since a Semidiameter contains about } 4000 \text{ Miles, we have}$$

$$\frac{60 \times 4000}{41} = 5854 = AC, \text{ from whence taking one Semidiameter } AG = 4000, \text{ there will remain } GC = 1854 \text{ Miles, for the Distance of the Point } C \text{ from the Earth's Surface.}$$

3. Since the Earth and Moon act on each other by Attraction, 'tis evident they must both revolve about the common Centre of Gravity C ; whence this Point C , and not the Centre of the Earth A is that which the Moon regards in her periodical Revolutions, and were there no other Bodies in the Heavens but the Earth and Moon, this common Centre of Gravity C would be at Rest, or a fix'd Point.

4. But since the large Body of the Sun commands (by the same Power of Attraction) the Earth and Moon

of

of those Bodies can fall; which is the Reason of many very surprizing Appearances

to revolve about itself, it will follow that the Point C is that which must describe the Circle (or *Orbis Magnus*) about the Sun; because no other Point between A and B can keep always at the same Distance from the Sun, on Account of the mutual Revolutions of those Bodies about that Point at the same time they are carried about the Sun.

5. But to illustrate this Matter further, let S be the Sun, and C D E F a Part of the annual Orbit; A and B the Earth and Moon in her Conjunction at C, in her Quadrature at D, in her Opposition at E, and in her last Quarter at F; during all these Motions from C to F, the Centres of the Earth and Moon will describe Curves of the *Cycloid* kind, every where concave to the Sun; and though this may appear a Paradox, yet it is capable of a Geometrical Demonstration, and may be also shewn by Experiment. In the common Astronomical Tables the Centre of the Earth is supposed to describe the *Orbis Magnus*, and the Place and Distance of the Sun or Earth is computed accordingly; but these will be different from the true ones, which must be computed from the Circle which the Earth does monthly describe about the common Centre of Gravity; and this Difference is call'd the MENSTRUAL PARALLAX.

Pl. VII.
Fig. 14.

6. The Point C is that in which the Force of each Body A. and B, is exerted; and therefore every Action of that Point is the same as it would be if both the Bodies were there united in one; thus if A and B were two Chain-Shots, the Point C in the Chain, and no other, would strike an Obstacle with the greatest Force, or that of a Body equal to the Sum of A and B.

7. If therefore a Wire be inserted in the Point C, and at the other End any Body D were affixed, the common Centre of Gravity of all the three Bodies A, B, and D may be found as before; for let the Body A = A + B; then will C + D : D :: CD : CE; or A + B + D : D :: CD : CE; and thus E will be

in

Fig. 13.

in Nature, as that common Experiment of suspending a Bucket of Water at the End of

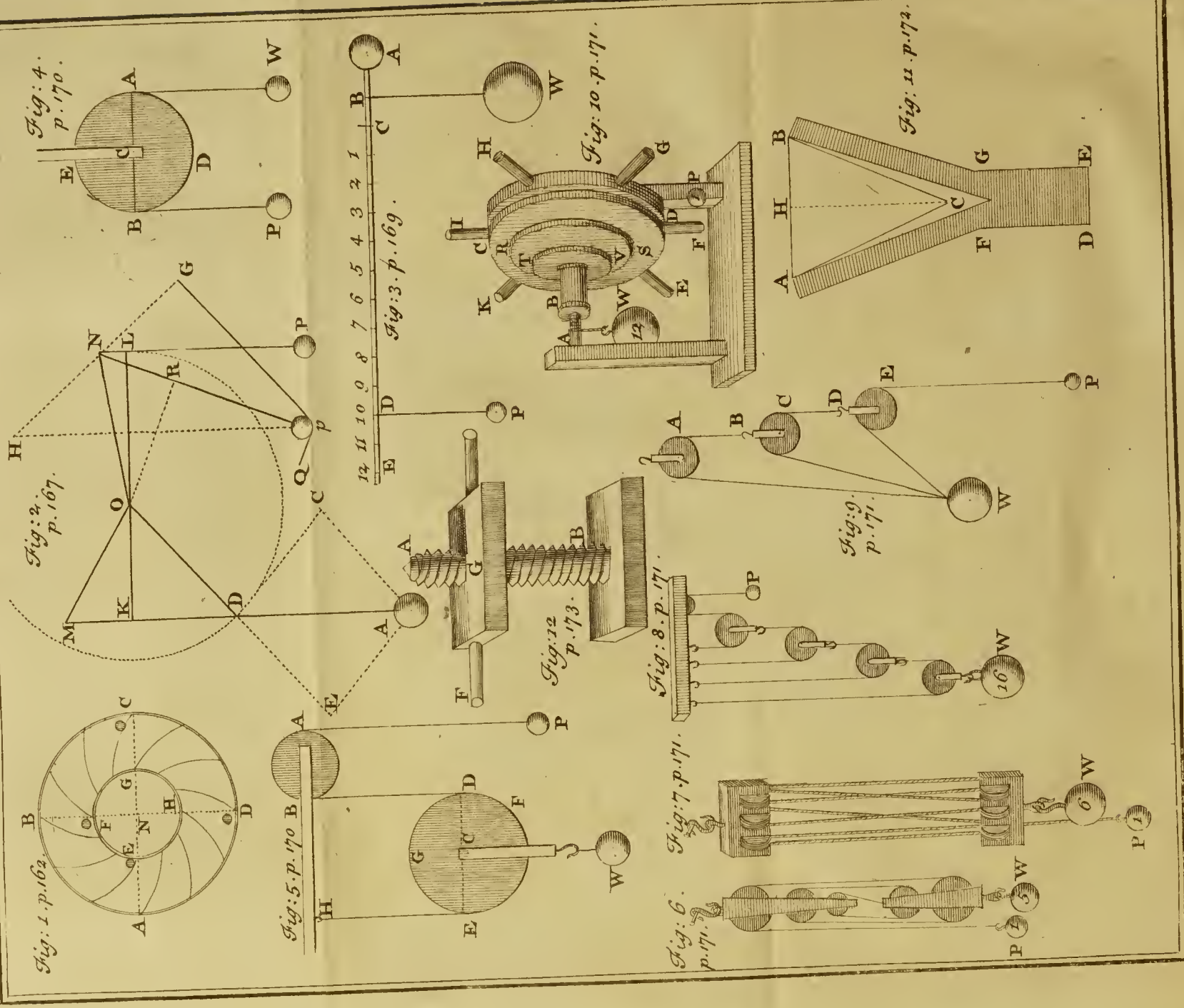
the Point sought. And in the same Manner the common Centre of Gravity in a System of any Number of Bodies may be found.

8. Hence the Common Centre of Gravity of the Sun and Planets may be easily found by comparing their Quantities of Matter and Distances; thus let ABC represent the Sun's Body, and S its Centre. Now since the Bulks and Distances of the four first Planets, *Mercury*, *Venus*, the *Earth* and *Mars*, are very inconsiderable in regard of the Sun, they would not (if placed in the Right Line CG) remove the common Centre of Gravity between the Sun and themselves far from the Centre of the Sun, suppose to L. But when we come to *Jupiter*, his Bulk and Distance give him a considerable *Momentum*, which will (if placed in the same Line CG) remove the Centre of Gravity from L to I, a Point without the Surface of the Sun's Body. Lastly, if we consider *Saturn* placed in the Line CG, with all the rest; though his Quantity of Matter falls short of, yet his Distance far exceeds that of *Jupiter*, and therefore his *Momentum* will be considerable enough to bring the common Centre of Gravity from I to K, at such a Distance CK from the Sun's Surface as is equal to $\frac{3}{10}$ of the Sun's Semidiameter SC; or $SC : CK :: 10 : 8$. Now it is this Point K which is the fixed and immoveable Centre of the System, about which the Sun, as well as all the Planets, continually move. But the Sun being always very near it, and its Distance therefrom varying with the different Positions of the Planets, the Motion of the Sun about this common Centre K will be very irregular and unequal; while that of the Planets, on account of their great Distance, may be esteem'd nearly uniform and circular.

9. If the two Bodies A and B move to or from each other in the same Right Line AB, with Velocities proportional to their Quantities of Matter inversely, the common Centre of Gravity C will remain at Rest, because their Distances from it will by that means be always in the same Ratio, viz. of their Masses inversely, which

a Stick

RPJCB



a Stick off the Table without falling; the Toy of the Man a sawing, &c.

WE have now premised every thing necessary for understanding the Nature of those Instruments which are commonly call'd MECHANICAL POWERS or MACHINES: They are Six in Number, as follow, *viz.* The *Lever*, the *Pulley*, the *Wheel and Axle*, the *Inclined Plane*, the *Wedge*, and the *Screw*. They are call'd *Mechanical Powers*, because they increase our Power of moving or raising heavy Bodies, which are often unmanageable by any natural human Strength, not thus assisted; and of two or more of these all other compound Instruments and Machines are contrived and composed.

I. A LEVER is any inflexible Line, Rod, or Beam, moveable about or upon a fix'd Point, (call'd the Prop or *Fulcrum*); upon one End of which is the Weight to be raised, at the other End is the Power applied to raise it, as the Hand, &c. Since (as we have before proved) the *Momen-*

is the only thing that makes that Point the Centre of Gravity between them. But if both Bodies were to move in two different Planes, either in the same or different Directions, the Centre of Gravity C would describe a Right Line, and with a Velocity of the same Sort as that with which the Bodies themselves do move.

tums of the Weight, and Power, are as the Quantities of Matter in each multiplied by their respective Celerities; and the Celerities are as the Distances from the Centre of Motion, and also as the Spaces pass'd through in a perpendicular Direction in the same Time; it must follow, that there will be an Equilibrium between the Weight and Power, when they are to each other reciprocally as the Distances from the Centre, or as the Celerities of the Motions, or as the perpendicular Ascent or Descent in the same Time; and this universally in all Mechanical Powers whatsoever, which is therefore the fundamental Principle of all Mechanics (XXXVII.)

(XXXVII.) 1. The Nature of this Proposition being not understood by Smatterers in Mechanics, gave them Occasion to imagine the Possibility of a *Perpetual Motion* from one Part of it, which they did not see was utterly impossible from another Part of it.

Pl. VIII. 2. That Part which seem'd to promise the Possibility
Fig. 1. thereof, was this, viz. *That the Momenta of equal Bodies were as their Distances from the Centre of Motion.* Hence, say the Perpetual-Motion-Men, if a Wheel were constructed of the Form of that in the Figure ABCD, with circular Cells going from the inner Part EFGH to the outer, containing equal Balls C, D, E, F, then upon turning the Wheel, they must move towards the Centre N on one Part, as the Ball E, and from it on the opposite Part, as the Ball C; and by this Means the Ball C will have a greater *Momentum* than the Ball F, and so will determine the Wheel to move round; and since this must be the Case of all the Balls E and C that come into the Situation E, C, the Wheel

To illustrate this, let AB be the Lever Plate V.
 supposed without Weight, and F its *Ful-* Fig. 9.
crum or Prop; let W be a Weight sus-
 pended from the End A , and P the Power
 applied to the other End B . Then let the
 Lever be moved into the Situation CD ,
 'tis evident the Velocities of the Points
 A and B will be as the Arches AC and
 BD described in the same Time: Also the
 perpendicular Distances, through which the

must necessarily move continually, because it will con-
 tinually bring two Balls into that Situation.

3. 'Tis true, were there but two Balls E and C , the
 Ball C would by this Contrivance move the Wheel
 one Quarter round, *viz.* while it descended from C to
 D ; and by this Means would raise the Ball E to F ;
 and there they will abide in the Situation DF ; but, say
 the Gentlemen of this Persuasion, two other Balls suc-
 ceeding to the Places E and C , will still keep the Wheel
 moving.—Yes, so they would, if the Balls at D and E
 could be taken away the Moment they come into that
 Position, not else; for the Balls C and E , in order to
 move the Wheel, must move the Balls D and F , which
 have equal *Momenta*, (as being at the same Distance each
 from the Centre, as are the other two respectively)
 which is absurd by the *general Proposition*.

4. The Absurdity of a Perpetual Motion will still
 farther appear, if we consider, that the *Momenta* of
 Bodies are always proportion'd to the perpendicular
 Descent or Ascent to or from the Centre of the Earth.
 Since, therefore, in the Wheel, the Bodies are all
 equal by Supposition, and the perpendicular Spaces
 through which they *descend and ascend* below and above
 the Horizontal Line or Diameter AC , are equal; it
 follows, that an *Equilibrium* must necessarily ensue.
 Thus so far is this Wheel from producing a Perpetual
 Motion, that it admits of none at all.

Weight W and Power P move in the same Time, are CE and GD , which are as the Arches AC and BD ; and these are as the Radii CF and DF , which are equal to AF and BF . Therefore in order to produce an *Equilibrium*, it must be $W \times AC = P \times BD$, or $W \times CE = P \times DG$, or $W \times AF = P \times BF$: Consequently, $P : W :: AC : BD :: CE : DG :: AF : BF$. Note, that in estimating the Effects of Machines, we regard only the Distances of the Power or Weight which are perpendicular to the Lines of Direction in which those Powers act, as FB , or FM , which are perpendicular to the Directions PB and LM .

THE Lever is of five Kinds. (1.) The common Sort, where the Prop is between the Weight and the Power, but nearest the former. (2.) When the Prop is at one End, the Power applied at the other, and the Weight between both. (3.) When the Prop is at one End, the Weight at the other, and the Power applied between both. (4.) The bended Lever, which differs only in Form from the first Sort. (5.) When the Prop is placed at an equal Distance between the Weight and the Power, and this is commonly call'd

THE BALANCE, whose Use is, with a Pair of Scales, to bring one Body to an equal

equal Weight with another that is a Standard. The *Proportional Balance* is without Scales, and is used for discovering or assigning any Proportion of Weight in Bodies. The *False Balance* makes Bodies of unequal Quantities of Matter appear to have equal Weight. Lastly, the *Statera*, or *Roman Balance*, commonly call'd the *Steelyard*, is a most useful Kind of universal Balance, the Structure and Use whereof will be easy to understand from the above Principles.

II. THE PULLEY is an Instrument well known; if single, it is reduced to the Lever of the fifth Sort, or Balance, and so affords no Advantage in raising Weights. If two or more be combined together in the common Way, *Then the Power is to the Weight as Unity to the Number of Ropes which goes to the Pulleys of the lower Box.* But there are different Ways of applying Pulleys, whose *Advantage or Power* will be obvious from a View of the Structure of the several Sorts of *Tackles*, and the Experiments with them.

III. THE WHEEL and AXLE (commonly call'd the *Axis in Peritrochio*) is the third Mechanical Power. We easily see by its Make it is reducible to a Lever of the first Sort; and that therefore *the Power is*

to the Weight, as the Diameter of the Axle to the Diameter of the Wheel, in an Equilibrium in this Machine.

IV. THE INCLINED PLANE is the fourth Mechanical Power, and from a due Consideration of it, it will appear, that, for an *Equilibrium*, the Power must be to the Weight, as the Height of the Plane to the Length.

V. THE WEDGE is only a double Inclined Plane, intended to separate the Parts of Wood, &c. which strongly cohere together; whence, in the common Form of it, the Power will be to the Resistance to be overcome, as half the Thickness of the Wedge to the Length thereof.

VI. THE SCREW is the last mention'd Mechanical Power, whose Use is both for Pressure and raising of Weights, but chiefly the former. The Power is to the Weight, as the Velocity of the Weight to the Velocity of the Power, that is, as the Distance between two Threads of the Screw to the Circumference of a Circle described by the Power (XXXVIII.)

(XXXVIII.) 1. Notwithstanding the Demonstration of the Fundamental Principle of Mechanics, as deliver'd in the Lectures, is most natural and perspicuous, and easily results from what has been said of the *Momentum*

WE have here consider'd the Action or Effect of each of these Machines, as they

of Bodies and their common Centre of Gravity: Yet as Sir *Isaac Newton* has demonstrated the same Thing in a different and most extensive Manner, which also is the Consequence of a different Principle, *viz.* the *Composition and Resolution of Forces and Ratios*, it will be proper to exhibit and explain that also, which is as follows.

2. Suppose two Weights A, P, appended by the Strings M A, N P, at the Ends of unequal Radii O M, O N, issuing from the Centre O of any Wheel, in a State of *Equilibrium*; those Weights will be to each other reciprocally as O L to O K, that is, $A : P :: O L : O K$, or as the nearest Distances of their Lines of Direction from the Centre reciprocally. Pl. VIII. Fig. 2.

3. For on the Centre O with the Radius O L describe an Arch cutting the Thread M A in D, and draw O D which continue out to E, to which draw A E perpendicular and compleat the Parallelogram A E D C. Now since K L passes through the Centre O, and is perpendicular to each String in the Points K and L by Supposition; it matters not whether the Body A and P be suspended from the Points M and N, or K and L, or D and L, since the Weight of the Bodies is the same in either of those Points respectively.

4. Therefore let A D expound the whole Force of the Weight of the Body A, and let it be resolved into the two Forces D E and A E, of which the former drawing directly from the Centre avails nothing in moving round the Wheel; but the other Part A E or C D, acting perpendicularly upon the End of the Radius O D, has the same Force or produces the same Effect as if it had acted perpendicularly at the End of the equal Radius O L.

5. Therefore the Weight of P will be express'd by D C (or A E) because of the *Equilibrium*; and hence the Weight of A is to that of P as A D to A E; and because of the similar Triangles A E D and D O K,

M 4

would

would answer to the Strictness of the Mathematical Theory, were there no such

we have $AD : AE :: DO (= LO) : OK$; therefore $A : P :: OL : OK$.

6. If the Weight p , (equal to P) suspended by the String Np , does at the same time in part rest upon the inclined Plane pG ; let pQ be drawn perpendicular to pN , pH perpendicular to the Horizon, and HG through the Point N perpendicular to pG : Then will the Tension of the String $P N$ be to that of the String pN , as pH to pN . For suppose the Body p wholly supported by the two Planes pG , pQ , it will press them with its whole Force, which may be express'd by pH , and which is resolvable into the two Forces HN and pN , of which the first presses the Plane pG , and the last the Plane pQ ; if therefore the Plane pQ be removed, the same Force will stretch the String pN ; but since $p = P$, the Force with which P stretches the String $P N$ is as pH ; therefore the Tension of the String $P N$ is to that of the String pN as pH to pN .

7. Let the Force of p to move the Wheel (as supported on the Plane) be called x . Then $P : x :: pH : pN$; and so $P = \frac{x \times pH}{pN} = \frac{A \times OK}{OL}$ (by 5th),

whence we have this Equation, $x \times pH \times OL = A \times OK \times pN$; therefore $A : x :: pH \times OL : pN \times OK$. But here 'tis to be observed that as the String PN is changed into the Position pN , its nearest Distance OL will become OR , so that the Analogy between A and x corrected will be $A : x :: pH \times OR : pN \times OK$.

8. From what is demonstrated it follows, (1.) That the Line KOL is a LEVER, whose Power is always express'd by the Ratio $\frac{OL}{OK}$. (2.) That the Lines M

OL , MON , DOL , are also Levers of equal Force, because the nearest Distance of their Lines of Direction from the Centre O is the same in all. (3.) Whenever the Lever KL is moved, the Velocity of the Points thing

thing as Friction or rubbing of Parts upon each other, by which means *one Third*

K and L will be proportional to the Distances from the Centre K O and O L. (4.) Therefore the Velocities also, and perpendicular Spaces pass'd through by the Bodies A and P. will be proportional to the said Distances from the Centre.

9. Since $P : A :: K O : L O$; 'tis evident the Power P may be in any Proportion less than the Weight A, if its Distance from the Centre of Motion O L be in the same Proportion greater than the Distance of the Weight K O, and yet its Force or *Momentum* shall be equal to that of the Weight A. Whence the Nature of a Lever for increasing a Power sufficiently appears.

10. If the Distances from the Centre O K = O L; then will the Lever become a *common Balance*, because then, in order to an *Equilibrium*, the two Bodies A and P must have equal Weight.

11. If the Arms of the Balance K O and O L differ but a very little in Length, there will be the same Difference in the Weights A and P. Whence if O K : O L 31 : 32 :: P : A; and therefore if A is an *Averdupois* Pound, P will want $\frac{1}{2}$ an Ounce of it and yet be in *Equilibrio*. Hence the Nature of a *False Balance* is evident; and in order to detect it we need only interchange the Weights, for then, in order to an *Equilibrium*, we must have $K O : O L :: (31 : 32 ::) 32 : 33 \frac{1}{11}$; but of this there wants $2 \frac{1}{11}$ half Ounces, which will render the Fraud very notorious.

12. The Arms of a Balance being nicely equated in Weight and Length, and divided into an equal Number of equal Parts, becomes the *Proportional Balance* to be used without Scales. For any two Bodies hanging on the Arms of this Balance in *Equilibrio*, will have the Proportion of their Weights express'd by the Numbers at the Divisions (whence they hang) alternately.

13. If A C the shorter Arm of a Balance be made Pl. VIII. equal in Weight to C E the longer Arm, by the Addition of the Ball A, so that the whole Beam A E

Fig. 3.

Part

Part of the Effect of the Machine is, at a Medium, destroy'd, as is evident by an ex-

may be nicely equipoised on the Centre of Motion C, it then becomes the *Statera* or STEEL-YARD in common Use. For if B be taken as a fix'd Point from whence to hang any Weight W, and P be any constant Weight, which by moving backwards and forwards on the longer Arm CE, comes to a Point D of the *Equilibrium*; then will $P : W :: CB : CD$; but because P and CB are given Quantities, or always the same, therefore W will always be as CD. Consequently if the Arm CE be divided into Parts each equal to CB, and number'd; the Number at which P hangs will always shew how many times P is contained in W, and thence its Weight will be known.

Fig. 4.

14. If A D B E be a Pulley, upon which hang the Weights P, W; then since the nearest Distances of the Strings A W and B P, from the Centre of Motion C, are A C and B C, the Pulley will be reduced to the Lever or Balance A B with respect to its Power; and from thence it appears that since $AC = BC$, we shall always have $P = W$ for an *Equilibrium*; and therefore no Advantage in raising a Weight, &c. can be had from a *single Pulley*.

Pl. VIII.
Fig. 5.

15. In a Combination of two Pullies A B and D F E G, the Power is doubled; for the Pulley D F E G is reducible to the Lever E D, which must be consider'd as fixed in the Point E to the immoveable String H E; and the Power acting at D is equal to P, and the Weight W is sustain'd from the Centre C of the Pulley, but $P : W :: CE : DE$; therefore since $DE = \frac{1}{2} CE$, it is $W = 2 P$, or $P = \frac{1}{2} W$.

16. The Force of the PULLIES may also be easily shewn by comparing the Velocities of the Power and Weight; for 'tis evident, if the Weight W be raised one Inch, each String H E, B D, will be shorten'd one Inch, and consequently the String A P will be lengthen'd *two Inches*, and so P will pass through twice the Space that W does, in the same Time, whence its Velocity

periment

periment of the Inclined Plane. And farther, concerning Friction we are to observe, that it is not proportional to the

will be twice as great, and therefore it will be equipollent to a Body W of twice its Weight.

17. In the two other Forms of Pullies 'tis evident Pl. VIII.
the Power P is to the Weight W as Unity or 1 to the Fig. 6, 7.
Number of Ropes going to the lower Pullies, because if the Weight be raised *one Inch*, each Rope belonging to the lower Pullies will be shorten'd one Inch, all which will go into the Rope to which the Power P is applied, which therefore must descend through so many Inches in the same Time. Consequently the Tackle of Pullies in the Form of *Fig. 6.* will increase the Power five Times; and that of *Fig. 7.* will increase it six Times.

18. In the Disposition of Pullies according to *Fig. 8.* Fig. 8.
'tis plain, since each Pulley has a fix'd Rope, it must be consider'd as a Lever of the second Sort, and so will double the Power of the foregoing Pulley, and so four Pullies will increase the Power sixteen Times.

19. For the Force and Conveniency of a Tackle Fig. 9.
of Pullies all together, none is equal to that in the Form of *Fig. 9.* where the uppermost Pulley is fix'd, and each has a Rope annex'd to the Weight; its Power is therefore thus estimated. When the Weight W is raised one Inch, the Rope AB will be lengthen'd as much, and so the Pulley C will descend one Inch, by which means the Rope CD will be lengthen'd two Inches, and one by the Rising of the Weight W ; wherefore the Pulley E will descend three Inches; and thus the Rope EP will be lengthen'd six Inches by that means (*viz.* three on each Side) also the Rising of the Weight will cause it to lengthen one Inch more; so that the Power P goes through seven Inches while the Weight W rises one; therefore $P : W :: 1 : 7$; and thus you proceed for any other Number.

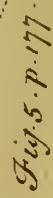
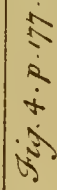
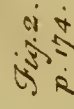
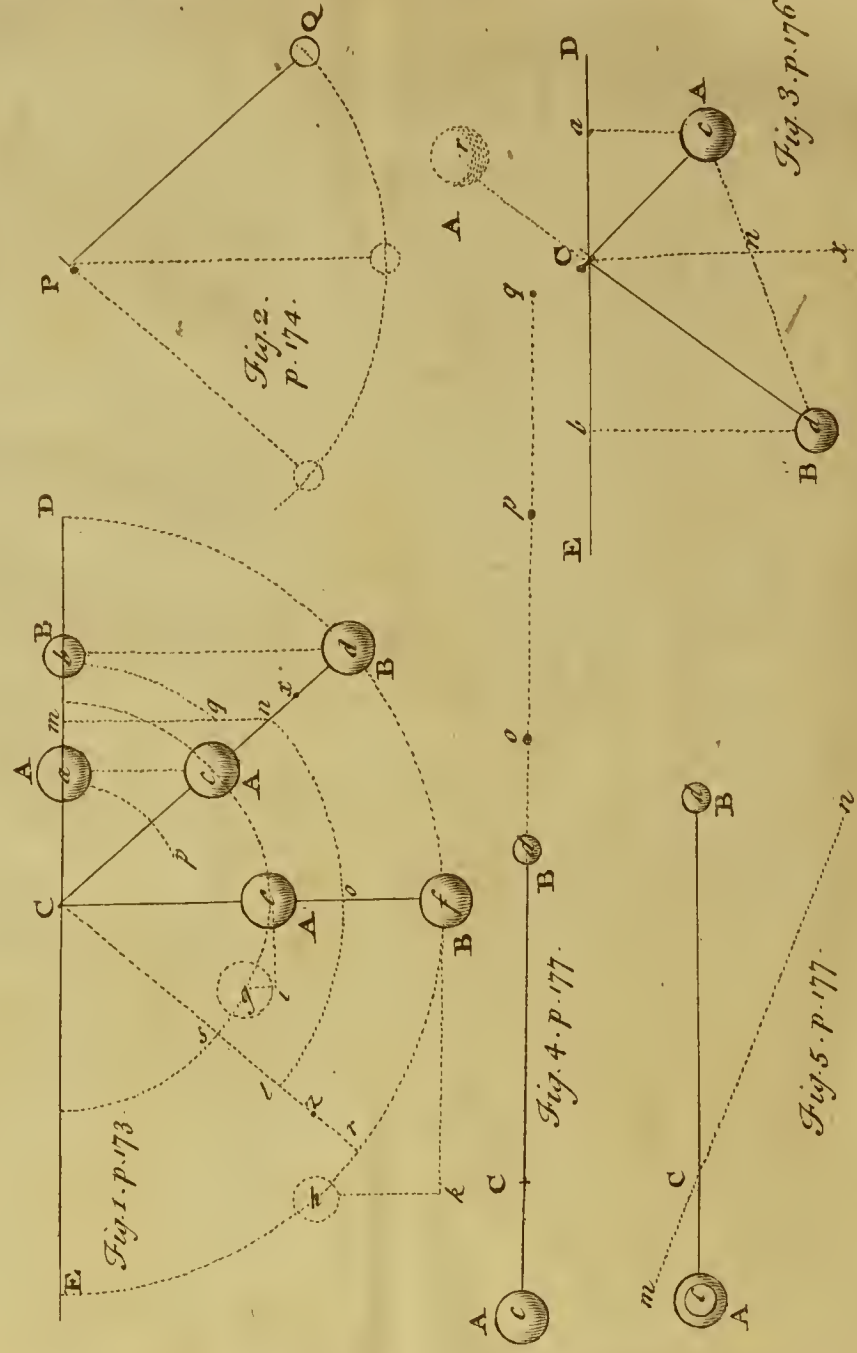
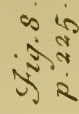
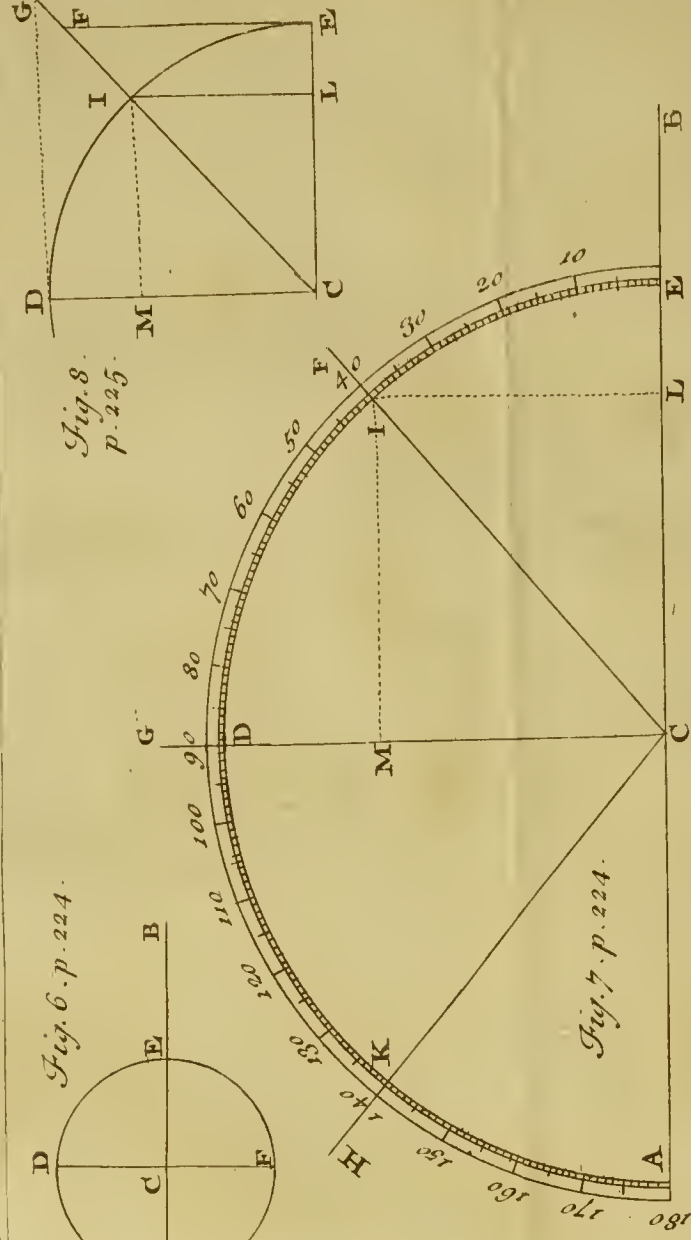
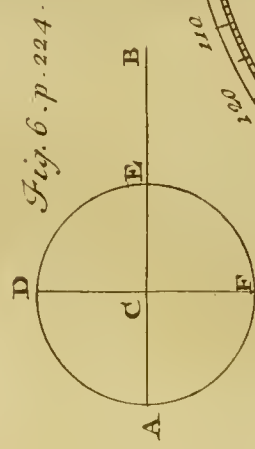
20. The WHEEL and AXLE is a Mechanical Power Fig. 10.
upon the same Principle; for the Weight W hanging from the Axis A will be to the Power P , which keeps it in *Equilibrio*, inversely as their Velocities; but the
Veloci-

Velocities are as the Circumferences of the Wheel and Axle; which again are as their Diameters or Semidiameters, that is, as their Distances from the Centre of Motion. If the Spokes or Handles F, G, H, I, K, &c. be added, the Power of the Machine is still farther augmented in Proportion to their Lengths.

Pl. VI.
Fig. 1. 21. That the INCLINED PLANE is a Mechanical Power appears sufficiently in its diminishing the Weight of a Body laid upon it in regard to the Power which holds it in *Equilibrio*. Let A be a Body sustain'd on the Inclined Plane BD; from the Centre C, draw CF perpendicular to the Horizon or Base DC; and CE perpendicular to the Plane, then CF will represent the whole Weight or Force of Gravity of the Body A, which is resolvable into the two Forces CE and EF; but the Force CE being perpendicular to the Plane acts wholly upon it, and is equally re-acted on or sustain'd by the Plane; the other Force EF, being parallel to the Plane, is that by which the Body descends, or is kept from descending by an equal Power acting in a contrary Direction. Therefore the whole Weight of the Body is to the Power which keeps it in *Equilibrio* on the Plane as CF to FE, or (because the Triangles CFE. and BDC are similar) as BD to BC, that is, as the Length of the Plane to its Height, as was more particularly shewn in *Annot. XXVIII. 1, 2.*

Pl. VIII.
Fig. II. 22. The Power of the WEDGE ACBH is evident from its consisting of two equal Inclined Planes AHC and BHC; as it is chiefly of use to separate the adhering Parts of Wood; and since the Power of Cohesion in Wood is every where variable and uncertain, 'tis evident there can be no regular Calculation of the actual Effect of the Wedge; but if we suppose the Power of Cohesion in the Wood ADEB to be uniform, or to make every where an equal Resistance to the Wedge ABC, dividing its Parts AF and BG, then the Power of the Wedge would be to the Resistance of the Wood, as their Velocities inversely, that is, as the Spaces moved through in the same Time, that is, as the Height of the Wedge HC to half its Width AH. But the Wedge being only a double inclined Plane, is not really a *different Mechanical Power*, tho' usually reckoned as such.

RPJCB



23. In the SCREW A B, suppose the Distance of the Pl. VIII. Spiral Threads $\frac{1}{10}$ of an Inch, and the Length of the Fig. 12. Lever F G = $16\frac{1}{2}$ Inches, then will the Circles described by the Hand at F be = $16,5 \times 6,3 = 103,9$, or 104 Inches nearly, that is, 1040 Tenths of an Inch; whence the Velocity of the Power is to that of the Weight, as 1040 to 1; therefore the Screw is a great Mechanical Power, either for raising Weights or Pressure, even though we allow the greatest Part for Friction; but it is properly a *Compound Power* consisting of the *Lever and Inclined Plane*; and therefore we can find but *four* such as we can properly call SIMPLE MECHANICAL POWERS.

SCHOLIUM:

24. Having treated of the *Simple Pendulum* in the preceding Lecture, and of the Nature of the Lever in this; I shall here let the Reader see how nearly these two Instruments are allied, or rather shew him that the *Lever* is in reality nothing but a *Compound Pendulum*; from whence many Particulars concerning the Properties of each, not hitherto mention'd, will occur worthy of Observation.

25. Let C D be a fine Rod or Wire (whose Weight Plate IX. is inconsiderable) and moveable about the Point C. If Fig. 1. A and B be two Bodies (of the same Kind) fixed at any Distance C a, C b on the Rod; and then the Rod permitted to descend freely; the Bodies A and B will begin their Motions with Velocities proportional to C a and C b, and so their respective *Momenta* will be A $\times$ C a and B $\times$ C b, in the Points a and b (by Annot. XXXVIII.)

26. Now if A : B :: m b : m a, then will A $\times$ a m = B $\times$ b m; and consequently the Point m will be the common Centre of Gravity between the Bodies A and B (by Annot. XXXV.) Wherefore since the Velocity of the Point m is as C m, its *Momentum* will be as C m $\times$ A + B; and from the Nature of the common Centre of Gravity, we have A $\times$ C a + B $\times$ C b = C m $\times$ A + B; and therefore $\frac{A \times C a + B \times C b}{A + B} = C m$; or

by

by putting $Ca = a$, $Cb = b$, and $Cm = d$; then $\frac{Aa + Bb}{A + B} = d$. Suppose the Moment the Rod CD began to descend, the Bodies A and B were to be disengaged from it in such a Manner as to descend upon it freely; then will they descend (not in the Curves ap and bq , but) in the Tangents ac and bd , describing the Spaces ac , bd , which will be as the Inceptive Velocities Ca , Cb ; and of course when the Rod comes into the Situation Cd , the Bodies will be at c and d ; it being in this Case $Ca : Cb :: ac : bd :: Cc : Cd$. And mn will be the Line described by the common Centre of Gravity.

27. At c and d , let us suppose the Bodies again fix'd to the Rod, they will then constitute a *Compound Pendulum*, which will now vibrate forwards and backwards through the Angle dCr , the Body A describing the Arch ces , and B the Arch dfr , and the common Centre of Gravity n the Arch nol . As this Pendulum vibrates, the Velocities of the Bodies will be everywhere as their Distances, and greatest of all in the Perpendicular Situation Cef : Where the Velocities of the Points e , o , f , may still be represented by a , d , and b .

Plate IX.
Fig. 2.

28. Now this *Compound Pendulum* is isochronous to (*i. e.* vibrates in the same Time with) some simple Pendulum of a determinate Length PQ . If then in the *Compound Pendulum* Cd there be taken $Cx = PQ$, the Point x is called the Centre of Oscillation in this *Compound Pendulum*; for if the Bodies A and B were there fixed, the Times of Vibration would be the same then as now: To determine this Point therefore, is to determine the Time of Vibration in any *Compound Pendulum* whatsoever.

29. In order to this; let us once more suppose the Moment the Bodies A and B come to the Points e and f in the Perpendicular, they were disengaged from the Rod, and were to ascend at the End of a String separately, with the Velocities they had acquired by their Descent in the Points e and f . Then the Altitudes ig and kh , to which they will ascend above the Horizontal Line ei and fb , will be as the Squares of the Velocities

locities at e and f ; that is, as aa and bb ; as is evident from what has been elsewhere shewn (*Annotat.* XXVI. 2, 3.)

30. The Bodies A and B will in this Case arrive to the Points g and h before their Motion will be destroy'd. And 'tis plain that eg is less than ec or es , because the Body A is retarded in coming from C to e by the Body B. Also fb is greater than df or fr , because B has its Motion quicken'd by the Body A. And as when in passing from d to f , the perpendicular Spaces descended thro' are as the Velocities singly, and the *Momenta* as the Velocities and Masses of Matter conjointly; so on the other Side, in passing from f and e to h and g , the perpendicular Spaces ascended through are as the Squares of the initial Velocities, the *Momenta* will be as the Masses of Matter, and those perpendicular Spaces conjointly; that is, as Aaa and Bbb .

31. Now in all Cases, the Sum of the *Momenta* divided by the Sum of the Masses gives a Quotient expressing the Velocity, or Ascent or Descent of the Centre of Gravity. (See *Art.* 26.) Therefore $\frac{Aaa + Bbb}{A + B}$

$= d =$ the Ascent of the Centre of Gravity; for the *Momentum* of this Centre of Gravity is on both Sides the Perpendicular Cf the same at equal Distances, and therefore it will always ascend or descend through the same perpendicular Heights.

32. Having thus obtain'd the Expression of the Descent of the Centre of Gravity, we have that for the Centre x of Oscillation of course, for we have $As Cn : Cx :: d : x :: \frac{Aaa + Bbb}{A + B} : \frac{Aaax + Bbbx}{Ad + Bd} =$

Descent of the Centre of Oscillation. But the Descent is equal to the Ascent; for the Point x is actuated and moved in the same Manner as the Body Q, which in one Oscillation descends and ascends through equal perpendicular Spaces, and those Spaces are always as the Squares of the Velocities in the lowest Point; and therefore for the Point x , it will be as $Cx^2 = xx$. Whence

we have $\frac{Aaax + Bbbx}{Ad + Bd} = xx$; and therefore

A au

$\frac{Aaa + Bbb}{Ad + Bd} = x = Cx$, the Distance of the Centre of Oscillation from the Centre of Motion.

33. Because (in *Art.* 26.) we had $\frac{Aa + Bb}{AB} = d$; if we substitute this Equivalent for d in the above Equation, we have $\frac{Aaa + Bbb}{Aa + Bb} = x$; and therefore $Aaa + Bbb = Aax + Bbx$. Hence $Aa - Aax = Bbb - Bbx$; and so $Aa : Bb :: b - x : x - a$; that is, $A \times Cc : B \times Cd :: xd : xc$. From whence it appears, that the Momenta of the Bodies or their Power to move the Pendulum, are inversely proportional to their Distance from this Centre; which is the true Definition of the Centre of Oscillation.

34 Therefore when $A : B :: b : a$, and so $Aa = Bb$; then also $b - x = x - a$, or $xd = xc$; or the Centre of Oscillation would bisect the Distance between the Bodies A and B.

35. If $a = 0$, or the Body A be removed to the Centre C, then $Aaa = 0$, and $Aa = 0$; whence $x = \frac{Bbb}{Bb} = b$, that is, the Instrument then becomes a simple Pendulum Cd .

Plate IX.
Fig. 3.

36. If the Bodies A and B are not placed both on one Rod, but on two Rods Cc and Cd , making an Angle cCd , being united in the Angle C. Then these Rods suspended on the Angle C will be the same Compound Pendulum as before, and the Centres of Gravity and Oscillation n and x , when the Bodies are at rest, will be both in the Perpendicular Cx , as in the other Case, and express'd by the same Equations, viz. $Cn = d = \frac{Aa + Bb}{A + B}$; $Cx = x = \frac{Aaa + Bbb}{Aa + Bb}$. And it will oscillate in the same Time with the simple Pendulum PQ , when $Cx = PQ$.

37. If the Body A be supposed removed on the other Side the Line ED , its Distance $Cr = a$, is now negative, or it will be $-a$ in the Equations for the Centres; thus $d = \frac{-Aa + Bb}{A + B}$; and $x = \frac{Aaa + Bbb}{-Aa + Bb}$ because

because $-a \times -a = aa$. Hence in this Case 'tis very obvious that the *Compound Pendulum* is now become a *Lever*, whose *Fulcrum* is the Point C; and A may be consider'd as a Weight to be moved by the Power B. And hence because when Bb is greater than Aa, d will remain affirmative, therefore n the Centre of Gravity will be in the Part Cd, and so will cause B to preponderate.

38. On the contrary, if Aa be greater than Bb, then will d be negative, or the Centre of Gravity n will be in the Arm Cr, and cause the Weight A to descend. But when Aa = Bb, then $-Aa + Bb = 0$, and $d = \frac{0}{A+B} = 0$, that is, n or the Centre of Gravity will fall upon the Point C, and produce an *Equilibrium*; and $A : B :: b : a :: Cd : Cr$. As was before shewn to be the Property of the Lever. (See *Annot. XXXVIII.*)

39. Thus also, with respect to the Centre of Oscillation, $x = \frac{Aaa + Bbb}{-Aa + Bb}$; 'tis plain, in case of an

Equilibrium where $Aa = Bb$, we have $x = \frac{Aaa + Bbb}{0} = \text{Infinite}$; for since the Lever in this Case has no Motion, it can only be isochronous to a simple Pendulum of an infinite Length, whose Times of Vibration are infinitely great, and its Motions, of course, not sensible. Plate IX.

40. It is farther evident, that the Centres of Oscillation and Gravity are both on the same Side of the Centre of Motion C; and that when Bb is greater than Aa, the Centre of Oscillation will be somewhere on the Side of B in the Line Cd continued out; if Bb exceeds Aa but a little, it will be at a great Distance, as at q; if the Excess be greater, it will be nearer, as at p. If it be very great, the Centres will be very near, as at o; and when Bb is infinite with respect to Aa, then the said Centre will be in the Centre of the Body B. Fig. 4.

41. Let the Lever cCd be in *Equilibrio* with the Weights A and B; and let it be required to raise the Weight A from c to m, by the Power B; then if Bb Fig. 5.

be ever so little greater than Aa it will descend, till the Lever comes to the Situation mn , as required. The whole Action of the Machine consists in raising the given Weight A to a given Height H , in a given Time; and this Effect of the Machine will (*ceteris paribus*) be greater as the Time is less. But the Time is the Time of half the Vibration of the Lever; and the Times of Vibrations are less as the Intensity of the Power B is increased, while its Distance continues the same, because the Distance of the Centre of Oscillation is diminished.

42. Now putting $A =$ Body to be raised, $H =$ Height, and $T =$ Time, then $\frac{AH}{T} = E =$ the Effect, or Intensity of the Power B ; whence $AH = ET =$ whole Action of the Machine; and therefore when ET is least of all, the Machine will be in its greatest Perfection; because as E increases, T will decrease, and though it be not in the same Proportion, yet it will cause that ET will become a *Minimum* at a certain Limit; as will be evident by the following Example.

43. Suppose $A = 100lb.$ and $B = 10lb.$; and if $Cc : Cd :: 10 : 100$; then there will be an *Equilibrium*, in which Case T will be infinite, and therefore also TE , and consequently cannot be express'd in Numbers.

44. If $B = 11lb.$ then $\frac{Aa + Bb}{-Aa + Bb} = \frac{100 + 1100}{-100 + 110} = \frac{1200}{10} = 120 = x$, the Distance of the Centre of Oscillation, the Square Root of which 10,95 will be proportional to the Time (T) of Vibration, (see *Annotat. XXIX.*) This multiplied by $E = 11$, the Intensity of the Power, gives 120,45 for the Expression of the Action of the Lever in this particular Case.

45. If $B = 15lb.$ then, as before, we shall find $x = 32$, whose Square Root $5,66 \times 15 (= T \times E) = 84,9$ the Lever, which is now less than before. Again, suppose B or $E = 20lb.$ then we shall have $x = 21$, and its Square Root 4,6; then $4,6 \times 20 = T \times E = 92$, which is again greater than the last; the

Quantity of Surface, but to the Weight of the incumbent Part; as we shall also shew by Experiment (XXXIX.)

the limit then is between 11 *lb.* and 20 *lb.* and is very near 15 *lb.* as will appear by the following Table; where the Numbers in the second Column express the whole Action of the Lever in Parts, of which the least contains 100000, all which corresponds to the Intensities of the Power increased from 10 to 20 *lb.*

<i>Powers.</i>	<i>Actions.</i>	<i>Powers.</i>	<i>Actions.</i>
10 ———	Infinite.	15, 16 —	100000
11 ———	142360	16 ———	100368
12 ———	114036	17 ———	101611
13 ———	104677	18 ———	103397
14 ———	101053	19 ———	105575
15 ———	100016	20 ———	108330

46. This Consideration of the Power is but of little Use in the Lever; but as that is the most simple Machine, it was best adapted to explain and exemplify this Doctrine. In the Axis in *Peritrochio*, and Pulley, it is of greater Use, and should not be neglected; but as it is there a Business of great Difficulty and Labour, I shall refer the Reader to *S^r Gravesande's* third Edition of his *Principia*, where he may see the Computations at large; the Result of which in general is, *That the Power which sustains the Weight in Equilibrio should be increased by one half in the Pulley and Axis in Peritrochio, that the Action of these Machines may be a Minimum.*

(XXXIX) The Doctrine of FRICTION is contain'd under the following Particulars, *viz.*

1. When one Body insists on another upon an horizontal Plane, it presses it with all its Weight, which being equally re-acted on (and therefore the whole Effect of its Gravity destroy'd) by the Plane, it will be absolutely free to move in any lateral or horizontal Direction by any the least Power applied thereto, provided both the touching Surfaces be perfectly smooth or even.

N 2

As

As to *Compound Engines and Machines*, they are as *numerous*, as they are *various*

2. But since we find no such thing as perfect Politure or Evenness in the Surfaces of Bodies (at least such as are produced by Art) but an evident Roughness or Unevenness of the Parts in the Surface, arising from the Porosity and peculiar Texture of the Body, it is easy to understand that when two such Surfaces come together the prominent Parts of the one will in some Measure fall into the concave Parts of the other, and therefore, when an horizontal Motion is attempted in one, the fix'd prominent Parts of the other will give more or less Resistance to the moving Surface by holding and detaining its Parts, which is what we call *Friction*.

3. Now since any Body will require a Force proportional to its Weight to draw it over a given Obstacle, it follows that the Friction arising to the moving Body will always be in proportion to its Weight only, and not the Quantity of the Surface, by which it bears upon the resisting Plane or Surface. Thus if a Piece of Wood four Inches wide, and one Inch thick, be ground, and thereby made exactly fit to the Surface of another fix'd Piece of the same Wood, it will require the same Weight to draw it along on the same, whether it be laid on its broad or narrow Side.

4. For though on the broad Side there be four times the Number of touching Particles (*cæteris paribus*) yet each Particle is press'd with but $\frac{1}{4}$ of the Weight that those are on the narrow Side; and since four times the Number multiplied by $\frac{1}{4}$ of the Weight is equal to $\frac{1}{4}$ of the Number multiplied by four times the Weight, 'tis plain the Effect, that is, the Resistance, is equal in both Cases, and so requires the same Force to overcome it.

5. The Reason why the Friction is proportional to the Weight of the moving Body, is, because the Power applied to move the Body must *raise* it in some Measure *upon and over* the prominent Parts of the Surface on which it is drawn; and this Motion of the Body, as it

in their Structure; concerning all which we may observe in general, that they con-

is not upright, so it will not require a Power equal to its whole Weight; but being in the Nature of the Motion on an *Inclined Plane*, (since the Body bears on the prominent Parts all the while) the Power which moves it will be proportional to but a *Part of its Weight only*; and this will vary with the various Degrees of Smoothness or Asperity between the rubbing Surfaces, and the other concurring Circumstances.

6. I find by Experiment, that a Body A B C D (of Plate X. Wood, Brass, &c.) laid on the Surface E F G H, will Fig. 1. be drawn along by a Weight P, nearly equal to *one third* of its own Weight; if the Surfaces be hard and well polish'd, it will be less than a *third Part*; but if the Parts be soft or rugged. it will be much greater. Thus also the Cylinder of Wood A B, if very smooth, Plate X. and laid on two well-polished Supporters C, D, (having Fig. 2. been first oil'd or greased) and then charged with the Weight of two Pounds in the two equal Balls G, H, it will require an additional Weight x (equal to about a third Part of the two Pounds) to give Motion to, or overcome the Friction of the said Cylinder.

7. Now this additional Weight, as it causes a greater Pressure of the Cylinder, will likewise encrease the Friction, and therefore require the Addition of another Weight y , equal to the third Part of its own; for the same Reason the Weight y will require another z , a third Part less; and so on *ad infinitum*. Hence upon Supposition that the Friction is precisely equal to a *third of the Weight*, the first Weight with all the additional ones, viz. $2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \&c.$ will be a Series of Numbers in Geometrical Progression decreasing. Now the Sum of all those Terms, except the first (*i. e.* the Sum of all the infinite Number of additional Weights $x + y + z, \&c.$) is found (by a well known Theorem in Arithmetic) to be equal to *one Pound*. So that if the Weight of the Cylinder be inconsiderable, the Way to

fist of one, two, or more of the Simple Powers combined together; that in most

overcome the Friction would be to double the Power G, or H at once.

8. But though we may at a Medium allow about a *third Part* of the Weight with which any simple Machine is charged for the Friction arising from thence, yet this is very precarious and seldom the Case; for if
 Plate X. A B C D be a Piece of Brass of six Ounces, and E F G H
 Fig. 1. be also a Plate of Brass, and both the Surfaces well ground and polish'd, the Weight P of near two Ounces will be required to draw along the Body A C alone; but if A C be loaded with 6, 8, or 10 *lb.* then a sixth Part of the Weight will be sufficient to draw it along the Plane. If the Plane be cover'd with a linen or woollen Cloth, then a third, or half Part, and sometimes more, will be requisite to draw it along on the Plane.

9. Yet notwithstanding the Difficulty and Uncertainty attending the Estimation of the Quantity of Friction, 'tis still a most useful and necessary Enquiry how, and by what Means the Friction of any Machine may be abated or diminished. In order to this we must consider it mechanically, that is, we must consider Friction as a Force acting against a Power applied to overcome it. Thus suppose A B an upright Stem or Shaft turning freely in the Socket B fix'd in the Table or Plane I K L M; and A C, D E two Arms fix'd in the said Shaft, the latter of which D E has three Pins going into a Socket in the Middle of a heavy Weight at F, G, or H, in such a Manner, that when a Power applied at C moves the Lever A C, it causes the Lever D E to protrude or thrust along the Weight at F, G, or H, in a circular Manner upon the Table.

10. Now since we suppose the Weight all the while it is in Motion is freely and wholly supported by the Plane, it follows that all the Resistance it can give to the Power applied at C, is only what arises from its Friction on the Plane. What this Friction is, will be

of

of them the *Axis in Peritrochio*, the *Lever*, and the *Screw*, are the constituent Parts;

found by applying the Weight at G, so that BG be equal to AC; for then the Power applied to C, acting in a Tangent to the Circle CRS, that shall just move the Weight G, will be equal to its Friction (by *Annot. XXXVIII. 10.*) But if the Weight be applied at F, because BF is greater than AC, the same Power at C, as before, will not move it, because here its Force is increased by having a greater Velocity than the Power; as on the other Hand, if placed at H, a less Power at C shall move it, because of its having there less Velocity than the Power. All which is plain from the Properties of the Lever, demonstrated in *Annotat. XXXVIII.*

11. Hence we understand, that though the Weight of a Machine remains the same, yet the Friction may be diminished by contriving that the Parts on which it moves and rubs shall have less Velocity than the Power which moves it. Thus if the Cylinder AB were to move on the two small Pins or Gudgeons E, F, the Friction would be abated in the same Proportion as the Diameter of those Gudgeons is less than the Diameter of the Cylinder.

12. The Friction on these Gudgeons is still farther diminish'd by causing them to move on the Circumference of a Wheel; thus let F be the Gudgeon of the Cylinder revolving on the Wheel CDE; the Velocity of the Wheel's Circumference will be the same with that of the Gudgeon; but the Velocity of the Wheel's Axis AB (which is now to be consider'd as the rubbing Part) is less than that of the Wheel in Proportion as its Diameter is less than that of the Wheel. For Example, if the Friction of the Cylinder moving on its Surface be $\frac{1}{7}$ Part of the Weight, and the Gudgeons be to the Cylinder as 1 to 10, they will reduce the Friction to $\frac{1}{70}$ Part; and if again the Axis of the Wheel be to the Wheel as 1 to 10, the Wheel will reduce it to $\frac{1}{700}$ Part; and if the Axis of this Wheel be laid on the

Plate X.
Fig. 4.

that in all, a certain Power is applied to produce an Effect of much greater Moment; and that, in the last Place, it is known, that the greatest Effect or Perfection of the Machine is then, when it is set to work with *four Ninths* of that Charge

Perimeter of another Wheel, the Friction will be reduced to a still lesser Part of the Weight, and so you may proceed to diminish the Friction *ad infinitum*. Hence Wheels applied in this Manner are call'd **FRIC-TION-WHEELS**.

Plate X.

Fig. 5.

13. Besides what we have now premised, somewhat farther is necessary to be understood to diminish Friction by *Wheel-Carriages*. It was before observ'd that Friction arose chiefly by lifting the Body over the prominent Parts of the Plane on which it moved; now if we can contrive to move the Body along without lifting or sustaining its Weight, we shall move it without much Friction, and this may be done by laying the Body on any moveable circular Subject, as Rollers, Wheels, &c. Thus let A B be the Section of an heavy Body laid on a Roller E F, upon the Plane C D, and drawn by the Power P; 'tis evident when A B moves, the Asperity of its Surface will lay hold on that of the Roller, and move it likewise; and 'tis as plain, that when the Body A B is drawn against the prominent Parts of the Roller, they immediately give way, and make no Resistance; thus the perpendicular Diameter *ab* yields into the Situation *ef*, and *cd* succeeds in its Place. By this circular Motion of the Roller, its prominent Parts below do only descend and move upon, or over, and are not drawn against the fix'd prominent Parts of the Plane, and so receive no Resistance from them; hence the Body A B is convey'd along without being lifted up, in the Manner as a Wheel is moved by a Pinion, without any considerable Resistance. And this is the true Foundation of the *Theory of Wheels*, or Doctrine of *Wheel-Carriages*.

which

which is equivalent to the Power, or will but juſt keep the Machine *in Equilibrio* (XL.)

(XL.) 1. To demonſtrate this Propoſition, I ſhall chuſe a Water-wheel A D E F, driven round by a Current of Water G A, ſtriking the lower Float-Boards A in a perpendicular Direction, in the Manner of an underſhot Mill. Now if the Wheel be not loaded or charged with any Weight, but moves freely on the Gudgeons of its Axis C, then the Water, coming on the Floats, will put the Wheel in Motion, and acting upon it continually will ſoon accelerate its Motion ſo far, as to give it a Velocity equal to its own.

Plate X.
Fig. 6.

2. But if the Axle of the Wheel C be charged with a Weight P, which it is obliged to raiſe, this will give Reſiſtance to the Wheel, and diminifh its Velocity, or cauſe it to move ſlower than the Water; as the Weight P is increas'd, the Motion of the Wheel will be proportionably retarded; till the Weight P, coming to have an equal *Momentum* with the Water, the Wheel will loſe all its Motion, or be reduc'd to a State of *Equilibrium*.

3. Now let F = Force of the Water, V = its Velocity, v = Velocity of the Wheel, P = Weight that holds the Wheel *in Equilibrio*, z = Weight raiſed by the Wheel in Motion. Then the Difference of thoſe Velocities, viz. V — v will be that with which the Water ſtrikes the Wheel; and ſince the Force of Striking Fluids is always as the Square of the Velocity, (as will hereafter be ſhewn) and Cauſes are proportional to their Effects, we ſhall have $\overline{V - v}^2$ always proportional to z; and when v = 0, we ſhall have z = P; and then V² will be as P, ſo that it will always be V² : P

: $\overline{V - v}^2$: z; and ſo $\frac{V\sqrt{z}}{\sqrt{P}} = V - v$ and v =

$$\frac{V\sqrt{P} - V\sqrt{z}}{\sqrt{P}}.$$

THE

THE *Common* JACK is a compound Engine, where the Weight is the Power applied.

4. But from the Principles of Mechanics, we have the Radius of the Wheel AC ($= R$) to the Radius of the Axle CB ($= r$) as the Velocity of the Wheel v to the Velocity of the Weight z ; that is, $R : r :: V \sqrt{P} - V \sqrt{z} : \frac{rV}{R} \times \frac{\sqrt{P} - \sqrt{z}}{\sqrt{P}}$ = Velocity of z .

Which Velocity of the Weight being multiplied by the Weight z , gives an Expression $\frac{rV}{R} \times \frac{z \sqrt{P} - z \sqrt{z}}{\sqrt{P}}$ for the Effect of the Engine.

5. This Expression is to be determined to a *Maximum* by making the Fluxion of $\frac{z \sqrt{P} - z \sqrt{z}}{\sqrt{P}}$

equal to Nothing, (the Part $\frac{rV}{R}$ being constant, we neglect.) Let $\frac{z \sqrt{P} - z \sqrt{z}}{\sqrt{P}} = y$, then $z \sqrt{P} -$

$\sqrt{z} = z \times P^{\frac{1}{2}} - z^{\frac{3}{2}} = y \times P^{\frac{1}{2}} = y$, (because $P^{\frac{1}{2}}$ is constant) therefore $\dot{z} P^{\frac{1}{2}} - \frac{3}{2} z^{\frac{1}{2}} \dot{z} = \dot{y} = 0$; hence $\dot{z} P^{\frac{1}{2}} = \frac{3}{2} z^{\frac{1}{2}} \dot{z}$, therefore $P^{\frac{1}{2}} = \frac{3}{2} z^{\frac{1}{2}}$; whence $P = \frac{9}{4} z$, and $z = \frac{4}{9} P$; that is, the Weight z , when the Machine is in its greatest Perfection, is equal to $\frac{4}{9}$ of the Weight P that will keep it in *Equilibrio* Q. E. D.

6. Hence, if in the Expression $v = \frac{V \sqrt{P} - V \sqrt{z}}{\sqrt{P}}$ we substitute the Value of z , viz. $\frac{4}{9} P$, we shall have $v = \frac{V \sqrt{P} - V \sqrt{\frac{4}{9} P}}{\sqrt{P}}$, or $v \sqrt{P} = V \sqrt{P} -$

$\sqrt{\frac{4}{9} P} = V \times \frac{1}{3} \sqrt{P}$, therefore $v = \frac{2}{3} V$. That the Velocity of the Wheel (v) is equal to the *Third Part* of the Velocity of the Water, when the Machine is in the greatest Perfection.

plied

plied ; the Friction of the Parts, and the Weight with which the Spit is charged, is the Force to be overcome ; and a steady, uniform Motion, by means of the Fly, is

7. When the Wheel is kept *in Equilibrio* by the Weight P, we have $F : P :: CB (= r) : CA (= R)$ by the Principles of Mechanics ; whence we have $\frac{FR}{r}$

$= P$; whence $z = (\frac{4}{9} P =) \frac{4}{9} \frac{FR}{r}$; which Values of z

and P, substituted in the Expression $\frac{r V}{R} \times \frac{z \sqrt{P} - z \sqrt{z}}{\sqrt{P}}$

will give $\frac{4}{27} V F$ for the Exponent of the greatest Effect of the Engine.

8. To illustrate this by Example ; suppose G A a Body of Water issuing from an Aperture in the Penstock *one Foot Square*, and four Feet below the Surface of the Water Head ; in this Case the Force of the Water $F = 250 \text{ lb.}$ and the Velocity $V = 16 \text{ Feet per Second}$ (as will appear hereafter) ; whence $\frac{4}{27} V F = \frac{1}{27} \times 250 \times 16 = 592,6$, and this is the greatest Effect

of the Engine ; now if $R : r :: 8 : 1$; then $P = \frac{R}{r} = 2000$; and $z = \frac{4}{9} P = 888,9 \text{ lb.}$ And its Ve-

locity will be $\frac{1}{8}$ of $\frac{1}{4}$ of 16, or $\frac{2}{3}$ of a Foot *per Second*. To prove the Truth of all that has been said, one need only take z either greater or lesser than 888,9 *lb.* and

then the Expression for the Effect of the Engine $\frac{r V z}{R} \times$

$\frac{\sqrt{P} - \sqrt{z}}{\sqrt{P}}$, will give a Number in each Case less

than 592,6 ; which therefore is the *Maximum*, as above found.

the

the End or Intention of the Machine
(XLI.)

(XLI.) 1. The FLY is the only Part I shall here take Notice of in the Chimney-Jack, and shall explain its Nature and Use both there and in other Mechanical Machines. For it may be applied to any Sort of Engines to very good Purposes, which have a quick and circular Motion, and where the Power or Resistance acts unequally in the different Parts of a Revolution.

2. The Use of the Fly is to facilitate the Motion of Engines, by accumulating and retaining the Power communicated to it, and exerting it gradually and equally in each Revolution of the Machine; whence it comes to pass that the Motion of the Machine is render'd very nearly uniform and of an equal Tenour in all Parts of the Revolution, and therefore more easy, pleasant, and convenient to be acted by the impelling Power. For neither the Strength of Men, nor other Powers or Resistances affecting the Engine, can or do act equally and uniformly in every Part of a Revolution. Thus Meat on a Spit gives always more Resistance in one Part than another, by which means the Motion would be so irregular and jolting, that it would soon become shaken and loos'd from the Spit, and so not be carried round, were not this Irregularity prevented by the Fly.

3. The Fly does not add any new Power to an Engine, as some have imagin'd; as is evident for the following Reasons. (1.) The Fly has no Motion but what it receives at first from the Machine. (2.) A Degree of Force is always necessary to maintain the Motion of the Fly, which must be supplied from the Machine. (3.) The Friction of the Pivot, Screw, &c. of the Fly is a Resistance to the impress'd Force, and must abate it. (4.) The Air likewise makes Resistance to the Weights at the End of the Fly. Upon all which Accounts 'tis easy to understand, that the Fly, instead of adding, does very much decrease or lessen the Power impress'd on the Machine.

IN CLOCKS, WATCHES, &c. the Power is the Weight or Spring; the Force

4. The best Form for a Fly is that of a heavy Wheel or Circle, of a fit Size; for this will meet with less Resistance from the Air; and being continuous, and the Weight every where equally distributed through the Perimeter of the Wheel, the Motion will be more easy, equable, and regular. In this Form the Fly is most aptly applied to the perpendicular Drill, where it not only gives Weight and Regularity of Motion, but contributes to keep the Drill upright by its Centrifugal Power.

5. In this Form it is also best applied to a *Windlass* or common *Winch*, where the Motion is pretty quick; for when a Man turns the bended Handle of the Winch, his Strength is not, nor can be equally exerted in every Part of the Revolution; for in pulling upwards from the lower Quarter, he can exercise more Power than in thrusting forward in the upper Quarter, where, of course, part of his former Force would be lost, were it not accumulated and conserved in the equable Motion of the Fly. By this means a Man may work all Day in drawing up a Weight of 40 *lb.* whereas 30 *lb.* would create him more labour in a Day without the Fly.

6. The Fly is sometimes made use of to increase the Force of Mechanical Engines, as of the Lever and Screw, in that for *Stamping* of Money in the Mint. Here the Power exerted by the Man is accumulated by the large Weights at each End of the Lever; the Lever increases and communicates it to the Cylinder, upon which it is fix'd; the Screw does again increase or condense it upon the Medal, by means of which the impression of the Image is made.

7. The great Power of this (or any other Machine of its Kind) may be thus computed. Suppose the Arms of the Fly 15 Inches each, or the Length of the Lever 30 Inches, and each Weight to be 50 *lb.* and the Diameter of the Cylinder at the Screw be *one Inch*. If each stroke be made in a Half-Circle, which will be four feet, and in half a Second of Time, the Velocity (be-

to

to be overcome is the Friction of the numerous Parts, which are chiefly a Combination of Wheels and Axes, whose Use is to divide a large Portion of Time, as a *Day* or *Hour*, into very minute equal Parts, as *Minutes*, *Seconds*, &c. and to point out those Divisions by an equable Motion of a proper Hand or Index round a graduated Circle (XLII.)

ing equably accelerated by the continued Action of the Man) will at the Instant of the Stroke be at the Rate of eight Feet *per* Second; therefore $8 \times 100 \text{ lb.} = 800 \text{ lb.}$ for the *Momentum* of the Lever or Fly. But since the Diameter of the Screw is but $\frac{1}{30}$ of the Length of the Lever, we shall have $30 \times 800 \text{ lb.} = 24000 \text{ lb.}$ for the *Momentum* of the Engine, which is equal to that of 100 lb. falling 120 Feet; which prodigious Force is still farther increased for coining larger Pieces of Money, by increasing the Weights and Arms of the Fly.

(XLII.) 1. I shall here shew the Principles of Watch and Clock-Work, and how to calculate the Numbers for the Movements, in as clear and compendious a Manner as I possibly can. In all *Automata*, or Machines of Clock-Work, there is a natural Agent or Principle of Motion; which, by acting on one Part, gives Motion to that, and all the other Parts depending upon it, and thus becomes the *Primum Mobile*, or first Mover to the whole Machine.

2. In common *Clocks* and *Watches* this is of two Sorts, *viz.* A *SPRING* or a *WEIGHT*; either of which may be made to act with any determinate Force; the Spring by its *Elasticity*, and the Weight by its *Gravity*. In these Machines this Force is requir'd to be such as will overcome the *Vis Inertiæ* and Friction of all the Parts in Motion; which in *Watches* is very inconsiderable, but in *Clocks* is much greater, and that in Proportion as they are more compounded.

3. Th

RPJCB

Fig. 1. p. 191.

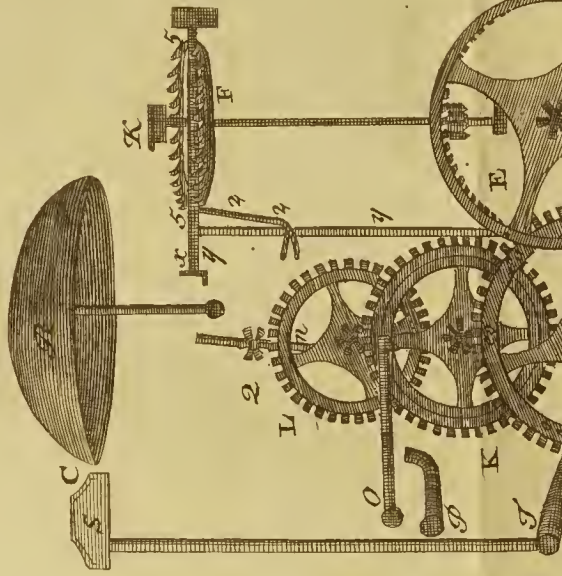


Fig. 2. p. 196.

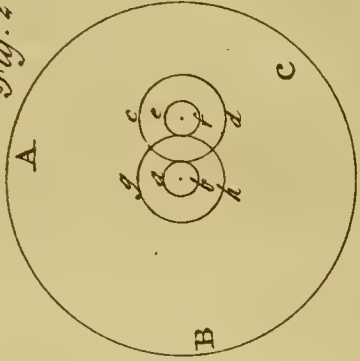


Fig. 4. p. 198.

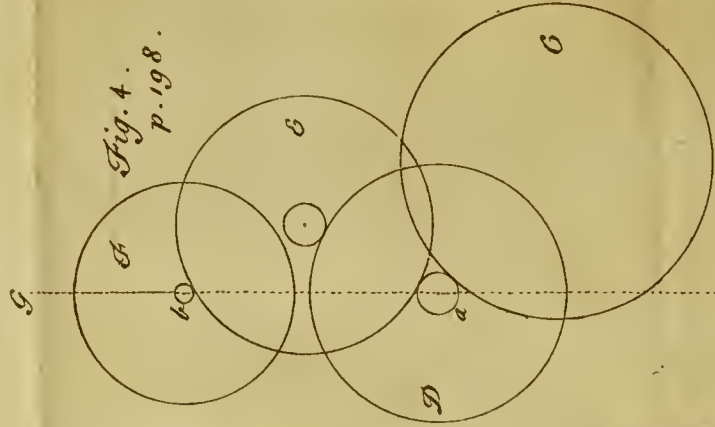


Fig. 3. p. 197.

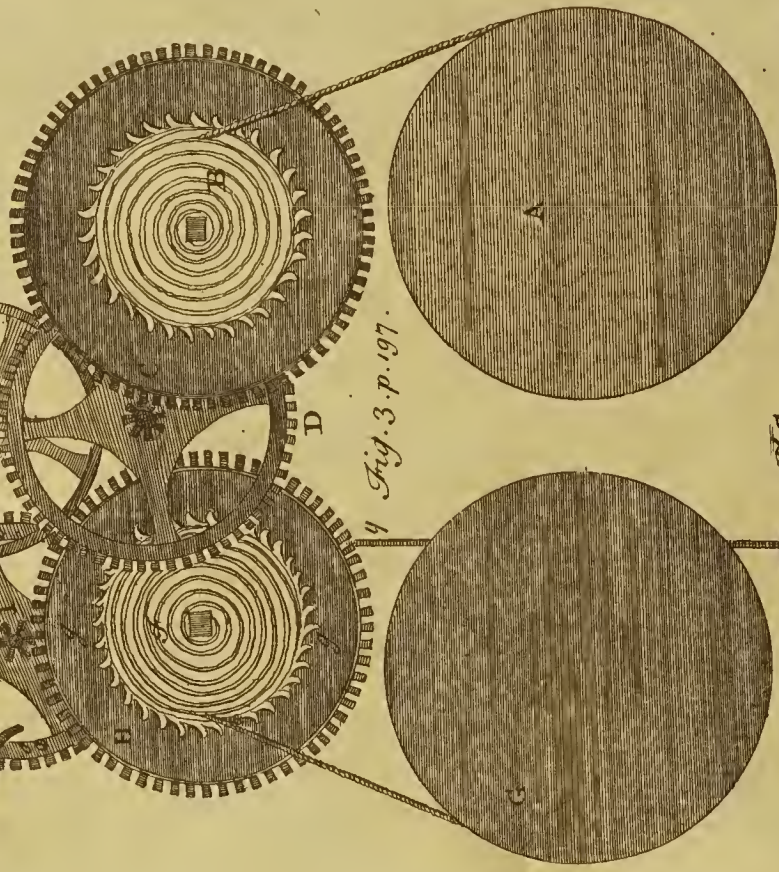


Fig. 5. p. 234.

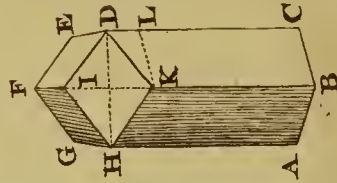
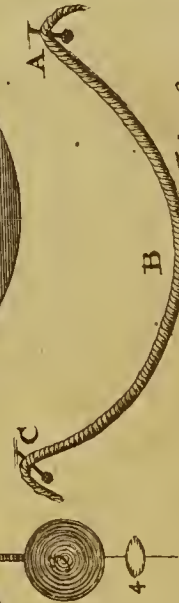


Fig. 6. p. 234.



3. The Manner in which a Weight acts upon the Cylinder, about which the Line or Cord (to which it hangs) is wound, is easy to be understood by all; but the Action of the Spring coil'd up within the Cylindric Barrel or Box of a Clock or Watch, is somewhat more nice and mysterious; and the Manner how it acts upon the Fusee always with an equal Force by means of the Chain, and the proper Figure of the Fusee for that Purpose, is next to be explain'd.

4. The Chain being fix'd at one End to the Fusee, and at the other to the Barrel, when the Machine is winding up the Fusee is turn'd round, and of course the Barrel; on the Inside of the Barrel is fix'd one End of the Spring, the other End being fix'd to an immoveable Axis in the Centre. As the Barrel moves round, it coils the Spring several Times about the Axis, thereby increasing its elastic Force to a proper Degree; all this while the Chain is drawn off the Barrel upon the Fusee; and then when the Instrument is wound up, the Spring by its elastic Force, endeavouring constantly to unbend itself, acts upon the Barrel, by carrying it round; by which means the Chain is drawn off from the Fusee, and thus turns the Fusee, and consequently the whole Machinery.

5. Now as the Spring unbends by degrees, its elastic Force, by which it affects the Fusee, will gradually decrease; and therefore unless there were some Mechanical Contrivance in the Figure of the Superficies of the Fusee to cause that as the Spring is weaker, the Chain shall be removed farther from the Centre of the Fusee, so that what is lost in the Spring's Elasticity is gain'd in the Length of the Lever; I say, unless it were for this Contrivance the Spring's Force would always be unequal upon the Fusee, and so would produce an unequable Motion of the Parts of the Machine.

6. The Figure of the Curve, which shall form the Superficies of the Fusee by a Revolution about its Axis, Plate XI. may be investigat'd as follows. Let B C D be the Fig. 1. Curve, A L the Axis of the Fusee produced; let D be the Point where the End of the Chain is fix'd on the Fusee when the Watch is down, or the Spring uncoil'd, and B the Point where it touches it when the Spring

Spring or Machine is wound up. From the Points B and D let fall the Perpendiculars to the Axis BA and DH; in which produced, let there be taken AE and HI proportional to the Force or Strength of the Spring, when the Chain is at B and D. Through E, I, draw the Right Line EIK intersecting the Axis somewhere in K; and from any Point C in the Curve, draw CF perpendicular to the Axis in G; then will FG be as the Strength of the Spring when the Chain is as G.

7. Now, since the Force acting on the Fusée ought always to be uniformly the same; and this Force being always as the Strength of the Spring express'd by FG, and the Distance at which the Chain acts from the Axis of the Fusée conjointly: Therefore the Force at any Point C will be as the Rectangle $FG \times GC$, and since this is a given Quantity it may be made $FG \times GC = ab$, and so we have $FG = \frac{ab}{GC}$.

8. Therefore to determine the Equation of the Curve BCD, let $KH = a$, $HI = b$, $HG = x$, and $GC = y$. Then because of the similar Triangle HKI and GKF, we have $HK : HI :: GK : FG = \frac{ab}{y}$; that is, $a : b :: a + x : \frac{ab}{y}$; whence we have $ay + xy$, which is the Equation of the Curve, and shews it to be of the *Hyperbolic Kind*.

9. The Form or Figure of the Fusée being thus determined by the given Force of the Spring in the Points B and D, and the Points G taken at Pleasure successively from H towards A; we next consider that it is acted upon, or put into Motion, by an uniform Force; by which means also the *Great Wheel*, which is fix'd to it, is put into Motion, and that drives the Pinion of the *Centre Wheel*, which *Centre Wheel* drives the Pinion of the *Third Wheel*, and this drives the Pinion of the *Contrate Wheel*, and this the Pinion of the *Balance Wheel*, which plies the two Pallets on the Axis of the Balance, and keeps the Balance in Motion.

10. The Balance in a Watch is instead of the Pendulum in the Clock; both serving to govern the Motion of the whole Machinery. To this Balance is fix'd a small Steel spiral Spring which regulates the Motion thereof,

thereof, and makes it equable; whence its Name of *Regulator*. N. B. *The Reader will do well to have his Eye upon the internal Parts of the Watch as he reads this Description.*

11. When the Watch is wound up, the Chain from the Spring exerts a Force upon the Fufee, which gives Motion to all the Parts of the Machine in the following Manner; which will be easy to understand when the Number of Teeth in each Wheel, and Leaves in the Pinions which they drive, are specified. And these in Modern Thirty-Hour Watches are as follows.

	Teeth.	Leaves.
Great-Wheel	48 ———	12
Centre-Wheel	54 ———	6
Third-Wheel	48 ———	6
Contrate-Wheel	48 ———	6
Balance-Wheel	15 ———	2 Pallets.

12. Hence 'tis easy to understand how often any one Wheel moves round in the Time of one Revolution of that which drives it. Thus the Great-Wheel on the Fufee, having 48 Teeth, and driving the Centre-Wheel by a Pinion of 12, must cause the Centre-Wheel to move round 4 Times in one Turn of the Fufee. And so for all the rest, as below.

- 12)48(4 = Turns of the Centre-Wheel.
- 6)54(9 = Turns of the Third-Wheel.
- 6)48(8 = Turns of the Contrate-Wheel.
- 6)48(8 = Turns of the Balance-Wheel.

13. Whence it follows that the Turns of each of those Wheels respectively in one Turn of the Fufee, will be had by multiplying those several Quotients together successively, as follows:

- 1 Turn of the Fufee or Great-Wheel.
- $4 \times 1 = 4$ Turns of the Centre-Wheel.
- $9 \times 4 \times 1 = 36$ Turns of the Third-Wheel.
- $8 \times 9 \times 4 \times 1 = 288$ Turns of the Contrate-Wheel.
- $8 \times 8 \times 9 \times 4 \times 1 = 2304$ Turns of the Balance-Wheel.

14. The Balance-Wheel having 15 Teeth, and each striking a Pallet twice in one Revolution, there will be

30 Strokes upon the Axis of the Balance, which we call the *Beats* of the Balance; and therefore there will be $2304 \times 30 = 69120$ Beats in one Turn of the Fusee or Great-Wheel.

15. The Centre-Wheel is that which we must have the principal Regard to in the *Division of Time*; the Wheels beyond this, towards the Balance, serving only to multiply the Strokes of the Balance, and cause it to move with an insensible Power, and be thereby subject to a more perfect Regulation. But the Centre-Wheel is that upon which both the *Hour and Minute Hand* is moved or carried round upon the Face of the Watch to indicate the Time, viz. the Hour of the Day, or Minute of the Hour.

16. Since the Time of the Watch's going is 30 Hours, and the Minute Hand, and consequently the Centre-Wheel, goes round once in an Hour, the said Centre-Wheel will have 30 Turns in the Time of the Watch's going; and because it has four Turns in one of the Fusee, therefore $4 \times 30 = 120 = 7\frac{1}{2}$ the Number of Turns of the Fusee in winding up the Watch. Whence $69120 \times 7\frac{1}{2} = 518400 =$ the Number of Beats during the whole Time of the Watch's going.

17. If then we divide 518400 by 30, we shall have the Quotient 17280 = Number of Beats in an Hour, which is call'd the *TRAIN* of a Watch; and it is said to be a *swifter or slower Train* as the Number of Beats in an Hour is greater or lesser. If we divide the Train 17280 by 3600, the Seconds in an Hour, the Quotient will be almost 5; that is, there will be near 5 Beats *per Second* in such a Watch.

18. From this Analysis of a Watch, it will be easy to form an Idea of the Manner of Calculation for the Numbers of the Teeth and Leaves for the several Wheels and Pinions throughout the Work; and I shall endeavour to facilitate and illustrate this by an Example of the Numbers for a Watch whose *Train* is 14400, and which therefore will beat Quarter-Seconds, because such an one will be useful for many Philosophical Purposes, as well as the common Measure of Time.

19. The Time which this Watch shall go, may be 32 Hours; then $14400 \times 32 = 460800 =$ the Beats of the Balance in the whole Time. Suppose the Num-
ber

ber of Turns in the Fusee be 8; then $8)460800(=57600$
 $=$ the Beats in one Turn of the Fusee. Again, let the
 Number of Teeth in the Balance-Wheel be 15, there
 will be 30 Beats in one Turn of this Wheel; then
 $30)57600(=1920$, which will be the Number arising
 from the continued Multiplication of all the Quotients
 of the Wheels, divided by the Pinions they drive from
 the Great-Wheel to the Balance-Wheel, as will be easy
 to understand by *Art.* 12, 13.

20. The Business is now to break this Number into
 4 convenient small Numbers, which multiplied together
 shall make the same Number 1920. This may soon be
 done by a few Trials; thus suppose I take the Num-
 ber 4 for one of them; then $4)1920(480$. This
 Number, 480, I plainly see can be divided by 6 with-
 out a Remainder; therefore $6)480(=80$; and as
 I plainly see that $80=8 \times 10$, therefore the four
 Numbers sought are 4, 6, 8 and 10. For these, multi-
 plied together, make 1920, *viz.* $4 \times 6 \times 8 \times 10=1920$.

21. Having thus got the Quotients, it will be very
 easy to find what large Numbers, divided by small ones,
 will produce the said Quotients; thus $12)48(=4$;
 wherefore, if I allow 48 Teeth to the Great-Wheel on
 the Fusee, it must drive a Pinion of 12 on the Centre-
 Wheel.

22. In the next Place, for the Quotient 6, I chuse
 the Numbers 54 and 9; thus $9)54(=6$; which shews
 that the Teeth of the Centre-Wheel may be 54, and it
 must then drive a Pinion of 9 on the Third Wheel.
 Or, if instead of 54 and 9, I rather make choice of
 48 and 8, it will answer the same End; for $8)48(=6$,
 as before.

23. Then for the Quotient 10, 'tis easy to see, that
 50 and 5 will produce it, *viz.* $5)50(=10$. That is,
 the Third Wheel having 50 Teeth must drive a Pinion
 of 5 on the Contrate-Wheel. If the said Wheel has
 40, or 60 Teeth, and drive a Pinion of 4 or 6, the
 same Number of Turns will be obtain'd.

24. Lastly, for the Quotient 8, we have the Num-
 bers 48 and 6; for $6)48(=8$; or $7)56(=8$; or
 $5)40(=8$. Whence if the Contrate-Wheel be al-
 low'd 40, or 48, or 56 Teeth, it must drive a Pinion
 of 5, 6, or 7 Leaves on the Balance-Wheel.

25. By this Means all the Wheels and Pinions are determined, and adjusted in the Body of the Watch from the Fusee to the Balance; but all that we have hitherto done shews only the *Minutes of an Hour*, and *Seconds*, or *Quarter-Seconds* of a Minute; but nothing has yet been mention'd relating to the Mechanism for shewing the *Hour of the Day*, of which we shall treat in the next Place.

26. This Part of the Work lies conceal'd from Sight between the upper Plate of the Watch-Frame and the Dial-Plate; and since few People have the Opportunity of viewing this Work, I have here represented it by a Figure; wherein A B C is the uppermost Side of the Frame-Plate, as it appears when detached from the Dial-Plate; the Middle of this Plate is perforated with a Hole receiving that End of the Arbor of the Centre-Wheel which carries the Minute-Hand; on this End of the Arbor, near the Plate, is fix'd a Pinion *ab* of 10 Teeth; this is call'd the *Pinion of Report*; it drives a Wheel *cd* of 40 Teeth; this Wheel *cd* carries a Pinion *ef* of 12 Teeth, and this drives a Wheel *gh* with 36 Teeth.

27. As in the Body of the Watch the Wheels every where drive the Pinions, and so quicken or increase the Motion; here, on the contrary, the Pinions drive the Wheels, and by that means decrease the Motion, which is here necessary; for the Hour-Hand, which is carried by a Socket fix'd on the Wheel *gh*, is required to move but once round while the Pinion *ab* moves 12 Times round. To this End the Motion of the Wheel *cd* is $\frac{1}{4}$ of the Pinion *ab*, because $10 \div 40 (= \frac{1}{4})$. Again, while the Wheel *cd*, or the Pinion *ef*, goes once round, it turns the Wheel *gh* but $\frac{1}{3}$ Part round, for $12 \div 36 (= \frac{1}{3})$. Consequently the Motion of *gh* is but $\frac{1}{3}$ of $\frac{1}{4}$ of the Motion *ab*; but $\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$, that is, the Hour-Wheel *gh* moves once round in the Time that the Pinion of Report on the Arbor of the Centre or Minute-Wheel makes twelve Revolutions, as requir'd.

28. Having thus shewn the Nature and Manner of the Mechanism of a *Watch*, the Structure of that Part of a *Clock*, which is concern'd in shewing the Time, will easily be understood, especially as it is represented in the Draught. The Mechanism of a *Clock* consists of

of *two Parts*, one to *shew the Time*, the other to *report it*, by striking the Hour upon a Bell. How each of these is effected, I shall shew by the following Steps.

29. Each Part is actuated or moved by *Weights*, as in common Clocks, or *Springs* included in Boxes or Barrels, as A; this Cylinder moves the Fusee B, and the great Wheel C, (to which it is fix'd) by the Line or Cord that goes round each, and answers to the Chain of the Watch. The Method of Calculation is here nearly the same as before. For suppose here the Great-Wheel C goes round once in twelve Hours; then if it be a *Royal Pendulum Clock*, swinging Seconds, we have $60 \times 60 \times 12 = 432000$ Seconds or Beats, in one Turn of the Great-Wheel.

Plate XI.
Fig. 3.

30. But because there are 60 Swings or Seconds in one Minute, and the Seconds are shewn by an Index on the End of the Arbor of the Swing-Wheel (which in those Clocks is in an horizontal Position) therefore 'tis necessary that the Swing-Wheel should have 30 Teeth. Whence $60 \nmid 43200 (= 720)$, the Number to be broken into Quotients for finding the Number of Teeth for the other Wheels and Pinions, as before, in *Art. 20*.

31. But since the Minute-Wheel D goes 12 Times round in one Turn of the Great-Wheel C, we know one of the Quotients must be 12; whence $12 \nmid 720 (= 60)$; and 60 is compounded of 6×10 , or, $8 \times 7,5$. So that either 6 and 10, or 8 and $7\frac{1}{2}$ are the two other Quotients; but the latter Numbers are preferable to the former, as affording a greater Uniformity of the Parts and Ease in the Execution. Therefore $8 \nmid 60 (= 7\frac{1}{2})$; hence if we give 60 Teeth to the Middle or Minute-Wheel D, it drives a Pinion of 8 on the Contrate-Wheel E.

32. The other Quotient 8 will give us $8 \nmid 64 (= 8)$, or $7 \nmid 56 (= 8)$; that is, if we give 64 Teeth to the Contrate-Wheel E, it must drive a Pinion of 8 on the Arbor of the Swing-Wheel. But if only 56 be allow'd, then it must drive a Pinion of 7, and these last Numbers are now generally used. Thus much for the *Time Part*, as far as the Minutes and Seconds: As for the

Pinions and Wheels to shew the Hours, they are the same as was before shewn in the Watch, *Art.* 26, 27.

33. In Spring Clocks the Disposition of the Wheels in the Watch Part is such as is here represented in the Figure, where the Swing-Wheel F is in an horizontal Position, the Seconds not being shewn there by an Index, as is done in the large Pendulum Clocks, according to the foregoing Calculation: Whence in these Clocks the Wheels are disposed in a different Manner, as represented in *Fig.* 4. where C is the Great-Wheel, D the Centre or Minute-Wheel, both as before; but the Contrate-Wheel E is placed on one Side, and F the Swing-Wheel is plac'd with its Centre in the same perpendicular Line GH with the Minute-Wheel, and with its Plane perpendicular to the Horizon, as are all the others. Thus the Minute and Hour Hands turn on the End of the Arbor of the Minute-Wheel at *a*, and the Second-hand on the Arbor of the Swing-Wheel at *b*.

34. It remains now that we give an Account of the Machinery of the *Striking-Part* of a Clock. Here as in the *Watch-Part*, the *Primum Mobile* is a large Spring, in the *Spring-Barrel* G; but in long Pendulums it is a Weight, as is well known. This by its Cord and Fusee moves the *Great Wheel* H; that gives Motion to the *Pin-wheel* I; that continues it to the *Detent* or *Hoop Wheel* K; and that to the *Warning-Wheel* L; which at last is spent on the *Flying-Pinion* Q, which carries the *Fly* or *Fan*; and by its great Velocity it meets with great Resistance from the Air it strikes, and by this means bridles the Rapidity of the Clock's Motion, and renders it equable.

35. All these Wheels are *quiescent* or motionless, unless when at the Beginning of each Hour the Detent O is lifted up, by which means the Work is unlock'd, and the Whole put into Motion by Virtue of the Spring in the Box G. During this Motion the Pins *e, e, e, e*, of the Pin-Wheel I, takes the Tail of the Hammer T, and carrying it upwards removes the Head of the Hammer from the Bell R; then being let go by the Pin it is made by a strong Spring to give a forcible Stroke upon

upon the Bell, and this is repeated as often as the Hour requires, by Means of a Contrivance in another Part.

36. This consists of moveable Wheels, and several Levers, and other Parts, which cannot be understood by a bare Description, or even by a Representation in a Draught, so well as any Person may have an Idea of them by taking off the Face or Dial-Plate of a late-made Eight-Day Clock; for, within 8 or 10 Years past, great Improvements have been made in this Part of the Mechanism. Of which, perhaps, I may give a more ample Account in another Place, the Scope of my present Design not admitting it here.

37. To the Invention of Mr. *Maurice Wheeler* we owe the curious Contrivance of a *Clock descending on an Inclined Plane*; the Theory of which is worth any Gentleman's Knowledge, and may be seen in N^o 161. of the *Philosophical Transactions*. Also the Clock itself may be seen at *Don Saltero's* Coffee-house in *Chelfea*.

38. But since many People may have a Curiosity to be acquainted with this odd Kind of Machinery, who have neither of the Opportunities abovemention'd, I will here present them with a short Account of this Clock. DE is the inclined Plane on which the Clock ABC descends; it consists externally of a Hoop, and two Sides or Plates standing out beyond the Hoop about $\frac{1}{8}$ of an Inch all round, and indented that the Clock may not *slide*, but turn round as it moves down.

Plate X.
Fig. 7.

39. One of these Plates is inscribed with the 24 Hours, which pass successively under the Index, which is always in a Position perpendicular to the Horizon, and shows the Hour on the Top of the Machine: For this Reason the lower Part of the Index is heaviest, that it may preponderate the other, and always keep it upright as the Movement goes on.

40. For the internal Part or Mechanism, let LEDQ be the external Circumference of the Hoop, and ff the Frame-Plate on which is placed the Train of Wheel-Work 1, 2, 3, 4. which is much the same as in other Clocks, and is govern'd by a Balance and Regulator as in them. But for a Spring and Fusee, there needs none in this Clock, their Effects being otherwise answer'd, as we shall see.

41. In this Machine the great Wheel 1 is plac'd in
O 4 the

the Centre, or upon the Axis of the Movement, and the other Wheels and Parts towards one Side, which therefore would prove a Bias to the Body of the Clock, and cause it to move, even on a horizontal Plane, for some short Distance; this makes it necessary to fix a thin Plate of Lead at C on the opposite Part of the Hoop, to restore the *Equilibrium* of the Movement.

42. This being done, the Machine will abide at rest in any Position on the horizontal Plane HH; but if that Plane be changed into the Inclined Plane DE, it will touch it in the Point D; but it cannot rest there, because the Centre of Gravity at M acting in the Direction MT, and the Point T having nothing to support it, must continually descend and carry the Body down the Plane.

43. But now if any Weight P be fix'd on the other Side the Machine, such as shall remove the Centre of Gravity from M to the Point V in the Line LD, which passes thro' the Point D, then it will rest upon the Inclined Plane, as we have shewn in the Case of the *Rolling Cylinder*; for this Instrument is founded on the same Principle.

44. If now the Weight P be supposed not fix'd, but suspended at the End of an Arm, or *Vellis*; which Arm or Lever is at the same Time fasten'd to a Central Wheel 1, moving on the Axis M of the Machine, which Wheel by its Teeth shall communicate with the Train of Wheels, &c. on the other Side; I say, in this Case if the Power of the Weight P be just equal to the Friction or Resistance of the Train, it will remain motionless all one as before when it was fix'd; and consequently the Clock also will be at rest on the Inclined Plane.

45. But supposing the Weight P has a Power superior to the Resistance of the Train, it will then put it into Motion, and of course the Clock likewise, which will then commence a Motion down the Plane, while the Weight P, its *Vellis* PM, and the Wheel 1, all retain constantly the same Position they at first have when the Clock begins to move.

46. Hence 'tis easy to understand that the Weight P may have such an intrinsic Gravity as shall cause it to act

act upon the Train with any required Force, so as to produce a Motion in the Machine of any required Velocity, as suppose such as shall carry it once round in twenty-four Hours ; then if the Diameter of the Plates ABC be four Inches, it will describe the Length of its Circumference, viz. 12,56 Inches in one natural Day ; and therefore if the Plane be of a sufficient Length, such a Clock may go several Days, and would be a *perpetual Motion*, if the Plane were of an infinite Length.

47. Let SD be drawn through M perpendicular to the inclined Plane in the Point D. Also let LD be perpendicular to the horizontal Line HH, passing through D. Then is the Angle HDE = LDS = DMT. Whence it follows, that the greater the Angle of the Plane's Elevation is, the greater will be the Arch DT, and consequently the farther will the common Centre of Gravity be removed from M ; therefore the Power of P will be augmented, and of course the Motion of the whole Machine accelerated.

48. Thus it appears, that by duly adjusting the intrinsic Weight of P at first to produce a Motion shewing the mean Time as near as possible ; the Time may afterwards be corrected, or the Clock made to *go faster or slower* by raising or depressing the Plane, by means of the Screw at S. The Angle to which the Plane is first raised is about ten Degrees.

49. We have given the Theory of a Clock moving *down an Inclined Plane* : Let us now consider how a Clock may be made to *ascend* on the same Inclined Plane. To this End let ABD be the Machine on the Inclined Plane EDE, and let it be kept thereon at Rest, or in *Equilibrio*, by the Weight P, at the End of the Lever PM. The circular Area CF is one End of a Spring Barrel in the Middle of the Movement, in which is included a Spring, as in a common Watch.

50. To this End of the Barrel the Arm or Lever PM is fix'd upon the Centre M ; and thus when the Clock is wound up, the Spring moves the Barrel, and therefore the Lever and Weight P, into the Situation PM. In doing this, the Centre of Gravity is constantly removed farther from the Centre M of the Ma-

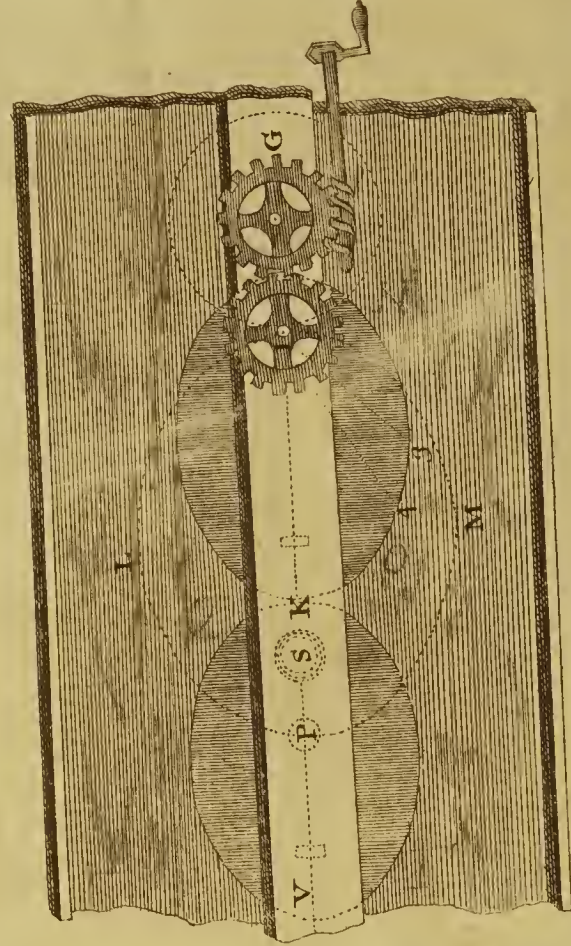
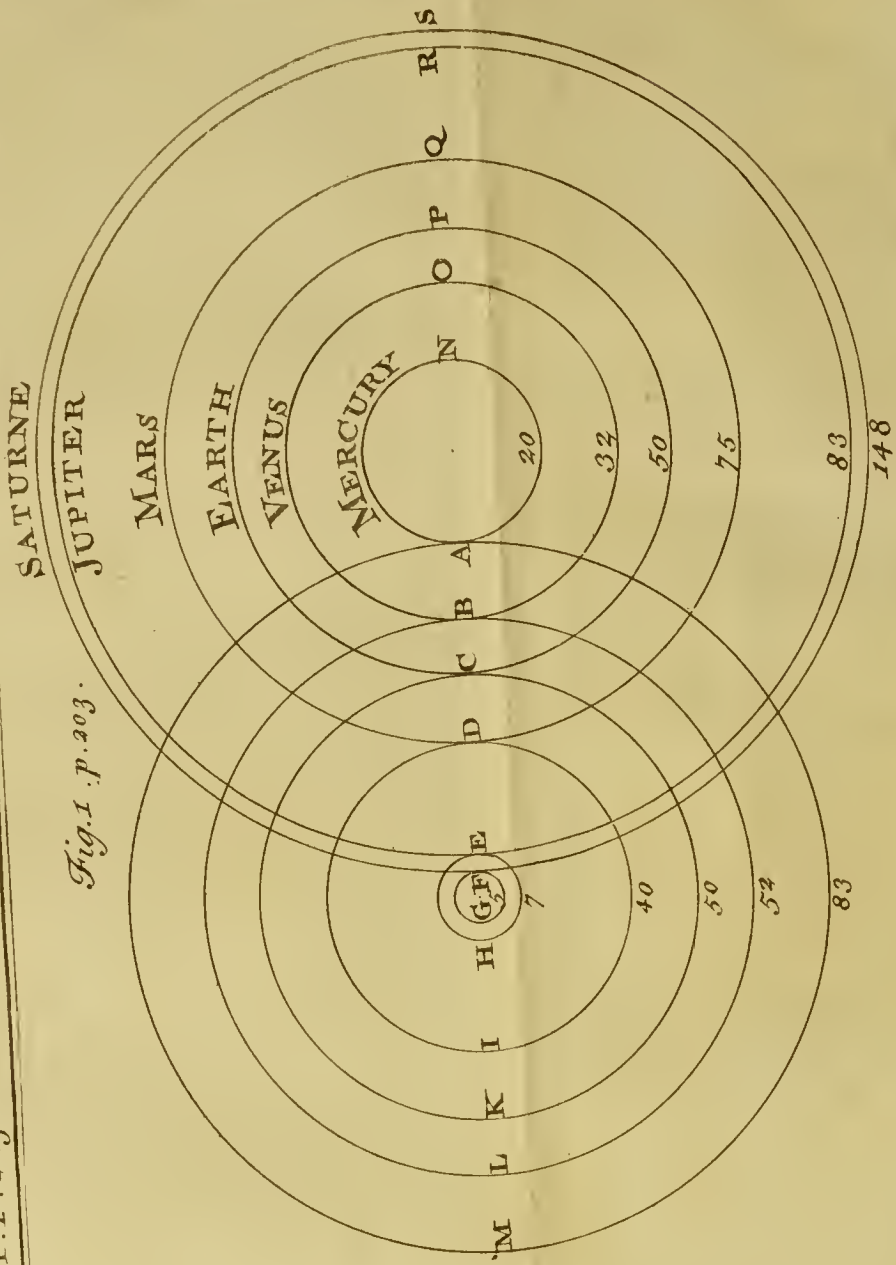
IN the ORRERY and COMETARIUM, the whole Machinery is a Compages of Wheels of various Sizes, suitably contrived and adapted to produce circular and elliptical Motions of Bodies representing the *Planets* and *Comets*, in such Periods of Time as are exactly proportional and correspondent to the respective Motions of the Heavenly Bodies which they represent; also their several Phases, Positions of the Orbits, and other Affections, so as to be a perfect *Microcosm*, or *Solar System* in Miniature (XLIII, XLIV.)

Machine, and therefore it must determine the Clock to move upwards; which it will continue to do, so long as the Spring is unbending itself; and thus the Weight and its Lever PM will preserve the Situation they first have, and do the Office of a Chain and Fusee. This is the Contrivance of *M. de Gennes*. See *Phil. Trans.* N^o. 140.

(XLIII.) It would be too great an Undertaking here to give an Account of the Mechanism of the larger Sort of Orreries, which represent the Movements of all the heavenly Bodies; nor indeed can it be done either by Diagram or Description, to render it intelligible to the most discerning Reader; but instead of that, I shall exhibit an Idea of the Theory and Structure of an useful, concise, and portable *Planetarium*, which any Gentleman may have made for a small Expence, and will exhibit very justly the Motions of all the primary Planets about the Sun by Wheel-Work; and those that have Secondaries or Moons, may have them placed about their Primaries, moveable by the Hand, so that the whole shall be a just Representation of the *Solar System*, or true State of the Heavens, for any given Time of the Year.

RPJCB

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2. In order to this we must first compare, and find out the Proportion, which the Periodical Times or Revolutions of the Primary Planets bear to that of the Earth; and they are such as are expressed in the Tablet below, where the first Column is the Time of the Earth's Period in Days, and Decimal Parts; the second that of the Planets; the third and fourth are Numbers in the same Proportion to each other.

As 365,25 : 88	8 :: 83 : 20, for <i>Mercury</i> .
365,25 : 224,7	8 :: 52 : 32, for <i>Venus</i> .
365,25 : 686,9	8 :: 40 : 75, for <i>Mars</i> .
365,25 : 4332,5	4 :: 7 : 83, for <i>Jupiter</i> .
365,25 : 10759,3	5 :: 5 : 148, for <i>Saturn</i> .

3. If now we suppose a Spindle or Arbor with six Plate XII. Wheels fix'd upon it in an horizontal Position, having the Fig. 1. Number of Teeth in each, corresponding to the Numbers in the third Column, viz. the Wheel A M of 83 Teeth, BL of 52, CK of 50 (for the Earth,) DI of 40, EH of 7, and FG of 5; and another Set of Wheels moving freely about an Arbor, having the Number of Teeth in the fourth Column, viz. A N of 20, BO of 32, CP of 50 (for the Earth,) DQ of 75, ER of 83, and FS of 148; then if those two Arbors of fix'd and moveable Wheels are made of the Size, and fix'd at the Distance from each other, as here represented in the Scheme, the Teeth of the former will take those of the latter, and turn them very freely when the Machine is in Motion.

4. These Arbors, with their Wheels, are to be placed in a Box of an adequate Size, in a perpendicular Position; the Arbor of fix'd Wheels to move in Pivots at the Top and Bottom of the Box; and the Arbor of moveable Wheels to go through the Top of the Box to a proper Height, on the Top of which is to be placed a round Ball, gilt with Gold, to represent the Sun. On each of the moveable Wheels is to be fix'd a Socket or Tube ascending above the Top of the Box, and having on the Top a Wire fix'd, and bent at a proper Distance into a Right Angle upwards, bearing on the Top a small round Ball representing its proper Planets.

5. If

5. If then on the lower Part of the Arbor of fix'd Wheels be plac'd a Pinion of Screw-Teeth, a Winch turning a Spindle, with an endless Screw, playing in the Teeth of the Arbor, will turn it with all its Wheels and these Wheels will move the others about with their Planets in their proper and respective Periods of Time very exactly. For while the fix'd Wheel CK moves its equal CP once round, the Wheel AM will move AN a little more than four Times round, and so will nicely exhibit the Motion of *Mercury*; and the Wheel FG will turn the Wheel FS about $\frac{1}{29,5}$ round, and will represent the Motion of *Saturn*. And the same is to be said of all the rest.

6. If on the Top of the Box be placed a Circle with the Signs of the Ecliptic, including all the Planets, it will be easy by this Machine to represent the Motion and various Phænomena of the heavenly Bodies, by those who have Skill in such Things. Having thus shew'd the Reason, Structure, and Parts of the portable *Planetarium*, the rest must be left to the Fancy and Pleasure of the Gentleman, and the Ingenuity of the Workman.

7. Since the above was wrote, I have had the Pleasure to find that Mr. J. NEALE (*Watch-Maker in Leaden-Hall Street, London*) has invented and made an Orrery upon the same Plan with mine above described; the Disposition of the Parts, and even the very Number of the Wheels (all but two) being the same except that in mine the Wheel of 148 Teeth for *Saturn* he thought too large, and therefore by breaking the Quotient of $5)148 (=29,6$ into two, he substituted two Wheels (as there is Room enough so to do) in its Stead, and has thus brought the Work into so small a Compass, that for such Gentlemen as require it, he can make an Orrery in the Compass of a Watch, that shall shew the Motions of all the Primary Planets in the Solar System very nicely, which, as such, may be properly call'd POCKET ORRERIES.

(XI IV.) 1. I shall here give an account of the Mechanism of that Instrument, which I call a COMETARIUM

IUM; for though Dr. *Desaguliers* has (the first of any I know of) described it under the Character of Part of a PLANETARIUM, and by it represented the Motion of the Planet *Mercury*; yet I conceive it is much more adapted to represent the Motion of a Comet than a Planet, because none of the Planets describe Orbits sensibly *Elliptical*, whereas those of the Comets are very much so. I have therefore alter'd the Doctor's Contrivance, and adapted it to the Motion or Theory of the Comet of the Year 1682, whose Period is $75\frac{1}{2}$ Years; and have moreover alter'd the Mechanism of the Parts, by which the Instrument is render'd much more elegant and useful.

2. The Structure and Rationale of this Instrument is Pl. XII. as follows. When the Lid is taken off of the Box, the Fig. 2.

Internal Parts appear as in the Figure. NO and QT are two Elliptic Wheels turning each other about their Foci I and S, by means of a Cat-Gut String in a Groove in their Edges, crossing at K. These oval Wheels are fix'd in Arbors or Axes which pass through the same Focus S and I in each; the Oval NO is moved by the circular Wheel I, fix'd also upon the same Axis, but above it on the Bar or long Piece GV; which Wheel is itself moved by another equal Wheel G, and that by an endless Screw, turn'd by a Winch, on the Outside of the Box. All which is evident in the Figure.

3. The Perimeter of the Oval QT, where it touches that of NO, will have a Velocity always proportional to the Distance from I, that is, in the Points K, 4, 3, 2, 1, &c. the Velocities will be as the Lines K, I 4, I 3, I 2, I 1, &c. which may be consider'd as Levers acting upon and moving the Oval QT in those Points. Now if the Ovals are such that SK is to SV, or IK to I 1, as 6 to 1, then will the Point K have 6 Times the Velocity turn'd by the Lever IK, as the Point V will have when it has made $\frac{1}{2}$ a Revolution, or is come under the Point S, where it is turn'd by the Lever I 1, than in the Situation I S.

4. If we take SP=SK=IS, and upon the Point S, as Foci, you describe the Ellipsis PLIM, that will represent the Orbet of the Comet, or the Figure of the Groove on the Lid of the Box, in which a round Brass Ball,

IN WATER-MILLS, the *Momentum* of the falling Water is the Power ; the Force to be overcome is the great Attrition of the two Stones in grinding the Corn, &c. which is effected wholly by a Complication of Wheels and Axles. A Query may here be put, Why, since the Power constantly acts upon the Wheel, the Motion of the Wheel should be equable, and not accelerated : The Answer is, The Increments of Velocity keep rising, till their *Momentum* is equall'd by the Resistance of

Ball, representing the *Comet*, is made to slide along on a Piece of Wire, call'd the *Radius Vector*, fix'd at one End into the Top of the Arbor at S, where we may suppose the Sun to be, and is accordingly represented by a silver'd Plate at Top.

5. The Place of the Comet at P is call'd the *Perihelion*, as being there *nearest the Sun* ; as I is its *Aphelion* or Point of *greatest Distance*. Since $SP = SK$, the Velocity of the Comet will be in the Point P equal to that of the Point K ; and were the Comet's Aphelion at s, its Velocity there would be equal to that of the Point V when under s, *viz.* six Times less than before ; but since the Comet's Aphelion is at I, and since the greater Arch described in the same Time must have a greater Velocity, the Velocity of the Comet at I will be about $\frac{1}{3}$ of that at P.

6. The silver'd Ellipsis on the Lid of the Box is divided into 100 Parts, shewing the *Anomaly* of the Comet ; and about the Axis of the Wheel G is placed a Silver Circle, divided into $75\frac{1}{2}$ equal Parts, representing the Years or Period of the Comet, with a proper Index, pointing to those Divisions, by which 'tis easy to shew the several Particulars relating to the Theory of Elliptic Motions, whether of a Planet or Comet.

the Machine ; after which Equilibrium the Wheel goes on with an uniform Motion (XLV.)

(XLV.) The Mechanism of a Water-Mill depends upon the following Principles.

1. The Action or Power of the Water which drives the great Wheel. Here it will be necessary to determine its Force issuing out of the Aperture of the Sluice or Pen-Stock, and also the Velocity of its Motion when the Height is known ; and the Height necessary to produce a given Velocity.

2. As to the Force of Water issuing through the sluice, that is, its momentary Impulse, we shall shew from Hydrostatic Principles, that it is always proportional to the Altitude of the Water above the Centre of the Hole through which it passes. And since we know by Experiment that a cubic Foot of Water weighs very nicely 1000 Ounces *Averdupois*, or 62,5 *lb* ; if we find the Area of the Aperture in Feet and Parts, and multiply that by the Number of Feet the Water has above the Centre, and lastly, you multiply that Product by 62,5 *lb*. this last Product will be the Force of the Water express'd in Pounds *Averdupois*.

3. For Example ; suppose the Width of the Sluice 2 Inches, or 1 Foot, and that it is drawn up to the Height of three Inches, or 0,25 of a Foot, the Area of the Aperture will then be $1 \times 0,25 = 0,25$; if the Height of the Water be 7,5 Feet above the Orifice, then $7,5 \times 0,25$ will be 1,875, which multiplied by 62,5 gives 116,1875, or about 116,2 *lb*. for the instantaneous Pressure of the Water on any Obstacle.

5. To find the Velocity, and consequently the Quantity of Water expended at the Orifice in a given Time, we must consider that a Body falls 16,2 Feet in the first Second, and acquires a Velocity which in a horizontal Direction is at the Rate of 32,4 Feet *per* second ; now let S be any other Space and the horizontal Velocity V ; then we shall have $16,2 : S :: 32,4^2$

V^2 ; therefore $32,4^2 S = 16,2 V^2$, and so $\frac{32,4^2}{16,2} \times S = V^2$,

V^2 , that is $\sqrt{64,8} S = V$. Now since $S = 7,5 =$ Height of the Water above the Centre of the Orifice, therefore $\sqrt{64,8 \times 7,5} = 22$ per Second, the Velocity of the issuing Water as required.

6. This Velocity might also have been found by finding the Time of the Fall thro' 7,5 Feet; thus $16,2 : 60'' :: 7,5 : t^2 = 1666,6$, the Square Root of which is $40'' ,8$; now since the Velocity of spouting Water is uniform, there will pass twice 7,5 or 15 Feet in $40'' ,8$; and therefore if $40'' ,8 : 15 \text{ F.} :: 60'' : 22 \text{ F.}$ and a little more, the same as before.

7. If the Velocity be given, and it be required to find the Height of the Fall necessary to produce it we have from the foregoing Theorem ($\sqrt{64,8} S = V$,

$S = \frac{V^2}{64,8}$; so that if the given Velocity be 22 Feet per Second, then $V = 22$, and $S = \frac{V^2}{64,8} = 7,5$ nearly, the

Height of the Water above the Centre of the Aperture. For the Velocity of Water is the same in the Fall thro' any Space or Height, and in the Orifice of a Sluice at the same Depth below the Surface.

8. The Quantity of Water issuing out is thus determined for any given Time; since a Column of Water $= 22$ Feet is produced in 1 Second, we shall have $6 \times 22 = 1320$ Feet in 1 Minute, or 158400 Inches, and the Area of the Aperture being $12 \times 3 = 36$ Inches, we have in that Column $158400 \times 36 = 5702400$ Cubic Inches; which divided by 282, gives 2022 Gallons, or 32 Hogsheds 6 Gallons, which spouts on the Wheel in a Minute.

9. If we now suppose this Column of Water flowing into the Buckets of an Over-shot Wheel, which is 12 Feet in Diameter, and has 30 Buckets on its Periphery (as is the Case of that at the *Bar-Pool*, by the Abbot in *Nun-Eaton* in *Warwickshire*, which is reckoned the best in *England*) then the Water will act on the Wheel by two Forces, viz. of Impulse and Weight. The Impulse or Stroke, were it made in a Tangent-Direction, and perpendicular to the Sides of the Bucket would be equal to 116 lb. (by *Art. 4.*) But since

runs some Distance in a *Trough* before it comes to the Wheel, and the Buckets are placed not at Right Angles, but nearly at the Angle of 45 Degrees with the Circumference of the Wheel, and so the Water must strike them very obliquely; I say, on these Accounts we must abate more than one half of the Force of Water, and may take about 50 *lb.* for a Medium.

10. The Weight of the Water is more or less in twelve or thirteen of these Buckets on the Fore-part of the Wheel, but most of all on that Bucket at the End of the horizontal Diameter, because there no Part of the Water rests upon the Perimeter of the Wheel or Sides of the Bucket, but gravitates with its whole Weight in a Tangent to that Part of the Wheel. What that Weight in that Bucket is, may be thus found. It was shewn that when the Engine is in the greatest Perfection, the Velocity of the Wheel ought to be equal to $\frac{1}{3}$ of that of the Water. (See *Annotat.* XL. 6.) Now since the Diameter is 16 Feet, the Circumference will be 50,3; and the Velocity of Water being 1320 Feet *per* Second, $\frac{1}{3}$ of that is 440, which divided by the Circumference gives $8\frac{7}{10}$ Revolutions of the Wheel in a Minute. The Aperture of the Sluice gives 2022 Gallons *per* Minute, which divided by 8,7 gives 232,4 Gallons, which again divided by 30, (the Number of Buckets) is 7,7 Gallons for a Bucket.

11. Now a Gallon of Water weighs 10,2 *lb.* (for $1728 : 62,5 :: 282 : 10,2$); therefore $7,7 \times 10,2 = 78\frac{1}{2}$ *lb.* for the Weight of Water in each Bucket. But as Part of this Water runs out of the Buckets in the lower Quarter of the Wheel, and what remains gravitates in various Degrees of Obliquity to the Radii of the Wheel, as does all the Water in the full Buckets above the horizontal one, we must allow for the total Weight about 450 *lb.* (as will be found near the Truth by Calculation) to which if we add the 50 *lb.* for Impulse, the whole Force of the Water on the Wheel will be 500 *lb.*

12. If the Cog-Wheel be 3,5 Feet Radius, or 7 Feet Diameter (as in the Mill above mention'd) then, as $7 : 16 :: 500 \text{ lb.} : 1143 \text{ lb.}$ = the Force of the Cogs to turn the Wallower or Trundle, which Force is applied to overcome the Resistance arising from the

Weight of the Stone, the Friction of the Geer in general, and the great Friction of the Stones and Corn in grinding.

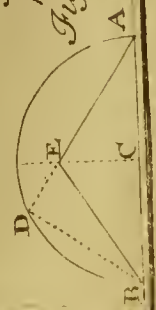
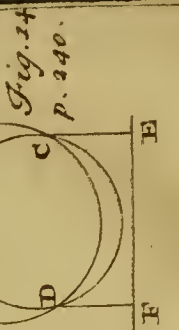
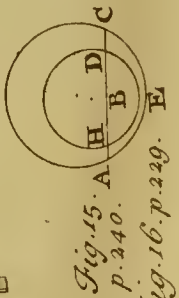
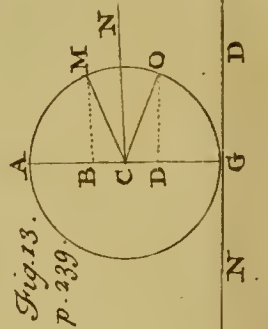
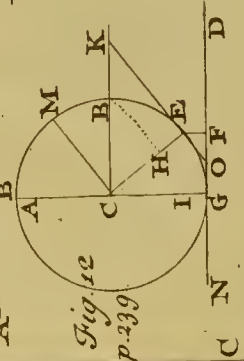
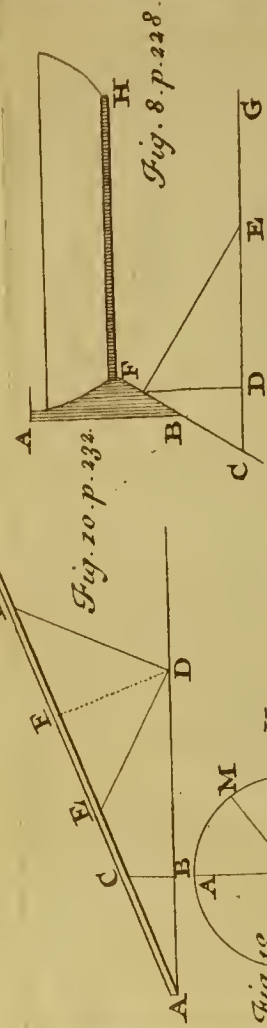
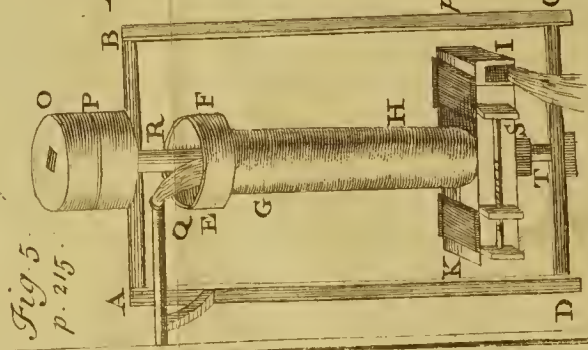
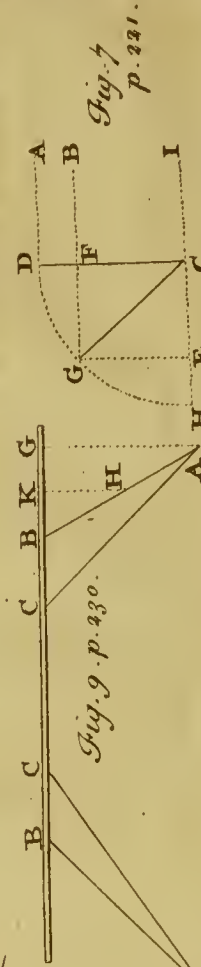
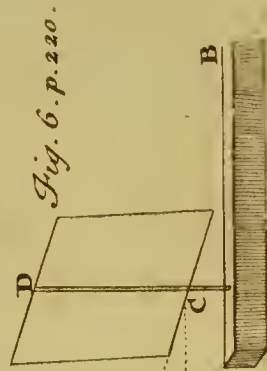
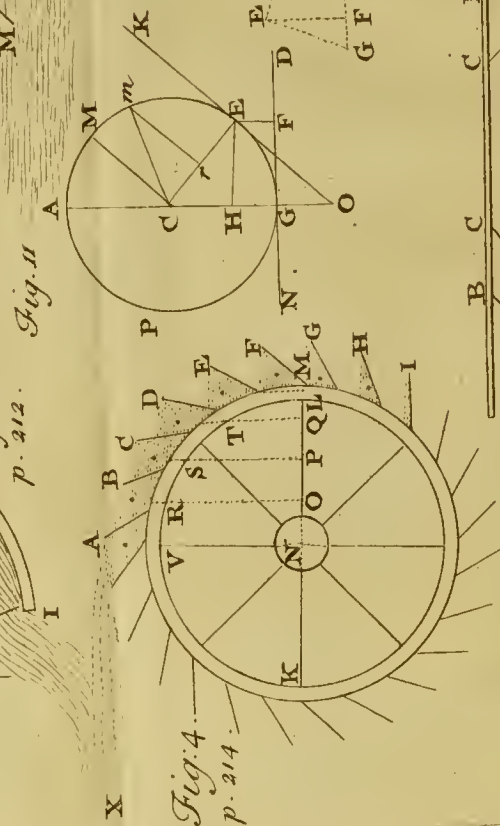
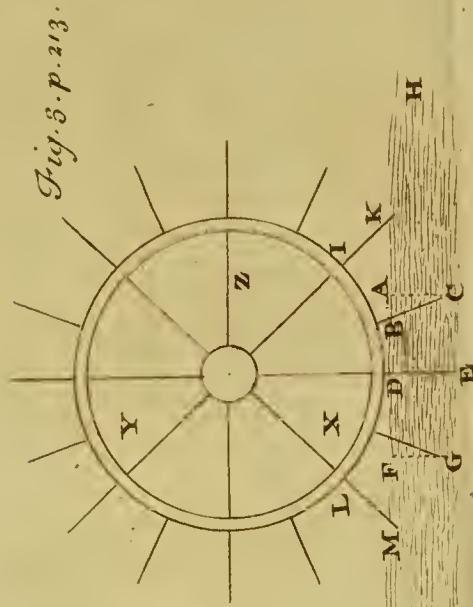
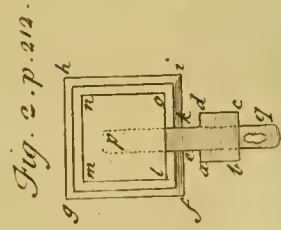
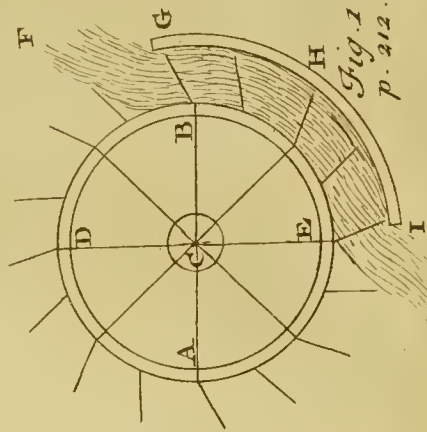
13. Suppose the Diameter of the Trundle 1,5 Foot, or 18 Inches, and that of the upper Stone 6 Feet; then as $6 : 15 :: 1143 : 285,7 \text{ lb.} =$ the Force at the Periphery of the Stone. If there be 48 Cogs in the Cog-Wheel, and nine Rounds in the Trundle, then $\frac{48}{9} = 5,33 =$

the Turns or Revolutions of the Trundle for one of the Water-Wheel; wherefore since the Wheel makes 8,7 in a Minute, the Trundle, and of course the Stone, will make $(8,7 \times 5,33 =) 46\frac{1}{2}$ Turns in a Minute. The Circumference of the Stone is 18,84 Feet; whence $18,84 \times 46,5 = 876$ Feet *per Minute*, for the Velocity of the Stone's Periphery.

14. Suppose the Stone contain 22,5 Cubic Feet, or its Weight 1912 *lb.* then the *mean Velocity* of the Stone being that Point of the Radius $\frac{2}{3}$ of its Length, consequently $\frac{2}{3}$ of the Velocity 876 at the Periphery, *viz.* 584, will be the mean Velocity of the Stone, which multiplied by 1912, *viz.* $1912 \times 584 = 1116608 \text{ lb.}$ would be the Expression of the *Momentum* of the Stone *per Minute*, were it to press upon the Corn with its whole Weight, which it does not; for nearly the whole Weight being supported by the Pivot of its perpendicular Spindle, a very small Part thereof is concerned in the Triture of the Corn, for that is principally effected by the violent Rotation of the Stone producing a Centrifugal Force.

15. This circular Motion of the Stone brings the Corn out of the Hopper by Jerks, and causes it to recede from the Centre to the Circumference in a special Manner; the Corn, while whole, causing the Stone to rise a little higher above the fix'd Stone than it would otherwise do, begins to be crushed by the Weight of the Stone gently pressing upon it, and the more so, as it approaches the Circumference, where being quite reduced to Flour, it is thrown out of the Mill by the Centrifugal Force of the Stone through a Hole for that Purpose.

16. When I say, *the Stone presses gently, and but with a very small Part of its Weight on the Corn*, it is implied,



Implied, that the Piece of Wood which supports the Stone on the Pivot of its Spindle must be somewhat elastic or springy, in order to admit of an easy and small Degree of Motion upwards and downwards. as the Stone is more or less resisted by the Grains of Corn, by which Means the Surfaces of the two Stones will have always a varying Distance, and so be adapted for Trituration and Comminution of the Corn in any Degree; whereas, if the supporting Piece were perfectly rigid or fix'd, the Stones must always be at an equal Distance, and the Flour very coarse, because it can't be affected by the upper Stone after the whole Grain has receiv'd its first Impression, or suffer'd its first Comminution. And the nicer this *Point of Support* is adjusted, so much the more nice and exact will be the Work done by the Mill, or so much the better will the Flour be that is produced thereby.

17. As the Water acts upon an *Overshot-Mill* both by Impulse and Weight, so does it likewise on a *Breast-Mill*, or that where the Water comes upon the Breast or middle Part of the Wheel; and here, though the Weight of the Water on the Wheel is not so great as before, being contain'd in the Buckets of the lower Quarter only, yet the Impulse of the Water is much greater, the Height of the Water being increased nearly the Semidiameter of the Great-Wheel, all other Things being equal.

18. If the Height of the Water remains the same, the Aperture of the Pen-stock must be enlarged to nearly twice the Area, that the Force of Water may be the same; and in this Case there will be twice the Expence of Water as before; so that as much more Water is necessary for a Breast-Mill than for an Overshot one, every Thing else being the same.

19. Since the Spout of Water is in the Curve of a Parabola, the Orifice through which it issues should not be just against the Middle of the Wheel, for then, if it were very near the Wheel, great Part of the Force would be spent in striking the Wheel directly against its Axis, and so would make the Motion round more difficult; and if it were not very near the Wheel, the Spout would reach only some of the lower Buckets,

and none go to those on the Middle, where the Effect would be rather greater. Therefore the Orifice of the Pen-stock ought to be situated some little Height above the Middle or Horizontal Diameter of the Wheel.

Pl. III.
Fig. 1.

20. In order that the Water may have the greatest Effect on the Wheel, Dr. *Barker* has constructed it with 24 Ladle Boards, (instead of Buckets) of 18 Inches Square; these Ladle-Boards pass through a quadrantal Channel of the same Dimensions nearly, or a little more than 18 Inches square, that the Motion may be free. The Water entering on the Top of this Channel is kept wholly on the Boards, till it comes to the Bottom, where it goes off without interrupting the Motion of the Wheel. The Contrivance is very useful, and may be easily apprehended by the Scheme, where A D B E represents the Wheel with its Ladle-Boards, C the Axis, F the Jet or spouting Water, GHI the square Channel or Trough in which the Ladle-Boards with the Water descend to I, where the Water is discharged from the Wheel.

Fig. 2.

21. In Fig. 2. you have a Section of the Sole or Perimeter of the Wheel, and the Channel with the Ladle-Boards in it; thus *a b c d* is the Sole of the Wheel, *f g h i* is the Section of the Channel open on the Part next the Wheel by a narrow Slit, in which *e k* is the Part call'd the Tongue, which projects from the Sole to fill up that Slit, that the Water may not run through it; *l m n o* is one of the Ladle-Boards put on upon the Supporter *p q*, which goes through the Tongue and Sole of the Wheel, with a Hole behind at *q*, through which a Key or Wedge, like a Piece of Wood, being driven, fastens it with the Board to the Wheel. Thus you see by this Means how great the Force of the Water must be on the Wheel, and how much less will suffice here than in the common Way.

22. As to an *Under-shot Wheel*, 'tis evident there can be only the Force of the Impulse from the Water on such a Wheel; and therefore the Height of Water remaining the same, there must be a larger Aperture of the Pen-stock, that so a greater Quantity of Water may come upon the Float-Boards in the same Time,

to

to have an equal *Momentum*, or to produce the same Effect as in the *Overshot* or *Breast-Wheel*. Whence a greater Expende of Water will be made here, than in any other Sort of Mill, and can only be supplied for a Constancy by a River ; and where there is not a Want of Water, this is the easiest, cheapest, and most simple Structure a Mill is capable of.

23. In this Case, the Float-Boards are to be placed perpendicular to the Sole or Periphery of the Wheel, because when they come into a vertical Position at the Bottom or lowest Part of the Wheel, they will then receive the horizontal Impetus of the Water directly, and therefore with the greatest Force. If the Water-Course be sufficient to cover the Float-Boards, that is enough ; all that is more, runs waste, either by the Sides or under the Floats, or both.

24. Let XYZ be an Undershot-Wheel, and HC Pl. XIII. the Water coming upon it in the Direction HC. Now Fig. 3. if there were so few Float-Boards, that when one, as DE, became vertical, the next on each Side IK, LM, should but just touch the Surface of the current Water, then would the Water strike the said Float-Board DE with all its Force. But when the two Floats K and E come into the Situation C and G, then will the Water strike but a Part of the Float C, viz. BC, and that too obliquely, which is to be estimated by AC, the Sine of the Angle of Incidence ; and since we suppose the Section of the Water equal to the Area of a Float-Board, 'tis plain the Float C will intercept the Water from the Float G, so that none can strike it ; and therefore the Force of Water will be every where less than upon the Float-Board in the Site DE. So that the Force upon the Wheel will always be fluctuating between AC and DE, which will be the two Extremes.

25. If the Float-Boards are so many, that while one is Vertical as DE, others on each Side, as C and G, are also partly in the Water-way ; then in this Case it is best to have the Stream of Water larger than the Floats, that some may run beside, and fill up the Space between, that so the Floats C, E, G, may all be impell'd at once ; for the Back-Water, here, having a greater Velocity than that of the Wheel, will still keep

the Float G forward; and though it be an oblique Force, it will always avail something so long as it touches the Float; and cannot be esteem'd a Negative Quantity, as some have asserted.

26. Since many may be curious to know how the Force of Impulse and Gravity of the Water in the Buckets of an *Overshot-Wheel* is to be computed or estimated to a Mathematical Exactness, I shall here give the Method, and illustrate it by a Scheme. Let ABC, &c. be the Buckets of an *Overshot-Wheel*, (having twenty-four in all) inclined to the Periphery in an Angle of forty-five Degrees: Those with Dots or Points represent the Buckets with the different Quantities of Water they contain, among which the largest Dots in the Middle represent the Centres of Gravity of the several Bodies of Water. Now suppose the Water comes upon the uppermost Bucket A in the horizontal Direction XA, though by its Curvature at entering the Bucket it cannot strike the Side of the Bucket directly, yet, since the Side of the Bucket obstructs and sustains the said whole horizontal percussive Force, and that under an Inclination of forty-five Degrees, we may conclude that half that percussive Force is spent in turning about the Wheel.

27. As to the Force arising from every descending Bucket of Water, it may be easily determined by finding the Bulk of the Water in each Bucket, and multiplying that into the perpendicular Distance of the Line of Direction of the Centre of Gravity from the Centre of Motion. Hence, with respect to the first Bucket A, since its Centre of Gravity acts in the Direction VN, that is, perpendicularly on the Centre of Motion, that Product, or the Weight of that Bucket of Water, will avail nothing to move the Wheel round, as being wholly supported on the Axis of Motion.

28. But the Water in the second Bucket B gravitating in the Direction RO, at the perpendicular Distance NO from the Centre, if we multiply its Mass or Weight into the Distance NO, we shall have its *Momentum* or Force to move the Wheel; and so of any other. Here we may observe, that as the Water in the Buckets decreases, the Distances increase from the Centre, in the upper descending Quadrant; and since the

the Distances increase much faster than the Quantities decrease, the Forces will increase till we come to the horizontal Bucket F, which is half full of Water; from thence it decreases to the Bucket I, where all the Water runs out, as being parallel with the Horizon, or Diameter KM. And hence we see four Buckets on the lower descending Quadrant carry no Water, when they are inclined to an Angle of 45 Degrees.

29. I shall here subjoin a Specimen of the Computation, supposing the Wheel and Buckets such as in the Scheme, and the Quantities of Water, and Distances of their Centres of Gravity, the same as in the Table below.

<i>Water in the Bucket.</i>	<i>Distances from the Centre N.</i>	<i>Moments of each Bucket.</i>
A = 1,00	× O	= 0,00
B = 0,9	× NO = 0,3	= 0,27
C = 0,8	× NP = 0,6	= 0,48
D = 0,7	× NQ = 0,85	= 0,595
E = 0,6	× NL = 0,95	= 0,57
F = 0,5	× NM = 1,00	= 0,5
G = 0,4	× NM = 1,00	= 0,4
H = 0,2	× NL = 0,95	= 0,19

The Total of all the Buckets = 3,005

That is, the *Momentum* of Water in all the Buckets is equal to the *Momentum* of three Times the Water contained in the Bucket A, and hanging at the End of the horizontal Diameter KM.

30. I shall conclude this Theory of *Water-Mills*, with a Description of Dr. Barker's new-invented Mill, of the most simple Structure of any yet made, performing its Effect without any Wheel, Trundle, Cog, or Round; the Nature of the Machine, and Manner of its Operation, will be easy to understand from the following Account of its several Parts.

31. ABCD is an upright Frame standing on a proper Base; EF is the wider Part of GH an upright hollow Pipe or Tube, fix'd at the Bottom to an horizontal Square Trunk IK; which Trunk, together with the Tube, is fix'd to an upright Spindle or Axis RS, by means of a Nut and Screw at S. The lower End of

Pl. XIII.
Fig. 5.

the Axis on a fine Point moves in the Pivot Hole in the Part of the Frame at T; on the upper Part of the Frame is a Hole thro' which the Spindle passes, as also thro' the round circular Piece P fix'd on the said Frame; on the upper Part of the Spindle is fix'd another round circular Piece O, which represents the upper moveable Stone of the Mill. Q is a Spout of Water filling the Tube or Trunk, and giving Motion thereto, and consequently to the Axis and upper Stone, by the horizontal Jets of Water from each End of the Trunk IK, thro' Holes made at each End on contrary Sides.

32. While the Holes continue stopp'd, the Trunk will be at rest, because then the Pressure is equal over all the Parts: but when the Holes are open, the Pressure of the Water (by its having Liberty to issue out) will be less on that Part where the Hole is, than on the other Part opposite to it; which stronger Pressure will prevail, and carry round the Trunk and Tube with the Axle and Stone, in a contrary Direction; and each Hole contributes to produce this Motion, which will be greater or lesser in proportion to the *Momenta* of the Jets of Water, or greater or lesser Aperture of the Holes.

33. For 'tis easy to understand, that the Power of this Machine is derived from, or depends upon, three Things: (1.) The Velocity of the Spouting Water; (2.) The Quantity thereof; and (3.) The Distance at which the Water spouts from the Axis of Motion. The two first make the *Momentum* arising from the Pressure of the Fluid, which is proportional to the Altitude, or Height of the Tube; the last is of a Mechanical Nature, for the Trunk is in this respect exactly of the Nature of the *Lever*.

34. In the Lecture of Hydrostatics it will be shewn, that the Velocity of the Spouting Fluid will be as the Square Root of the Altitude of the Fluid; whence this Part of the Force will be in the *Subduplicate Ratio of the Height of the Tube*. The Quantity of the Spouting Fluid will be also in the same Ratio, while the Apertures are the same; but if these vary their Magnitude, it will be directly as the Aperture, the Altitudes being given. Therefore the whole *Momentum* arising from these

these hydrostatical Principles will be *as the Altitude of the Tube, and Sum of the Apertures conjointly*; and if this be multiplied by the Distance of the Aperture from the Centre or Axis, we shall have the Expression of the whole Force of the Machine.

35. That is, if H = the Height of the Fluid, A = Sum of both the Apertures, and D = Distance of each from the Axis; then we shall have the *Momentum* M expressed by $M = H \times A \times D$. Hence we see that the Trunk IK is analogous to the great Wheel of a common Water-Mill, whose Force is in the same Manner computed from the Height of the Fluid H , the Aperture of the Sluice A , and the Distance or Radius of the Wheel D .

36. For both in the common Mill and this, if the Aperture A and Distance D from the Centre be the same, the Force of the Jet will vary with the Height of the Fluid H . Also if the Height of the Fluid H and Length of the Trunk or Diameter of the Wheel D remain the same, the Force will be as the Aperture A , or as the Quantity of the Fluid flowing out in a given Time. Lastly, if the Height of the Fluid H , and the Aperture A continue the same, the Force will be as the Diameter D of the Wheel, or Length of the Trunk of this new Mill.

37. I shall now give a Calculation of the Power of this Machine; and in the first Place, let us suppose the Height of the Tube to be 9 Feet, and always full of Water; the Velocity of the Spouting Water will be the same that a Body will acquire in falling 9 Feet, *viz.* an uniform Velocity of 18 Feet in the *Time of the Fall*;

which Time is thus found, as $16 : 9 :: 1^2 : \frac{9}{16}$ = the Square of the Time, whose Square Root is $\frac{3}{4}$, that is, $\frac{3}{4}$ of a Second. Therefore say, as $\frac{3}{4} : 18 :: 1 : 24$; the uniform Velocity is at the Rate of 24 Feet *per* Second, in the Spouting Water, from 9 Feet Altitude.

38. Let the Aperture at each End of the Trunk be 6 Inches Square, then in both there will be 72 square Inches, or half a square Foot in Area. The Water issuing out in a Second at both Orifices will be equal to a Column 24 Feet long, and $\frac{1}{2}$ a square Foot in Base; therefore the whole Water will be equal to 12 Cubic

bic Feet. The Weight of one Cubic Foot is $62\frac{1}{2}$ lb; wherefore $62\frac{1}{2} \times 12 = 750$ lb. is the Force acting on the Trunk, supposing the Depth of Water only one Foot; but since it is at the Depth of 9 Feet, we have $9 \times 750 = 6750$ lb. acting upon the extremc Parts of the Trunk *per* Second.

39. Let us now suppose the Length of the Trunk to be 6 Feet; then the Motion is made by a Power of 6750 lb. acting at the End of a Lever three Feet long; consequently $6750 \times 3 = 20250$ lb. the *Momentum* of the Wheel *per* Second.

40. Now the Velocity of the Trunk at the Orifices will be the same as that of the Water nearly, while the Mill is not charged; but that of the Water is 24 Feet *per* Second, and the Circumference which each Orifice describes is 18,84, or 19 Feet nearly; therefore the Trunk turns round a little more than once *per* Second. But when the Mill is charged with the Stone, Corn, &c. the Velocity will not then be so great; and we have shewn (*Annot.* XL. 6.) that it ought not to be above $\frac{1}{3}$ Part of that of the Water, *viz.* at the Rate of 8 Feet *per* Second; but let us suppose it a little more, *viz.* $9\frac{1}{2}$ Feet, then will the Trunk move round once in two Seconds.

41. If we suppose the Stone 6 Feet in Diameter, its Velocity in the Periphery will be the same with that of the Trunk; *viz.* $9\frac{1}{2}$ Feet *per* Second; but $\frac{2}{3}$ of this is the mean Velocity of the Stone, which is therefore but $6\frac{3}{4}$ Feet *per* Second. Suppose the Weight of the Stone 1912 lb. and that $\frac{1}{3}$ of this be allowed for Friction, *viz.* 637 lb. then $6,3 \times 637 = 4015$ lb. the Moment of the Stone by its Resistance arising from $\frac{1}{3}$ of its Weight; but this is only when the Stone is first put into Motion; after it is in Motion, this *Momentum* is greatly lessen'd by the Centrifugal Forces; and therefore if we allow 4000 lb. for the *Momentum* arising from the Weight of the Stone, Attrition of the Corn, &c. it will be but $\frac{1}{5}$ Part of the *Momentum* of the Trunk or Mill, which we shewed was 20250 lb.

42. Here we have supposed the Mill to carry the largest Stone that is used in any Mill, and that the Power of the Mill is five times greater than the Resistance to be

IN WIND-MILLS the Mechanism is the same, only the Vanes or Sails are to be consider'd as a *Wheel on the Axle*, actuated by the Power of the Wind. In this Machine we have only to consider the Position of the Sails, and the Power of the Wind. As to the former, if the Sails stand *right* before the Wind, it cannot affect them at all; if they stand *direct* to the

be overcome; the Quantity of Water therefore in the Jets may be diminish'd a *Fifth Part*, and consequently the Apertures; which therefore, instead of being 72 Square Inches, need be only 15 or 16. If therefore at each End of the Trunk there be an Aperture of four Inches long and two wide, the Jets from the Altitude of nine Feet, at three Feet from the Axis, will have a *Momentum* sufficient for turning the largest Stones.

43. I shall only observe further, that there are many mechanical Purposes to which this Invention might be made subservient, as it has so great a Power of Motion in so easy and simple a Structure: And also, that the Water-Wheel of a common Mill, if placed in a horizontal Situation, and the Sluice so ordered as to throw a Side-Jet in a Tangent-Direction on the Ladder-Boards, such a Wheel would be in the same Circumstances nearly as the Trunk of this New Mill, and therefore might be made to answer all the same Intentions, and capable of nearly all the same Advantages.

44. I need not observe to the Reader, that whatever Quantity of Water is expended at the Jets, as that must be supplied at the Top or Cistern of the Tube, so the Stream which supplies it must be as much longer than the Jets, as its Velocity is less; because what is deficient in one Respect must be made up in the other. There are several other Circumstances and Particulars, which may deserve to be consider'd, when it shall be found to answer any valuable End, beyond that of mere Speculation.

Wind,

Wind, the Mill will be blown down, at least the Sails can have no Power to move round; they must therefore be placed *oblique* to the Wind, and that under an Angle of 54 Degrees, and 44 Minutes, for the greatest Advantage (XLVI.)

(XLVI.) The internal Parts of a Wind-mill are the same with those of a Water-Mill nearly, of which I shall take no farther Notice, but shall confine myself to the Theory of the Sails in regard of their Position, Motion, and Figure.

1. In regard of the Position of the Sails, we must consider that if they are placed *Direct to the Wind*, or at Right Angles to the Axis of the Mill, they will receive the whole Force of the Wind, which in this Case will tend to blow them forward, and consequently to blow down the Mill; which Position of Course cannot be admitted.

2. If the Sails are set *Right to the Wind*, or parallel with the Axis of the Mill, 'tis plain that in that Position the Wind cannot act upon them at all, and therefore they cannot be turn'd round, nor the Mill put into Motion; which Position of the Sails must likewise be rejected.

3. Since neither the *Direct* nor *Right Position* of the Sails will do, an *Oblique Position* must, as there can be no other. Now to shew that an *Oblique Position* of the Sails will turn the Mill, let AB be the Axis, CD a Sail, and its Angle of Obliquity (*viz* that which it makes with the Axis) be ECG; then if GC be the Force of the Wind in the *Direct Position* of the Sail, GE will be the Force of the Wind in its *oblique Position* (as being the Sine of the Angle of Incidence GCE). But the Force GE is resolvable into two others, EF and GF; of which the latter, being parallel to the Axis, avails nothing in turning the Sails about it; but the other, EF, being perpendicular thereto, is wholly spent

Pl. XIII.
Fig. 6.

spent in compelling the Sail to turn round ; which was the Thing to be shewn.

4. The Force of the Wind on the Sail will be as the Square of the Sine of Incidence, or as $\overline{GE}^2$; for the Force of each single Particle of Air will be as the Sine GE, (by *Annot. XXIV. 6.*) and it will be also as the Number of Particles which strike it at the same Time, Pl. XII. Fig. 7. which Number of Particles is also as the Sine of Incidence GE. For let CD represent the Section of the Sail in a direct Position, and CG the same in an oblique Position, 'tis plain the Number of Particles striking it in the former Case will be to the Number striking it in the latter as CD to CF, which is equal to GE the Sine of Incidence ; for all the Particles between AD and BF will not come upon the Sail in the Oblique Position CG. Since then the Force of the Wind on the Sail is on two Accounts as GE, it will be as the Square of the said Line GE.

5. If we suppose the Velocity of the Wind to vary, the Force thereof will be as the Square of the Velocity ; for the greater the Velocity, the greater will be the Stroke of each single Particle, and also the greater will be the Number of Particles coming upon the Sail in the same Time ; the Force will be therefore as the Squares of the Velocity.

6. Again, if the Area of the Sail be variable, the Force of the Wind will be directly as the Area or Superficies of the Sail, because the Number of Particles of the Air coming upon it will always be proportional thereto, and consequently the Force with which they strike it. Hence, if A, S, and V represent the *Area*, *Sine* of Incidence, and *Velocity* of the Wind on one Sail, and *a*, *s*, *v*, those on another ; the Force compelling the former to turn round will be to that compelling the latter, as $A \times S^2 \times V^2$ to $a \times s^2 \times v^2$.

7. When the Area of the Sail and its Position in respect of the Wind continue the same, the Force which turns the Sail will be as the *Squares of the Velocity* ; and since the Wind scarce ever blows with one uniform Velocity, but varies with almost every Blast, the Force upon the Sail will be much more variable and unequal ; and therefore the Action or Working of a Wind-Mill cannot

cannot be so equal, uniform, and steady as that of a Water-Mill, whose Power is always of the same Tenor, while the Jet of Water is so.

8. If the Area of the Sail and the Velocity of the Wind be supposed constant, the Force of the Wind in the Direct Position will be to that in the Oblique one as $\overline{GC}$ to $\overline{GE}^2$, as we have before shewn; and it has been also shewn that that Part of the Force which turns the Sail is represented by EF , when GE is the whole Force: But $GE:EF (:: GC:CE) :: \overline{GE}^2: \frac{CE \times \overline{GE}^2}{GC}$

= to the Force which turns the Sail, when the whole Force is represented by $\overline{GE}^2$, as is here the proper Expression of it.

9. This Expression $\frac{CE \times \overline{GE}^2}{GC}$ begins from Nothing, when the Angle of Incidence begins to be oblique, and increases with the Obliquity of the said Angle to a certain Number of Degrees; because that Part of the Force which is parallel to the Axis becomes lesser in proportion to that which is perpendicular to it; but after it has pass'd this Limit, it again decreases, and becomes nothing, when the Angle of Incidence vanishes; as is easy to understand by considering that the Quantity of Wind on the Sail does in this Case continually decrease.

10. There is therefore one certain Position of the Sail, in which the Force of the Wind is greatest of all upon it, or a *Maximum*; and to find it, put Radius $GC=a$, $EC=x$, and we have $\overline{GE}^2 = aa - xx$, and

consequently the Force $\frac{CE \times \overline{GE}^2}{GC} = \frac{aax - xxx}{a}$, which must be a *Maximum*: Therefore its Fluxion $aa\dot{x} - 3xxx = 0$; whence $aa = 3xx$, and so $x = \sqrt{\frac{aa}{3}}$

which in Logarithms is $\frac{20,000000 - 0,477121}{2}$

9,761439, which is the Logarithm Sine of the Angle $35^\circ 16' = \text{Angle } CGE$; and therefore the Angle ECG is

is equal to $54^{\circ}44'$, when the Force of the Wind is a *Maximum*, as required.

11. The Angle now found, is only that which gives the Wind the greatest Force to put the Sail in Motion, but not the Angle which gives the Force of the Wind a *Maximum* upon the Sail when in Motion. What this Angle is Mr. *MacLaurin* has shewn in his Book of *Fluxions*, and which I have farther explained in a Treatise, intituled, *New Principles of GEOGRAPHY and NAVIGATION*.

12. Mr. *Parent* has also shewn that an Elliptic Form of the Sails is better than the Parallelogram or long Square; and that the best Position of the Sail is not that which is common, *viz.* with its longest Side or Diameter parallel to the Axis of the Sail; but on the contrary, it ought to be perpendicular to it; that is, they ought to be of such a Form, and placed in such a Manner, as represented in the Figure; and after the four Sails B, C, D, E, are thus placed in the Axis or Arm A, they are then to be turn'd about, and fix'd under the proper Angle of Obliquity above-mention'd.

13. There are three Things yet wanting to the Perfection of a Wind-mill. The *First* is, some Contrivance in the Nature of a *Fly* to regulate the Motion of the Train, under the irregular and unequal Impulse of the Wind. The *Second* is, some other Contrivance to supply the Hopper or Stones with more or less Corn, in Proportion to the greater or less Strength of the Wind.

And, *Thirdly*, a Method of altering the Angle of the Sail's Obliquity from its *Maximum* of $54^{\circ}44'$ at the Beginning of the Motion to its *Maximum* when in Motion.

14. By means of an *Anemoscope* (hereafter to be described) it will be easy to prove by Experiment what Form of the Sails, that is, whether Rectangular or Elliptical, whether the Vertical or Horizontal Position of the longest Diameter, and what Angle of Obliquity is best; also whether the Surface of the Sails should be plain or concave, with many other Things of this Kind. But this must be done by as many particular Anemoscopes, or in other Words, you must have an Anemoscope for every particular Experiment, and all disposed together in one common Frame; the Reason is evident, because

Plate X.
Fig. 9.

because they all require the same Strength of Wind, which cannot otherwise be had.

15. That these Things may be better understood, I shall premise the following Definition of some Geometrical Lines and Figures, which are absolutely necessary to a compleat Knowledge of the modern Mechanical Philosophy.

Plate IX. 16. I take it for granted, that the Reader knows, Fig. 6. that if on any Point C, taken in the Right Line AB, a Circle ADEFF be described, the Point C is call'd the *Centre*, and AE the *Diameter* of the Circle: To which I shall add, that AC, or CE, is call'd the *Radius* of the Circle, which is the same thing as the *Semidiameter*.

17. If the Circle be divided into four equal Parts, $AD=DE=EF=FA$, by the two Diameters AE, DF; then each of the Areas ACD, DCE, ECF, FCA, are call'd *Quadrants*, or Quarters of the circular Space; and the Parts of the Circle AD, DE, EF, FA, are call'd *Quadrantal Arches*, or Quarters of the Circle.

Fig. 7. 18. Every Circle, great or small, is supposed to be divided into 360 equal Parts, call'd *Degrees*; consequently each Quarter, AD, DE, &c. will contain 90 of those Degrees, as is evidently represented by the large Half-Circle of Fig. 7.

19. If two Lines BC and FC meet in a Point C, the Space FCB included between them is called an *Angle*; and the Measure of that Angle is the Number of Degrees contain'd in an Arch EI of a Circle described on the Angular Point C, and included between the said two Lines BC and FC. Thus the Angle in the Figure contains 40 Degrees.

20. If a Line, as GC, be drawn through the 90th Degree on the Point C, it will make the Angle on one Side GCB equal to the Angle GCA on the other Side, because each is equal to 90 Degrees. Such an Angle is called a *Right Angle*; and the Line GC is then said to be *perpendicular* to the Line AB.

21. The Angle FCB, which is less than a Right Angle or 90 Degrees, is call'd an *Acute Angle*; and the Angle HCB, which is greater than a Right Angle or 90 Degrees, is call'd an *Obtuse Angle*. Again; the Arch

Arch ID is called the *Complement* of the Arch EI to a Quadrant ED, or 90 Degrees; and the Arch AK is call'd the *Supplement* of the Arch EK to a Semicircle EDA, or 180 Degrees.

22. If from the Point I be let fall the Perpendicular IL to the Line or Radius EC, then is that Line IL call'd the *Sine* of the Angle ICE or FCB. In the same manner the Line IM is the *Sine* of the Complement Arch ID, or Angle ICD. But instead of *Sine-Complement*, we say, in short, *Co-Sine*: Thus we say that IL is the *Sine*, and IM the *Co-Sine*, of the Angle ICE. The Angle FCB is call'd the *Inclination* of the Line FC to the Line BC; and the Angle FCD is the *Inclination* of the Line FC to the Perpendicular DC: That is, FC is inclined to BC in an Angle of 40 Degrees, and to DC in an Angle of 50 Degrees. Hence IL and IM are call'd the *Sines of Inclination* respectively.

23. Hence, when we say, *The Force of a direct Stroke is to that of an oblique one as Radius is to the Sine of Inclination*, we only mean, that those Quantities are to each other as the Radius IC to the Sine IL, or IM, according as the Inclination is 40 or 50 Degrees. Also, when 'tis said that *the centrifugal Force decreases from the Equator towards the Poles, in proportion of Radius to the Co-Sines of the Latitude*; no more is meant than this, that if the Radius CE represent the said Force in the Equator E, and EI be any given Latitude, then will IM, the Co-Sine of the Latitude, represent the Force in that Latitude: Or, the Force decreases with the Length of the Line IM, as the Point I moves on from E to D.

24. In several Books we have Tables of Numbers which express the Length of the Sine of every Degree and Minute of the Quadrant, in such equal Parts as the Radius CE or CI contains 100000. And since it is of the greatest Use to know the Proportion of Radius to the Sine of every Degree, I have here subjoin'd a Table thereof, and a Specimen of its Use.

Deg.	Parts.	D.	Parts.	D.	Parts.	D.	Parts.	D.	Parts.
SINE of 1	1745	19	32556	37	50181	55	81915	73	95630
2	3483	20	34202	38	51566	56	82902	74	96126
3	5232	21	35831	39	52932	57	83867	75	96592
4	6971	22	37460	40	54278	58	84804	76	97029
5	8715	23	39073	41	55605	59	85716	77	97437
6	10452	24	40573	42	56913	60	86602	78	97814
7	12186	25	42261	43	58199	61	87461	79	98162
8	13917	26	43837	44	59465	62	88294	80	98480
9	15643	27	45399	45	60710	63	89100	81	98768
10	17364	28	46947	46	61933	64	89879	82	99026
11	19080	29	48480	47	63135	65	90630	83	99254
12	20791	30	50000	48	64314	66	91354	84	99452
13	22495	31	51503	49	65470	67	92050	85	99619
14	24192	32	52991	50	66604	68	92718	86	99756
15	25881	33	54463	51	67714	69	93358	87	99862
16	27563	34	55919	52	68801	70	93960	88	99939
17	29237	35	57357	53	69863	71	94551	89	99984
18	30901	36	58778	54	70901	72	95105	90	100000

25. The use of this Table will be obvious from two or three Examples. It was observed, that the Power is to the Weight it sustains on any Inclined Plane IC, as the Height of the Plane IL to the Length thereof IC; that is, as the Sine of the Plane's Inclination to the Radius. Suppose the Angle of Inclination ICE = 40 Degrees, then will the Sine IL be equal to 64278, and the Radius CI = to 100000, which Numbers are as 64 to 100; therefore 100 Pounds will be sustained on the Inclined Plane by a Power equal to 64 Pounds nearly.

26. Again; since EC = 100000 represents the centrifugal Force under the Equator, then will IM = 76604 (the Sine of 50 Degrees, and Co-Sine of 40) be as the said Force in the Latitude of 40 Degrees: Which Numbers are as 1000 to 766; and such is the Proportion of the Forces in those two Places.

27. In the same manner, if the Radius CD = 100000 express the Force of any direct Stroke, then will the Sine IL = 64278 be expressive of the Force of an oblique Stroke in the Direction FC, every Thing else being equal.

28. Again; since the Force of a direct Stroke is express'd by CD = 100000, if it were required to find the Angle of Obliquity, such that the Force of the Stroke

Stroke shall be but one fourth Part so great, or 25000, look in the Table for the Number nearest to this, and you will perceive it to lie between 14 and 15 Degrees, and therefore about 14 Degrees and a half is the Angle required.

29. In the last Place: It was said, That the Force of the Wind on the Sail is proportional to the Squares of the Sines of the Angle of Incidence. This may be illustrated by Numbers, thus: If the Sail be turn'd to the same Wind, first under an Angle of 45 Degrees, and then under an Angle of 30 Degrees; the Sine of the first Angle is (by the Table) 70710, and of the latter 50000, the Squares of which are 4999904100 and 2500000000, which Squares are as 50 to 25 or as 2 to 1; and therefore the Power of the Wind is twice as great upon an Angle of 45 Degrees, as upon an Angle of 30.

30. Because the Square of the Sine of 45 Degrees is 5000000000, twice that Square will be 10000000000, which is equal to the Square of Radius 100000; 'tis evident the Sum of the Squares of the Sines of any two Angles above 45 Degrees will be greater than the Square of Radius; and therefore the Force of the Wind upon two oblique Sails, in that Case, will be greater than upon one Sail set direct before the Wind.

31. After the same manner, the Table of Sines may be applied to Calculation in any other Case of the like Nature, where the Proportion of Radius and Sine of an Angle is required to be express'd or stated in Numbers. And since each Degree is divided into 60 equal Parts or Minutes, therefore the Sine of any Number of Degrees and Minutes also may be easily found by the foregoing Table, by those who understand the Rule of Proportion.

32. I shall only further observe here, that as I L is the Sine, and L M the Co-sine, of the Arch I E: so if on the extreme Point E of the Radius C E we raise a Perpendicular which shall cut the Line C I, continued, in F, then is the said Perpendicular E F call'd the TANGENT of the Angle I C E, and the Line F C the SECANT of the same Angle. In like manner the Perpendicular D G is the Tangent of the Angle D C I, and

Q 2

G C

IT is to be observed, that in order to turn a Ship about in the least Time, or with the greatest Celerity possible, the Rudder ought to make an Angle with the Stern of 54 Degrees, 44 Minutes: And also, that this is the Angle which the *Gates of a Lock* upon a River ought to make with the Sides of the River, in order to resist the Water with the greatest possible Force (XLVII.)

GC the *Secant* thereof; and therefore DG is the *Co-Tangent*, and GC the *Co-Secant* of the Angle ICE. I thought it necessary to acquaint the Reader with these Definitions, because they sometimes occur in Treatises of this Kind.

(XLVII.) I. If ABF be the Rudder of a Ship, Pl. XIII. AH placed in the oblique Situation FC, and the Water striking against it in the Direction GC; let CE Fig. 8. be Radius, then the Sine of the Angle of Incidence will be FE, and so the Force of the Water against the Rudder will be as $\overline{FE}^2$; but EF is resolvable into the two Forces FD and DE, of which the latter is parallel, the other perpendicular to the Direction of the Ship's Course, and therefore FD is the only Part of the Force that compels the Ship to turn round. But EF is to FD ($:: CE : CF$) $:: \overline{EF}^2 : \frac{CF \times \overline{EF}^2}{CE}$, that is (putting $CE = a$, $CF = x$) as $\frac{aax - x^3}{a}$, and so x will be found (by the Method *de Maximis & Minimis*) equal to $\sqrt[3]{\frac{aa}{3}}$; and therefore the Angle of Incidence $ECF = 54^\circ 44'$, as before, when the Force of the Water against the Rudder to turn the Ship is a *Maximum*.

I SHALL

I SHALL conclude this Lecture with a few Words concerning WHEEL-CARRIAGES,

2. After a like Manner we determine the Angle of Position of the Gates A E, B E, of a Lock upon a River, (*viz.* the Angle $\angle B A E = \angle A B E$) in which the said Gates shall resist the Pressure of the Water with the greatest Force possible. For if upon A B we describe the Semicircle A D B, and continue A E to D; then the Pressure of the Water against the Gate A E will increase with the Length of the Gate, and the Resistance of the Gate will decrease as the Pressure increases, and therefore it will be on this Account inversely as the Length of the Gate, or as $\frac{1}{A E}$.

3. Again, the Resistance will be diminish'd as the Length of the Gate increases, inasmuch as the Strength Pl. XIII. of the Gate will be diminish'd in that Proportion, there- Fig. 16.

fore it will be on this Account, also as $\frac{1}{A E}$, consequently the Resistance of the Gate on both these Accounts will be as $\frac{1}{A E^2}$.

4. Join B D and E C perpendicular to A B; then we have $\overline{A E^2} : \overline{A C^2} :: \overline{A B^2} : \overline{A D^2}$; and here, because $\overline{A D^2}$ decreases as $\overline{A E^2}$ increases, we shall have the Resistance express'd by $\overline{A D^2}$.

5. But this Resistance is augmented by the Oblique Pressure of the other Gate, which let be represented by B E; this Oblique Force B E is resolvable into two Forces B D and D E; which latter, as it is parallel to the Gate A E, avails nothing, but the other B D being at Right Angles thereto, is wholly spent in resisting it; therefore the whole Resistance the Water meets with from the Gate A E, is as $\overline{A D^2} \times B D$.

6. This Expression, therefore, is to be determined to a *Maximum*; in order to which, let $A B = a$, $B D = x$, and then $\overline{A D^2} = a a - x x$, and so $\overline{A D^2} \times B D$

Q 3

the

the whole Doctrine whereof (as it stands on a *Mathematical Theory*) may be re-

$= aax - x^3$, whose Fluxion $aa\dot{x} - 3xx\dot{x} = 0$, gives $x = \sqrt{\frac{aa}{3}}$ which shews the Angle BAE =

35°, 16', as in the Examples above.

Pl. XIII. 7. Since we are upon the Subject of *Maximums*, I shall here add Examples of two or three other Cafes of the same Kind, which 'tis hoped will be acceptable to the Curious, and yet not besides the Purpose of Mechanical Gentlemen. Let BB be a Piece of Wood placed horizontally, and supported by the Pieces AB, AB, which make a given Angle ABC with the former; it is required to find the Positions of two other Pieces AC, AC, given in Length, such that they shall support the Piece BB with the greatest Force possible.

8. The Pieces AC, are fix'd in A and C so as not to slip, they are supposed to have no considerable Weight. Then make $BH = \frac{1}{2}AC$, and from the Points A and H draw the Lines AG, HK, at right Angles to BB. If AC expresses the absolute Strength of the Piece AC, then AG will express the Strength with which it supports the Piece BB, as being perpendicular thereto. Now AG multiplied by the Lever (or Distance) BC from the Centre of Motion B, (which expresses the *Momentum* or Force of the Piece AC) ought to be a *Maximum*.

9. To this End, put $AC = a, GA = x$; also $HK = n$, and $KB = m$; then $GC = \sqrt{aa - xx}$, and because of the similar Triangles HKB, AGB, we have $HK : KB :: AG : GB = \frac{m}{n}x$, and so $BC = \sqrt{aa - xx} - \frac{m}{n}x$, and

$AG \times BC = x\sqrt{aa - xx} - \frac{m}{n}xx$, whose Fluxion made

equal to nothing is $\dot{x}\sqrt{aa - xx} - \frac{xxx}{\sqrt{aa - xx}} - \frac{2m}{n}x\dot{x} = 0$.

duced

duced to the following Particulars, viz.

(1.) Wheel-Carriages meet with less Re-

10. Therefore $\sqrt{aa - xx} - \frac{xx}{\sqrt{aa - xx}} - \frac{2m}{n} x = 0$:

Whence $aa - 2xx - \frac{2m}{n} x \sqrt{aa - xx} = 0$.

And squaring each Part $\left\{ \begin{aligned} a^4 - 4a^2 x^2 + 4x^4 - \frac{4m^2}{n^2} a^2 x^2 + 4 \frac{m^2}{n^2} x^4 \\ = 0. \end{aligned} \right.$

And multiplying by n^2 $\left\{ \begin{aligned} n^2 a^4 - 4n^2 a^2 x^2 - 4m^2 a^2 x^2 + 4n^2 x^4 + \\ 4m^2 x^4 = 0. \end{aligned} \right.$

But it is $4n^2 + 4m^2 = aa$, because $\overline{BH}^2 = \overline{HK}^2 + \overline{KB}^2$,

Therefore $n^2 a^4 - 4n^2 a^2 x^2 - 4m^2 a^2 x^2 + a^2 x^4 = 0$.

And dividing by a^2 $\left\{ \begin{aligned} n^2 a^2 - 4n^2 x^2 - 4m^2 x^2 + x^4 = 0. \end{aligned} \right.$

Whence again $n^2 a^2 - a^2 x^2 + x^4 = 0$, (because $-4n^2 - 4m^2 = -aa$)

Hence by Transposition $\left\{ \begin{aligned} x^4 - a^2 x^2 = -a^2 n^2. \end{aligned} \right.$

And completing the Square $\left\{ \begin{aligned} x^4 - a^2 x^2 + \frac{1}{4} aa^2 = \frac{1}{4} aa^2 - a^2 n^2. \end{aligned} \right.$

But it is $\frac{1}{4} aa^2 - a^2 n^2 = aam^2 + a^2 n^2 - a^2 n^2 = a^2 m^2$.

Therefore $x^4 - a^2 x^2 + \frac{1}{4} aa^2 = a^2 m^2$.

And extracting the Square Root. $\left\{ \begin{aligned} x^2 - \frac{1}{2} aa = \pm am. \end{aligned} \right.$

Wherefore $x^2 = \frac{1}{2} aa \pm am$; and so $x = \sqrt{\frac{1}{2} aa \pm am}$

11. Hence 'tis evident if $m (=BK) = 0$, in which Case the Angle ABC is a right one, then $x = \sqrt{\frac{1}{2} aa}$, and therefore the Angle ACB will then be half a right one, or 45 Degrees.

12. Since the two Values of x , viz. $\sqrt{\frac{1}{2} aa - am}$ and $\sqrt{\frac{1}{2} aa + am}$ being squared, and the Sum of those Squares are equal to the Square of the Radius; viz. $aa - am + \frac{1}{2} aa + am = aa$, therefore the two Angles, of which x is the Sine, are Complements to each other. Thus suppose the Angle ABC obtuse, and = 120 Degrees, or the Angle ABG = 60 Degrees; then $BK = m = \frac{1}{4} a$, and so $AG = x = \sqrt{\frac{1}{2} aa - am} = \sqrt{\frac{1}{2} a a - \frac{1}{4} a a} =$

Q₄

distance

distance than any other. (2.) The larger the Wheels the easier is the Draught of the

$\frac{1}{2}a$, therefore the Angle ACB is = 30 Degrees. But if the Piece AC be placed on the other Side AB, then $x = \sqrt{\frac{1}{2}aa + am} = \sqrt{\frac{3}{4}aa}$, and so the Angle A C B would in that Case be = 60 Degrees, and consequently equal to the Angle A B G.

13. If the Angle ABC, instead of being obtuse, were acute, and the Complement to this, we should

have $AG \times BC = x \sqrt{aa - xx} + \frac{m}{n} xx$ a Maximum, which would in Fluxions, give the same Value for x as before, viz. $\sqrt{\frac{1}{2}aa - am}$, which if substituted for x in the above Expression, will give $\frac{2ann + aam - 2amm}{2n}$, where 'tis plain if $x =$

$\sqrt{\frac{1}{2}aa - am}$, the greatest Force will be $\frac{2an^2 + a^2m - 2am^2}{2n}$;

but if $x = \sqrt{\frac{1}{2}aa + am}$, then the greatest Force will be $\frac{2an^2 + a^2m + 2am^2}{2n}$: But $x = \sqrt{\frac{1}{2}aa - am}$

is the Sine of 30°, and $x = \sqrt{\frac{1}{2}aa + am}$ is the Sine of 60° : therefore the Piece BB is supported with the greatest Force by the Piece AC when placed on that Side B on which the acute Angle is.

Pl. XIII. 14. If AG be a Piece of Wood of an indefinite Length, and fixed in A, so as to make a given Angle GAD with the horizontal Line AD; let it be required to find the Position of another Piece DE, given in Length, such that it shall support the Piece AG with the greatest possible Force. To this End make AC = $\frac{1}{2}a$, and from B let fall the Perpendicular CB; then since the Angle A is given, the Ratio of CB to AB is given also, which let be as n to m ; that is, let CB = n , and AB = m ; from the Point D draw the Perpendicular DF (= x) to the Piece AG. Then if DE (= a) expresses the whole Force of the Piece DE press-

Carriage.

Carriage. (3.) A Carriage upon four large Wheels, of equal Size, is drawn

ing perpendicularly, D F will exprefs that with which it fupports A G; therefore D F multiplied by the Dif- tance from the Fulcrum A, or Lever A E, ought to be a *Maximum*.

15. Now from the fimilar Triangles A C B, A D F, we have C B : B A :: D F : F A = $\frac{m}{n} x$, alfo F E =

$\sqrt{a a - x x}$, therefore A E = $\sqrt{a a - x x} + \frac{m}{n} x$,

which multiplied by D F = x , is A E $\times$ D F = x

$\sqrt{a a - x x} + \frac{m}{n} x x$, whose Fluxion made equal to

nothing will give $x = \sqrt{\frac{1}{2} a a + a m}$, as before. If the Angle A D E be acute, the Point E will fall be-

tween F and A, and we fhall have A E = $\frac{m}{n} x -$

$\sqrt{a a - x x}$ and A E $\times$ D F = $\frac{m}{n} x x - x \sqrt{a a - x x}$,

which fluxed will give the fame Value of x as before found.

16. In each Cafe, 'tis plain $x = \sqrt{\frac{1}{2} a a + a m} =$ Sine of 60° , muft be the *Maximum*, which if fubftituted

in the Expreflion $x \sqrt{a a - x x} + \frac{m}{n} x x$, (where the

Angle A D E is oblique) will give $\frac{2 a n n + a a m + 2 a a m}{2 n}$

= Force of D E which fupports the Piece A G. But

if $\sqrt{\frac{1}{2} a a + a m}$ be put for x in the other Expreflion $\frac{m}{n}$

$x x - \sqrt{a a - x x}$, we fhall have for the faid Force (when the Angle is acute) $\frac{a a m + 2 a m m - 2 a a n}{2 n}$,

which is lefs than the other Forces; and therefore the latter Pofition is lefs advantageous than the former.

with

with less Force than with two of those Wheels, and two of a lesser Size. (4.) If the Load be laid on the Axle of the larger Wheels, it will be drawn with less

17. I can't conclude this Speculation of the *Maxima* and *Minima* of Quantities, without observing to the Reader, that though the Method of discovering them by Fluxions is a Part of Knowledge which the *Mathematicians* have but lately acquired, and which they esteem the Sublimity of human Science, yet this very Thing was imparted to the *Insect Tribe* at the first Creation of Things; for by this very Method it is that Bees construct the Cells of their Combs in which they deposit their Honey.

Plate XI.
Fig. 5.

18. Each Cell consists of six plain Sides, which are all *Trapeziums*, but equal to each other. The Bottom of the Cell is contrived with three Rhombus's HKDI, DEFI, and FIHG, so disposed as to constitute a Solid Angle at I, under the three equal Angles DIH, DIF, and HIF; each of which is double the *Maximum* Angle of $54^{\circ} - 44' = DIK = DKI$. Hence it comes to pass, that a less Quantity of Surface is sufficient to contain a given Quantity of Honey, than if the Bottom had been flat, in the Proportion of 4658 to 5550, as I have found by Calculation; that is, $\frac{8.12}{4658}$, or $\frac{1}{5}$ Part of the whole, so far as the Figure of the Ends of the Cells extends in each, which fifth Part of Wax and Labour saved amounts to a vast deal in the whole Structure of the Comb. And if those Creatures knew their Advantage, they could not more nicely keep up to the Rules of this sublime Geometry.

Fig. 6.

19. The last Thing among the *Maxima*, that I shall mention, is, that if a Chain ABC be suspended by its two Ends, it will sink down in such a Manner, by its Gravity, as to form the Curve ABC, call'd the *Catenaria*, which if inverted, would exhibit the best Form for an Arch of any other whatsoever. For the Demonstration of this, we must refer the Reader to the Inventor Dr. GREGORY, in *Philos. Trans.* N^o 231.

Force

Force than if laid on the Axis of the lesser Wheels; contrary to the common Notion of *loading Carriages before*. (5.) The Carriage goes with much less Force on *Friction-Wheels*, than in the common way; all which will be confirmed by Experiments (XLVIII.) (XLIX.)

(XLVIII.) The THEORY of Wheel-Carriages is as follows: Let A P G E M be a Wheel, N D the horizontal Plane on which it moves, E F the Height of an Obstacle over which it is to be drawn; the Wheel arriving at the Obstacle, and touching the Top E, stands upon the Point G, and presses it with its whole Weight. Draw O E K, a Tangent to the Wheel in the Point E, and meeting the vertical Diameter A G produced, in O. Draw the Radius E C, and E H perpendicular to A G; and M C, *m r*, perpendicular to C E, and consequently parallel to the Tangent O K. Lastly, draw the Radius C *m*.

2. Since the Wheel gravitates in the Direction C O, let C O express its Weight pressing the Point G; this may be resolved into two others C E and O E; of which C E presses the Top of the Obstacle, and is wholly sustain'd by it; the other Weight O E draws the Wheel down in a Direction parallel to the Tangent O K. Now let $W = C O$ Weight of the Wheel, $R =$ Radius, $H = E F$, the Height of the Obstacle, and $x = O E$; then since $O E : C O :: H E : C E$, we have

$$x : W :: H E : R, \text{ whence } x = \frac{W \times H E}{R}; \text{ but, from}$$

$$\text{the Nature of the Circle, } H E = \sqrt{A H \times H G} = \sqrt{A H \times E F} = \sqrt{2 R H - H^2}; \text{ therefore } x = \frac{W \times \sqrt{2 R H - H^2}}{R}.$$

3. A Force just equal to this Weight x , and acting in Opposition to it, that is, drawing the Wheel upwards in the Direction C M parallel to E K, will just be able

able to make the Wheel rest on the Top of the Obstacle at E, without suffering any Part of its Weight to rest on the horizontal Plane at G.

4. Now this Force must be increased if it acts in any other Direction but that of CM ; for let it draw the Wheel in the Direction Cm , between M and E , then the Force may be resolved into two others Cr , and rm , of which Cr draws the Wheel directly against the Top of the Obstacle E , and so is destroy'd by equal Re-action of the Point E ; what therefore remains to draw it upwards in a Direction parallel to OK , is mr , which is less than Cm or CM ; and to be made equal thereto, (as it must be to support the Wheel on the Top of the Obstacle E) it must be increased in the Ratio of Cm to rm , which let be as R to S (or as *Radius to the Sine* of the Angle which the Direction of the Force makes with CE). But 'tis plain, the Force rm cannot be increased, but the whole Force CM must be increased in the same Proportion; that is, when rm becomes $\frac{R}{S} \times rm$, CM will become $\frac{R}{S} \times CM = \frac{R}{S} \times$

$$\frac{W \times \sqrt{2RH - H^2}}{R} = \frac{W \times \sqrt{2RH - H^2}}{S}.$$

5. In order that the Wheels may be drawn over the Obstacle FE , it is necessary the Direction of the Force should lie between CE and CA ; for if it were in the Direction CE , it could only draw the Wheel upon or against, but not over the Obstacle; and if it acted in the Direction CA , it would not make it press against the Obstacle, and consequently, could never draw it over.

$$6. \text{ Let } F = \frac{W \times \sqrt{2RH - H^2}}{S} = \text{the Force suf-}$$

ficient to sustain the Wheel on the Top E of the Obstacle, 'tis evident if W , R , H , continue the same, $F = \frac{r}{S}$; that is, the Force will always be less as the Sine of the Angle ECm is greater, till $rm = CM$, when the said Force will be least of all.

7. If

7. If W and H be given, or always the same; then
 $F = \frac{\sqrt{2R - 1}}{R}$ (for here we suppose the Force applied to draw in the most advantageous Direction, *viz.* CM , where S becomes equal to R .) If therefore the Radii of four Wheels be 1, 2, 3, 4, then will $\frac{\sqrt{2R - 1}}{R}$

be 1, $\frac{\sqrt{3}}{2}$, $\frac{\sqrt{5}}{3}$, $\frac{\sqrt{7}}{4}$, or as the Numbers 1000, 866, 745, 661. From hence 'tis evident how much less Force is necessary to draw a large Wheel over any Obstacle than a lesser one, when the Weight of the Wheels are the same.

7. If the Height of the Obstacle H be indefinitely small and given, in which Case the Tangent OK will coincide with the horizontal Line ND , and the Point E with the Point G , very nearly; and the Direction of the Force be parallel to ND ; then because H^2 is inconsiderable we reject it, and the Expression for the Force

will be $F = \frac{W \times \sqrt{2R}}{R}$, (for H is given, and therefore not express'd). And if W be also given, the

Force will be $F = \frac{\sqrt{2R}}{R}$, or $F = \frac{\sqrt{R}}{R}$, because 2 is

a given Quantity; but $\frac{\sqrt{R}}{R} = \frac{1}{\sqrt{R}}$; therefore $F =$

$\frac{1}{\sqrt{R}}$; that is, in case of rough uneven Surfaces the

Force to draw the Wheel will be inversely as the Square Root of the Radius or Diameter of the Wheel. Thus if three Wheels are in Diameter as 1, 4, 9; the Force to draw them will be as 3, 2, 1.

8. If $H = 0$, that is, if the horizontal Plane on which the Wheel moves be perfectly smooth or plain, then the Quantity $\frac{W}{S} \sqrt{2RH - H^2} = 0$; whence it appears that no Force is required to move an heavy Body on an horizontal Plane which is perfectly even.

9. If

9. If the Height H of the Obstacle be proportional to the Radius of the Wheel, that is, if H be as R , and the Force draw in a Direction parallel to OK ; then because $\frac{\sqrt{2RH-H^2}}{R} = \frac{\sqrt{2RR-RR}}{R} = \frac{\sqrt{R^2}}{R} = 1$, therefore $F = W$, or the Force will be proportional to the Weight of the Wheel.

10. If the Direction of the Force be parallel to the horizontal Plane, that is, if Cm be parallel to ND , then because the Angle mCE is (in that Case) equal to the Angle CEH , their Sines will be equal, that is, $\sin m = CH = R - H$; therefore the Expression of the Force (*Art. 6.*) will become $F = \frac{W \times \sqrt{2RH-H^2}}{R-H}$; and if the Height H be given it will be $F = \frac{W \times \sqrt{2R-1}}{R-1}$.

11. From the Expression $F = \frac{W \times \sqrt{2RH-H^2}}{S}$,

we have this Equation $\frac{F}{W} = \frac{\sqrt{2RH-H^2}}{S}$, which

gives the following Analogy $F : W :: \sqrt{2RH-H^2} : S$. That is, *The Force is to the Weight as the Sine of the Angle ECH (viz. EH) is to the Sine of the Angle mCE , which the Line of Direction makes with the Line EC .*

12. If the Obstacle is capable of being depress'd or born down by the Wheel; the larger the Wheel the greater will be the Force to do this; for since CE represents the whole Force with which the Wheel bears upon the Obstacle, and this is resolvable into the two Parts CH and HE , of which the former CH being parallel to EF tends to press it down, it will be express'd by $R - H$, and since H is given, the depressing Force will be as $R - 1$, and therefore will increase with R , or the Radius of the Wheel.

13. If the Obstacle be such, as that it can neither be surmounted nor depress'd, but must be driven forward, the Force to do that will be express'd by $HE = \sqrt{2RH}$.

$\sqrt{2RH - H^2}$, which, since H is given, will be as $\sqrt{R - 1}$; but $\sqrt{R - 1}$ will be greater in Proportion to R when R is small than when it is greater. Thus if $R = 2$, then $\sqrt{R - 1} = 1 = \frac{1}{2}R$; but if $R = 5$, then $\sqrt{R - 1} = 2$, which is less than $\frac{1}{2}R$; and if $R = 10$, then $\sqrt{R - 1} = 3$, which is less than $\frac{1}{3}R$; so that in this Respect small Wheels have the Advantage of large ones. But this Case seldom happens.

14. The principal Advantage of small Wheels is, that in them the Line of Traction is not parallel to the Horizon as CK , but inclined thereto in a certain Angle, as CM , making with the Horizon the Angle MCK ; now if CM be parallel to the Tangent OK , the whole Force will be employed to draw the Wheel over the Obstacle EF ; whereas, if the Line of Traction were parallel to the Horizon, the Line CK might then express the Force, which being resolved into the two Forces CE and KE , shews that the Part CE draws the Wheel directly upon the Obstacle, and is therefore lost by its Re-action; and only the Part KE remains to draw the Wheel over the said Obstacle; and consequently the horizontal Direction is not the best, unless upon a smooth and even Plane, where no Obstacles and Ascents are to be surmounted.

Pl. XIII.
Fig. 12.

15. From what we have said, it is evident that a small Wheel, whose Radius is KE , and the Line of Traction parallel to OK , is equivalent to a large Wheel whose Radius is CK , and the Line of Traction parallel to the Horizon ND ; but $EK : CK :: HB : CB :: CI : CE$; that is, the Radius of the smaller Wheel is to that of the larger, as the Co-sine of the Angle ECG to Radius.

16. Though the Force employ'd be never wholly spent in drawing, but when the Direction is CN , parallel to the Plane on which the Carriage moves; yet if it be applied in that oblique Direction CM , where the Breast of the Horse is higher than the Axle of the Wheel C , in which Case only the Part BM is employ'd in drawing, the other Part CB is not however wholly lost, but is acting contrary to the Gravity of the Carriage, and by that Means lessens somewhat of the

Fig. 13.

the Weight of the Load, by lifting it (as it were) along, for in this Case the Horse not only *draws*, but also *carries* along (in some measure) the Load.

17. On the contrary, if the Axle of the Wheel be higher from the Plane than the Breast of the Horse, that is, if the Power be applied in the oblique Direction CO, then the Part DO draws the Load along, but the Part CD acting perpendicular on G draws the Load directly against the Plane, and thereby increases the Weight of the Load, or the Difficulty of drawing it; and is therefore the worst Direction in which the Force can be any how applied in drawing. Hence it follows, that (*cæteris paribus*) where the Wheels of a Carriage have their Radius equal to the Height of the Horse's Breast, or Traces, the Draught will be easiest of all; and Wheels, whose Radius's are less than that, are better than those Wheels whose Radius's exceed it.

Pl. XI. I. 18. A small Wheel BDC will descend farther down between two Obstacles DF and CE than a larger Wheel ADC, as is evident from the Figure; and therefore the Draught is more difficult, and subject to greater Shocks or Jolts, with the small Wheel, inasmuch as its Axis, and consequently the Weight of the Load, must be raised to a greater Height in order to get from between them.

Fig. 15. 19. Also in soft or yielding Ground, a small Wheel will sink deeper than a larger Wheel charged with the same Weight. Thus suppose ABC be the Plane of the Road, which is so soft as to permit the small Wheel to sink down to E, then the Weight must overcome the Resistance of as much Earth as the Wheel in sinking has displaced, that is, as much as is equal to the Segment HED; if now the larger Wheel were to sink to the same Depth, it must overcome the Resistance of so much Earth as is equal to the Segment AEC, which is greater than HED, which is impossible, because the same Weight can overcome but an equal Resistance in either Wheel, therefore the larger Wheel will not sink so deep as the smaller, and so will be drawn more easily.

20. Since the Ends of the Axles, and the Holes in the Naves of large and small Wheels are equal, and since

since, in passing along, the small Wheel (to measure the same Length of Road) must turn round upon its Axis oftener than a large one; it follows, that there will be a greater Quantity of Friction in the small Wheel than in the larger, and that in the same Proportion as it is less, or as its Velocity is greater. Hence on Account of this, and several other like Causes, small Wheels are much more subject to be out of Repair, to be at Fault, and to be worn quite out, than larger ones.

21. Next to the Conveniency mention'd *Art. 14*: that of turning the Carriage in a smaller Compass, with small Wheels, than can be done by large ones, has made them more necessary in Waggon and Coaches, for because of their Smallness they can be brought near to, and partly under the Sides of the Carriage, and so their Axles lying more obliquely under the Bed of the Carriage, admit it to be turned about with greater Ease.

(XLIX.) 1. To conclude these Mechanical Lectures, I shall give the Reader a short View of the famous Controversy that has so long (about 60 Years) subsisted between the *English* and *French* Philosophers on one Side, and the *Dutch*, *Germans*, and *Italians* on the other. The Subject was, *Whether the Force of Bodies in Motion, striking each other, be proportional to the Simple Velocity of the Motion, or to the Square of the Velocity?* The *English* and *French* maintain the former; the other Gentlemen the latter.

2. This Dispute first commenced between Mr. *Huygens* and the Abbot *Catalan*, about the Force of Oscillating Bodies; it continued some Time between these two Gentlemen; at last, another Subject of the same Kind engaged the said Abbot with the famous *Leibnitz*, who is to be esteemed the first Author, that plainly declared, in express Words, *That the Forces of Bodies were as their Masses multiplied by the Square of the Velocity.* *Catalan*, and afterwards Mr. *Papin*, answered *Leibnitz*; he reply'd again, and several Papers were written on the Subject.

3. It then became a Matter of general Enquiry, and the Philosophers of every Nation began to consider which Side to be of, and whether they should declare

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for the *old* or for the *new Opinion*. However, they did here, as they do in Religion, go by a whole Nation together; the common Herd of Philosophers following the Dictates of their Leaders. Thus *Leibnitz*, *Polemus*, *S'Gravesande*, and *Muschenbroek* lead the *German* and *Dutch*; *Papin*, *Mairan*, &c. the *French*; and *Pemberton*, *Eames*, *Desaguliers*, *Clarke*, &c. the *English*.

4. In this Controversy also, as in those of Religion, the Opponents disputed with very great Warmth, wonder'd at each other's Slowness of Apprehension, or Backwardness of Belief; and I wish I could say, that they had always observed such an impartial, free, and generous Behaviour and Stile of Expression, as the Dignity of Philosophy demands. We should then perhaps have found Dr. *Desaguliers* better employed than in blaming Dr. *Samuel Clarke* for *uncivil Treatment*, for *rude and impertinent Expressions*, &c. in regard to those who defend the new Opinion of the *Square of the Velocity*.

5. As it usually falls out in other Cases, so here when Men find themselves press'd with Difficulties and Absurdities in their Schemes and Notions, they have recourse to the Subtilties of Metaphysical Distinctions, though seldom to any good Purpose. Thus when it appear'd too plainly by all Experiments, and even to common Sense, that the natural Force of Bodies was proportional to the simple Velocity and Mass of Matter conjointly; we were told 'twas necessary to distinguish the Force of Bodies into two Kinds, *viz* the *Vis Viva* or *Living Force*; and the *Vis Mortua*, or the *Dead Force*.

6. By this *Vis Viva*, or *Living Force*, we were to apprehend that which resulted from the visible Action of one Body upon another, as that of a falling Weight; but the *Vis Mortua* or *Dead Force* was to be understood of that which was destroy'd by a contrary Agent, as a Weight in one Scale of a Balance is kept from descending by a Counter-Poise in the other Scale. But unluckily for the Author of this Refinement, both those Forces appear, even by the Balance, to be in the Ratio of the simple Velocity into the Mass.

7. Thus

7. Thus for the Living Force, if on the proportional Balance you place 4 *lb.* at the Distance of 6 Inches on one Side, and 2 *lb.* at the Distance of 12 Inches on the other Side, and if the Balance be put into Motion, they will each of them have a *Vis Viva*, or an active Force, because they will keep the Beam in Motion for some Time, till, by militating, they murder or kill each other; and then surely enough they become *Vires Mortuæ*, or *Dead Forces*.

8. But let us see how they exerted their Power while living. One (A) acted against the other (B), with the Gravity of every Particle in a Mass of 4 *lb.* and with the Velocity in each Stroke that 6 Inches Distance could give; but since all allow the Weight to be as the Mass of Matter, and the Velocity as the Distance from the Centre of Motion, therefore all the Force which A exerted was as $4 \times 6 = 24$. In the same Manner it is shewn the whole Force of B acting against A, was as $2 \times 12 = 24$; that is, in each Case, the Force was as the Velocity into the Mass of Matter. And because they were equal they destroy'd each other.

9. But had those Combatants A and B been armed with Forces proportioned to the Squares of the Velocities, that of A would have been but $4 \times 6 \times 6 = 144$; whilst B would have had a Force equal to $2 \times 12 \times 12 = 288$; by which he would have demolished A at one single Stroke, and been the surviving Conqueror.

10. Let us now consider those two Bodies A and B as dead, and see what Forces they exert (*Verbo detur Venia*) in that State. In order to this they must be hung upon the Arms of the Balance, one on each Side, till they are dead, *i. e.* motionless; but this will not happen till their Distances from the Centre of Motion become reciprocally proportional to their Masses of Matter; and then 'tis plain the Case is the same as before; for the dead Force of A will be as $4 \times 6 = 24 = 2 \times 12 =$ dead Force of B. Whereas if these Forces were as the Masses into the Square of the Velocities, A of 4 *lb.* and B of 2 *lb.* ought to die at the Distances 6 and 8,4 Inches from the Centre respectively; but if the Experiment be tried there, such strong Symptoms of Life

will be found in A, as manifest the Falsity of this Hypothesis.

11. Being driven from this Subterfuge, they seek another in a critical Distinction between *Force* and *Pressure*. *Pressure*, say they, is the Power with which Bodies act by Means of Instruments; thus a Weight in one Scale acts against the Weight in another by *Pressure*; but the Power by which Bodies act on each other alone, is properly call'd *Force*, as when one Stone strikes another by falling on it, or a Hammer strikes an Anvil. The former they allow is proportional to the Velocity and Mass of Matter conjointly; but the latter, they say, is as the Mass multiplied by the Square of the Velocity.

12. But this *Eclaircissement* boots them as little as the former, if they mean a momentary Impact or Stroke, or such whose Effect is produced in a Moment of Time. For in such a Case we say, the Stroke is in Proportion to the Mass of Matter, and also to the Degree of Velocity, and therefore as both conjointly; but we deny there is any other Source of Power from whence a Body can derive any Force for producing a momentary Effect: Nor have any of those Gentlemen been able to shew there is, though some (bewilder'd in the Labyrinth) have attempted it, but in how weak and ridiculous a Manner may be seen in a Piece intituled *De Conservatione Virium vivarum*, &c. And 'tis worth the Reader's while to see the jocular Confutation of this ludicrous Piece by *Phileleutheris Lond'nenfis*.

13. If they say, they would not be understood of momentaneous Effects, but such as are produced in Time, then they have no Antagonists. And 'tis plain from their Experiments, that this is their Meaning after all. For the Experiments which they so much insist on, are such as are made by letting Bodies fall on soft Substances, as Clay, Butter, &c. or by the Action of Bodies on yielding Springs, or springy Bodies; in all which Cases, the Effect is not produced instantaneously, but in Time; the Clay takes Time to recede, the Spring to bend, &c.

14. If the Time be taken into Consideration, then every Body must know that the Effect will be proportional

tional to the Intensity of the Cause, and the Time of its Continuance: Thus, for Example, the Effect of Gravity on Bodies left to themselves causes them to descend; their Descent therefore will be proportionate to the Power of Gravity upon them, and therefore greater in those which fall freely, than in others which descend on Inclined Planes in a given Time, where Part of the Force of Gravity is destroyed by the Reaction of the Plane. But since the Action of Gravity is not instantaneous, but continual, therefore its Effect, *i.e.* the Descent of the Body, will be greater in Proportion to the Time of its continuing to act on the Body, separately consider'd from its intrinsic Force. But this Force is as the Velocity produced by it in a given Time; also the Time is as the Velocity, when the Force is given; therefore the Effect, *viz.* the Descent of the Body, is as the Square of the Velocity.

15. Or in Symbols thus; let E = the Effect produced by any Power $= P$, acting in any Space of Time $= T$, upon any Body $= Q$, moving with any Degree of Velocity V . Then 'tis plain, E will be as $Q = \text{Mass of Matter in the Body}$, When P , T , and V , are given; also the Effect E will be as the Intensity of the Power P , when T , Q , and V are given; again, when P , Q , T , are given, E must be as V ; and lastly, we shall have E as T , when P , Q , V , are given. Therefore when neither of these are given $E : Q \times P \times V \times T$; and in case of any different Effect, let $e : q \times p \times v \times t$.

16. Then supposing the Bodies equal, *viz.* $Q = q$, we have $E : e :: P V T : p v t$. Thus the same or two equal Bodies, descending on two Planes unequally inclined, will descend thro' Spaces, which will be as the different Powers of Gravity P and p , as the different Velocities V and v in any Point of Time, and as the different Times T and t of their Descent, conjointly.

17. If not only the Bodies Q and q , but also the Powers P and p which actuate them, be supposed equal, or $P = p$; then $E : e : T V : t v$. Thus in equal Bodies falling freely by Gravity, the Effect, or the Space which they describe, will be as the Time T and t of the Falls, the Velocities V and v at any Point of Time

in the Fall. And because in this Case the Velocity is always proportional to the Time, that is, $T : t :: V : v$; if we multiply each Ratio by the same Ratio $V : v$, the Analogy will still be the same, $v/z \quad TV : tv :: V^2 : v^2$; whence $E : e :: V^2 : v^2$; or the Effect of falling Bodies in describing the Space, will be as the Square of the Velocities.

18. If we suppose the Times given, that is, $T = t$, then (the Rest as before) we have $E : e :: V : v$; that is, the Forces or Effects of equal Bodies, falling in equal Times, are proportional to the *simple Velocities*.

19. If $V = v$, or the Velocities given; then $E : e :: T : t$. Thus if two equal Bodies lying on an horizontal Plane, receive a Stroke each from Springs of equal Force, then will the Velocities be equal in every Part of the Motion, and the Effects E and e , in this Case, being the Spaces described, will be as the Times of their Motion.

20. Hence we have seen every possible Case wherein the Forces of Bodies or their Effects can be supposed to vary; and 'tis plain there is none where the Force is as the Square of the Velocity but where the Time is concerned; or where some one of the Factors Q , P , V , T , is proportional to the Velocity; thus in Spouting Water, because Q is always as V , therefore, tho' T be given, E will be as QV , or as V^2 ; for P also in this Case is given, the Power of Gravity which gives Motion to the Particles of the Fluid being always constant.

21. Nor have they any Experiment which shews the Force or Effects of Bodies proportional to the Squares of the Velocities, but where the Time in which the Effect is produced ought to be consider'd. Thus in their famous Experiment of Cavities, formed in soft Clay by falling Bodies, 'tis true, those Cavities are the Effect of the falling Bodies, and proportional to the Squares of the Velocity. But what is this to the Purpose, unless they will say, those Pits are instantaneously produced, which I believe none will pretend to do?

22. 'Tis evident, this Cavity must be proportional to the Quantity of Matter put in Motion in the Clay,
by

by the falling Body; but this is proportional to the Velocity on two Accounts; the first is, that every Particle which comes in Contact with the Striking Body, receives a Stroke proportional to the Velocity, therefore also the Number of Particles which each of these can move, will be as the Force of the Stroke, or as the Velocity of the falling Body. Secondly, among yielding Particles, the Number which the falling Body can apply to in a given Time, will be as the Velocity; consequently the whole Number of Particles which can be moved, both immediately by the Body itself, and by the Motion communicated to the Particles, will be as the square of the Velocity; and therefore the Cavity will be in the same Ratio also.

23. We hence observe, the same Method of Reasoning may be applied to soft and yielding Substances, as has always been used in the Case of unelastic Fluids; and for this Reason only, that in every Case of yielding Particles, the Effect of a Body in striking them is not destroy'd momentarily, but in Time, which Time (*cæteris paribus*) will be as the Velocity; and therefore E being as TV , and T , in all these Cases, being as V , it will be universally for all soft and yielding Substances $E : V^2$.

24. Hence we may observe that as Bodies are more or less yielding or soft, so the Effect will be more or less approaching to the Ratio of the Square of the Velocity. This Consideration is of great Use in mechanical Affairs. Thus a Hammer with a double Velocity will produce much more than a double Effect in driving a Nail; the same may be said of a Rammer driving a Pile: Hence the Reason why a small Hammer will by its Velocity do more Execution upon red-hot Iron, than the large Hammer by its *Momentum*. In short, all these Things are so plain, and easy to be understood, that 'tis surprizing to think how such a Dispute could subsist so long, and occasion so much to be said about it.

L E C T U R E IV.

Of HYDROSTATICS in general. Of the PARTICLES of a FLUID, the FORM, SIZE, &c. The HYDROSTATIC LAWS of FLUIDS. Of the GRAVITY and PRESSURE of FLUIDS. The HYDROSTATIC PARADOX demonstrated. The RATIONALE of the SWIMMING and SINKING of Bodies expounded. Of ABSOLUTE, RELATIVE, and SPECIFIC GRAVITIES of SOLIDS and FLUIDS. The NATURE and USE of the HYDROMETER or WATER-POISE. The NATURE, USE, and NEW STRUCTURE of the HYDROSTATIC-BALANCE. The COMMON BALANCE improved on HYDROSTATIC PRINCIPLES. A large TABLE of SPECIFIC GRAVITIES. The QUANTITY of PRESSURE how estimated. The CENTRE of PRESSURE calculated. Various HYDROSTATIC PROBLEMS in GEOMETRY, MECHANICS, PHILOSOPHY, &c. The USEFULNESS of HYDROSTATIC SCIENCE. The THEORY at large relating to the MOTION of BODIES

DIES *in resisting* MEDIUMS, as AIR, WATER, &c.

HYDROSTATICS is that Part of Philosophy which treats of the Nature, Gravity, and Pressure of Fluids. Fluid is a Substance whose Particles yield to the least partial Pressure, or Force impress'd. And hence 'tis reasonable to infer, that those Particles must be exceeding small, smooth, round, and ponderous Bodies; and observe all the Laws of Motion and mutual Action in common with those of all other Matter (L). The following are the Laws and Properties of Fluid Bodies.

(L.) 1. That the Characters of a Fluid intended in this Definition are all of them necessary, will appear from what follows. The first is, that they are *exceeding small*; for that the Smallness of the Particles conduces to *Fluidity*, is evident from hence, that the Points of Contact between Spheres are in Proportion to their Diameters, and therefore grow less with the Spheres, though not in the same Proportion. And the less the Points of Contact, the less will be the Cohesion, and of course the greater the Disposition of the Particles to *Fluidity*.

Pl. XIV.
Fig. 1.

2. To illustrate what I have said, let ABCD be a Sphere inscribed in a Cylinder EFGH; it is demonstrated by Geometers, that the Superficies of the Sphere is equal to the curved Surface of the Cylinder, and that the Superficies of any Segment of the Sphere LDEM is equal to the correspondent Surface IKGH of the Cylinder; if therefore the Line IK move on till it coin-

I. All

I. *All Fluids are incompressible, except Air; or, they cannot by any Force be compress'd in-*

cides with H G, the Superficies of the Segment of the Sphere L D M will become the Point of Contact with the Plane H G, and that of the Segment of the Cylinder I K G H will become the Periphery of the Base of the Cylinder; but this Periphery of the Base will be as the Diameter H G of the Cylinder, which is equal to the Diameter of the Sphere A C; therefore the Point of Contact of the Sphere with the Base of the Cylinder, or the Plane which it touches, will be proportional to its Diameter: whence it follows, that the Point of Contact between two Spheres will be in Proportion to their Diameters.

3. The *second Characteristic* of the Particles of a Fluid is, that they are *smooth*; by this means they become lubricous, and apt to slide or move by each other with the greatest Facility and Freedom; and therefore their Disposition to *Fluidity* is proportionably promoted and augmented; for want of this Quality, though the Particles of Matter were possess'd of all others mention'd in the Definition, yet would they never constitute a Fluid, since nothing is a greater Obstruction thereto, than Asperity or Roughness of the Superficies.

4. A *third Characteristic* is their *Roundness or Sphericity*; for the more spherical the Particles, the fewer Points they touch each other by, and the less the Points of Contact are; on both which Accounts the Attraction of Cohesion will be diminished, and their Disposition to Fluidity or Volubility will be increased.

5. The *fourth Characteristic* in the Definition of the Particles of a Fluid is, that they are *ponderous or heavy Bodies*. By this Property, Fluids do not only gravitate in common with all other Sorts of Bodies, but also they derive from hence a Power or Force of Pressure peculiar to themselves; and by that means act upon Solids in a very different Manner than that in which Solids act upon each other, as will be shewn farther on.

to a less Space than what they naturally possess; as is proved by the *Florentine Experiment* of filling a Globe of Gold with Water, which, when press'd with a great Force, causes the Water to transude or issue through the Pores of the massy Gold, in Form of Dew, all over its Surface (L.I.)

6. The surprizing Subtilty or Minuteness of the Particles of Fluids deserves our farther Speculation and Remark. So small are they, as to escape the Sight assisted by the best of Glass-s; that they freely pass through the Pores of the densest Matter, as *Gold*, &c. that they become invisible in Vapour; and so light as to rise in Air, yea, in Air that is greatly rarified and attenuated. And it is well known, that Water has its Particles so very subtile as to pass by Ways that Air will not, and perhaps is exceeded by nothing in this wondrous Property, but the Particles of Fire and ardent Spirits.

(L.I.) 1. This Experiment must be made with a Globe, for this Reason, because a Sphere contains a greater Quantity of Matter under the same Superficies, than a Body of any Figure whatsoever. This may be made appear in the following manner; let A B, B C, C D, and A D be four Lines, of which the three first are of a given Length, and the fourth variable; and let it be required to dispose them in such a manner as to comprehend the greatest Area possible. Pl. XII.
Fig. 2.

2. Let A B and B C make a given Angle A B C; then will the triangular Space A C B be given. Upon C, with the Radius C D, describe the Semicircle *a D e*, then 'tis plain the other Part of the Area, or the Triangle made by the Lines A C, C D, and A D, will be greater or lesser, according to the Position of the Line C D; for as 'tis made C D or *c d*, the variable Line will be A D or A *d*; and the Triangle form'd will be A C D or A C *d*, whose Area will be $\frac{A C}{2} \times C D$, or

II. *All*

II. *All Fluids gravitate, or weigh, in proportion to their Quantity of Matter ; and that not only in the Air, or in Vacuo, but in*

c d, (*c d* being drawn perpendicular to *A C* produced.)

3. Now, 'tis evident this Triangle will be greatest, when *c d* is a *Maximum*, that is, when it becomes *CD*; and consequently the Triangle *ACD* is the greatest possible when *CD* makes a Right Angle with *AC*; and in this Case the Point *C* is in a Semicircle described on *AD* as a Diameter. Therefore, also the other Angle *B*, being in the Semicircle described on the same Diameter *AB*, will make the other Part of the Trapezium a *Maximum*; and so the whole Trapezium *ABCD* inscribed in a Semicircle, will be greater than any other, whose three Sides *AB*, *BC*, *CD*, are the same.

4. Since what has been demonstrated of the Trapezium, is true of any other Polygonal Figures (because they may be resolved into *Trapezia*) and since the Sides of a Polygon, when infinitely small, do coincide with the Circle; therefore the Circle is the most capacious Figure, or contains the greatest Area under the same Periphery.

5. For Example; suppose a String *C* were disposed into the Form of a Circle; then as $22 : 7 :: C : \frac{7C}{22}$
 $=$ Diameter of the Circle; the Radius therefore is $\frac{7C}{44}$
 and since the Area of a Circle is equal to the Periphery multiplied into half the Radius, therefore $C \times \frac{7C}{88} =$
 $\frac{7CC}{88} =$ Area of the Circle. Again, suppose the same String disposed in the Form of a Square; the Side would be $\frac{C}{4}$, and the Area $= \frac{CC}{16}$; hence the Area of the

proprio

proprio Loco; or, a Fluid weighs the same communicating with a Quantity of that Fluid, as *in Vacuo*; which all Philosophers (till very lately) have denied (LII).

Circle would be to that of the Square as $\frac{7\text{ CC}}{88}$ to $\frac{\text{CC}}{16}$,

or as $\frac{7}{88}$ to $\frac{1}{16}$, that is, as 14 to 11.

6. Now since a Sphere may be consider'd as made up of circular Areas, it plainly follows, that a Sphere will contain a greater Quantity of Matter under the same Surface than any other Solid. Thus a Sphere will be found by Calculation to be more than twice as big as a Cube of the same Surface; and therefore the Water contain'd in the Globe must, when the Figure of the Globe was alter'd, be either compress'd into a less Space, or make its Way through the Pores of the Globe, as in the Experiment we find it will; which therefore evinces its *Incompressibility*.

7. Some Philosophers are inclined to think that Water is not absolutely incompressible, or that the Particles thereof do not touch each other; but are kept at a Distance by a Centrifugal Force superior to any Force we can apply in compressing them. But because this is an *Hypothesis* that has scarce any thing more than bare possibility to recommend it, at least nothing is offer'd to render it probable or necessary; therefore we are not to admit it among the Principles of the *Newtonian Philosophy*.

(LII.) 1. That Fluids gravitate, or are heavy, in the same Manner with Solids, is evident, because the Earth's Attraction, which is the Cause of Gravity, equally affects the Particles of all Sorts of Matter; and therefore excites the same Endeavour or Tendency towards the Centre of the Earth in the Particles of a Fluid, as in those of a solid Body; and this is what we call their *Absolute Gravity*.

2. Now since in Fluids of the same Kind, as Water, all the Particles are reasonably supposed equal and

III. From

III. *From the Gravity of Fluids arises their Pressure, which is always proportional thereto; and since we may suppose all the Particles of a Fluid to have equal Bulk and Weight, the Gravity of the Fluid, and consequently its Pressure, will be always proportional to the Altitude or Depth thereof.*

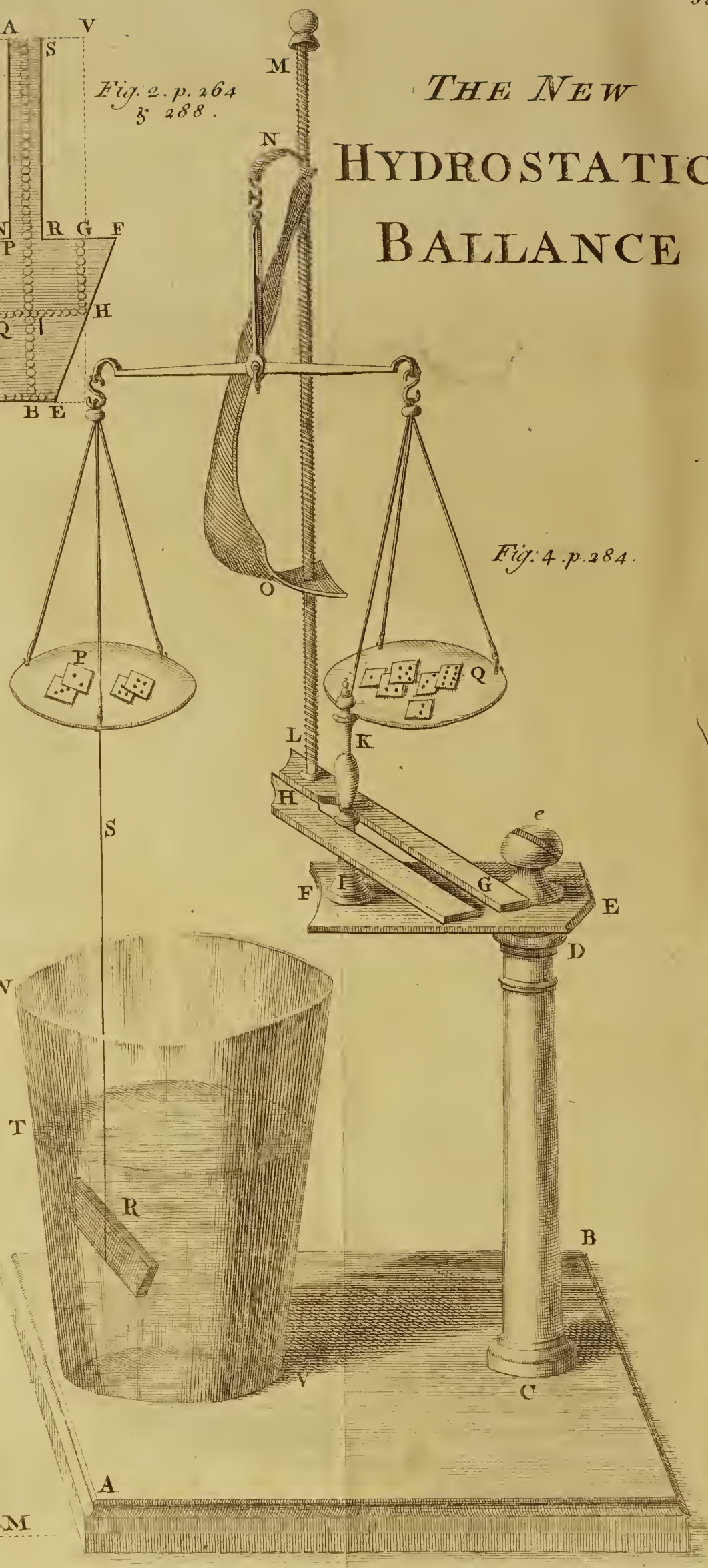
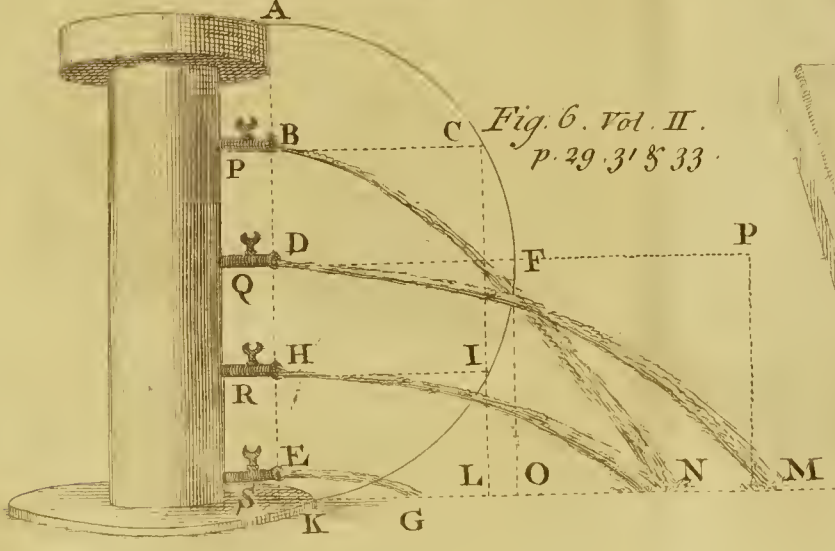
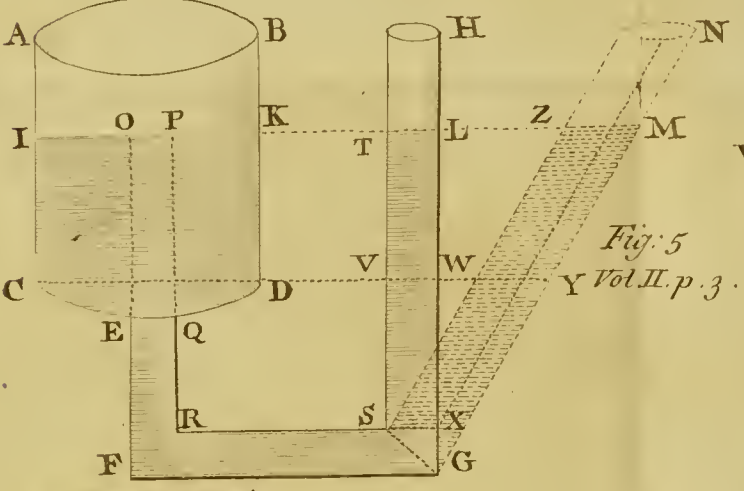
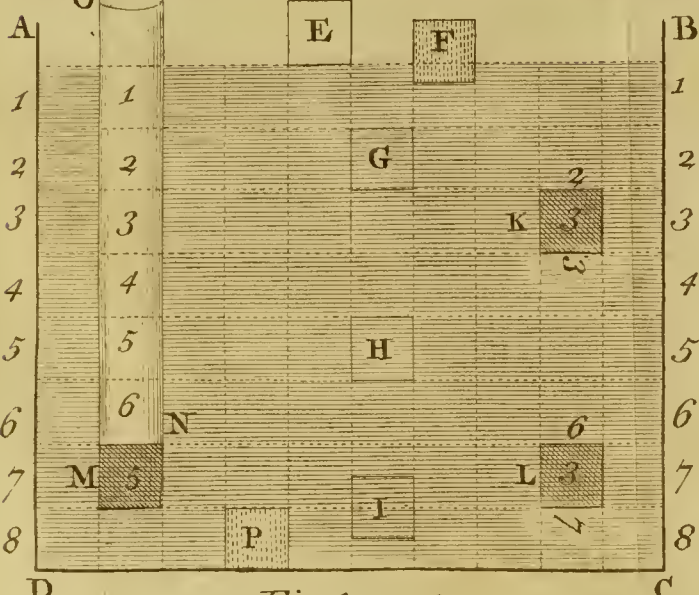
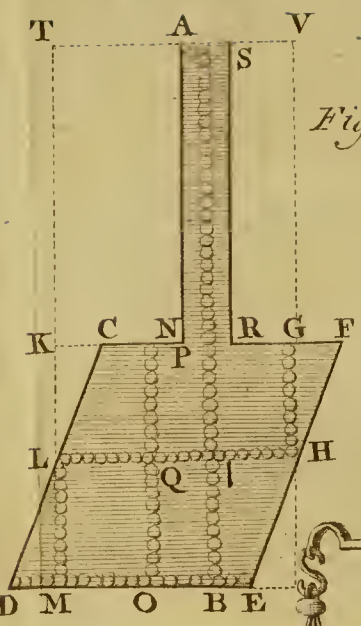
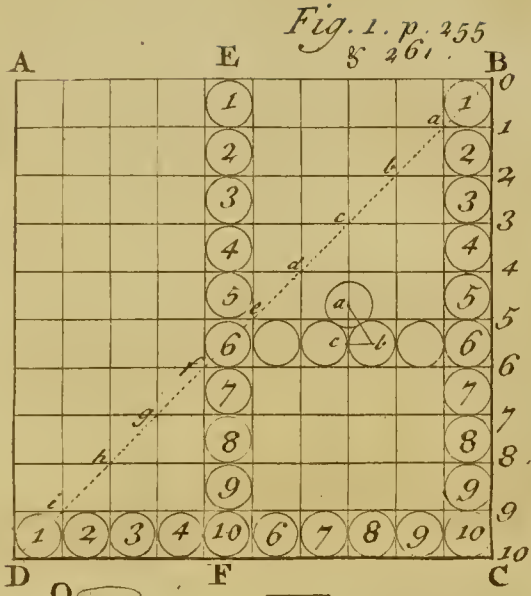
alike in all Circumstances, they will be all equally affected by Attraction, and therefore have among themselves an equal Tendency towards the Earth's Centre. Whence, since they gravitate equally, if they are equally obstructed in their Descent (as by the Bottoms of Vessels, &c.) they will all retain the same Position among themselves, as if they were affected by no Kind of Power at all; and thus they are said to be *relatively at Rest*, or in a State of *Quietus* among themselves.

3. Since no one Particle of the same Fluid has a greater Share of the attracting Power than another, no one will tend to descend before another; and therefore among the Particles of the same Fluid, there is no such Thing as we call *Relative* or *Residual Gravity*, which is nothing but *the Excess of Gravity, by which one Body tends downwards more than another*, as will be more fully explain'd hereafter (See *Annotat.* LVIII.)

4. Now because Philosophers found that a Bucket of Water in Water weigh'd nothing (that is, that there was *no Relative Gravity in Water*) they very strangely inferr'd there was no *Absolute Gravity* of any Part or Particle of Water, whilst it remain'd in Water, but only became heavy when taken out, or separated from the rest. But their Mistake is easily evinced by the following Experiment; let a Bottle or Vial with Shot in it to make it sink in Water (when close corked) be hung at the End of a nice Balance, and then immersed into a Jar of Water; while thus hanging in Water, let it be counterpoised very exactly by Weights put into the Scale at the other End. Then pulling out the Cork

Whence

RPJCB



Whence the Weight and Pressure of Fluids on the Bottoms of Vessels, &c. must be equal.

IV. *The Pressure of Fluids upwards is* pl. XV.
equal to the Pressure downwards, at any given Fig. 1.

Depth. To illustrate this, and the foregoing Proposition, let A B C D be a Vessel of Water, whose Altitude E F suppose to consist of a Column of 10 aqueous Particles: Then, 'tis evident, the first or uppermost Particle 1 can affect the next Particle 2 only by its Weight or Pressure, which therefore is as 1; and since that Particle 2 is immoveable, and Action and Re-action equal and contrary, the said Particle 2 will re-act upwards upon the Particle 1 with a Force which is as 1. In the same manner the Particle 2 acts on the Particle 3 by Pressure downwards, with 2

the Water will rush into the Bottle and destroy the *Equilibrium*, by causing the End of the Balance to descend; which will be a plain Proof *that Water has Weight in Water.*

5. But to carry this Experiment farther; let now the *Equilibrium* be again restor'd, by adding more Weights to those in the Scale: then taking the Bottle out of the Water let the Water be pour'd out, and weigh'd, and the Weight will be found exactly equal to that which was added to the Scale to restore the *Equilibrium* above-mention'd. This shews *that Water (and all Fluids) weigh the same in their own Element as out of it.*

Degrees of Force, arising from its own Weight and that of the Particle above it; and accordingly it is press'd upwards with an equal Force by the Re-action of the Particle 3: And so of all the rest. Wherefore the Propositions are manifest.

V. *The Pressure is upon all Particles of the Fluid at the same Depth equal in every Part; or, the Particles of a Fluid, at the same Depth, press each other every way, and in all Directions equally.* For if any Particle were press'd more on one Part than another, it must give way, or yield, till the Pressure became every way equal; otherwise an incessant intestine Motion of the Particles would ensue, which is absurd, and contrary to Experience (LIII.)

(LIII.) 1. There are several Ways to shew that the Pressure of Fluids is every Way equal; but the most simple and evident, I think, is by taking a very long Glass Tube hermetically seal'd at one End, and immersed in a perpendicular Position with the open End in another tall Tube or Jar of Water; the Air in the Tube being compressible will yield to the Pressure of the Water below, and admit the Water to rise in the Tube, to Heights which will be always proportional to the Altitudes of the Fluid above it; and this will be the Case when the lower Part of the immersed Tube is bent into a Right Angle, that the Water may come upon it laterally: and thence it will appear *that the Pressure of Fluids is every Way equal, and proportional to the Altitudes.*

VI. FROM

VI. FROM the mutual Pressure and equal Action of the Particles it follows, *that the Surface of a Fluid must be perfectly smooth*

2. This likewise may be shewn from mechanical Principles, for let the Sphere A be supported by two others in an horizontal Position B and C; join the Centres a, b, g ; and draw the Perpendicular af ; let ec be a Tangent to the Sphere B in the Point c , where the Sphere A touches and presses it, intersecting the Perpendicular af in e ; and let ae express the Force of A downwards; then since ae is resolvable into two Forces ac and ce , of which the latter being in the Tangent to B, does not at all affect it, and the other ac being perpendicular to the Surface; this other Force ac is that alone by which the Body A presses B.

Pl. XIV,
Fig 3.

3. But since ac is an oblique Force, that is, neither perpendicular nor lateral, let it be resolved into the two Forces ad and dc ; of which the former is perpendicular, and so does not affect the Body B in pressing it sideways; but the other dc being in an horizontal Direction is all the Force with which A presses B in a lateral Direction; but since dc is parallel to bf , we have $ac : dc :: ab : fb$; and since the Particles of a Fluid are equal, we have $fb = bc = ac$; therefore $fb = \frac{1}{2} ab$, and consequently $dc = \frac{1}{2} ac$; that is, *the Force with which A presses B laterally is just half the Force with which it presses it directly.*

4. And since by the third Law of Motion the Pressure of the Sphere D upwards is equal to that of A downwards, and so the Force upon the Bodies B and C in a lateral Direction is the same, that is, *half the direct Force*; therefore the whole Force with which the Body B or C is urged laterally by the joint Action of A and D is equal to the whole Force with which either of them act upon them singly. From what has been said, it appears, that the Force with which the Body A or D presses the Body B or C, is less than the Force of Gravity, or that by which they act perpendicularly, that is, ac is less than ae .

and even; for should any Part stand higher than the rest by any Force, as Attraction, &c. it would immediately subside to a Level with the other Part by the Force of its own Gravity, when that Force is removed.

VII. THE Figure of the Surface of all Fluids is spherical or convex; for all the Particles equally gravitating towards the Centre of the Earth, will take their Places from it at equal Distances at the Surface, and so form a Part of the Superficies of a Sphere, equal to the Bulk of the Earth (LIV).

Pl. XIV.
Fig. 4.

(LIV.)¹. Besides the Reason of the Thing, we know by Experiment, that the Surface of large Waters, as those of the Sea or Ocean, is convex; for a Person standing on the Shore, and viewing a Ship under Sail, directly before him, will lose Sight thereof by Degrees, the Hull or Body of the Ship first disappearing, then the lower Parts of the Masts, then the Tops of the lower Masts, and lastly the Top of the tallest Mast; as represented at A, B, C, where the Ship gradually descends below the horizontal Line HO, or Line which bounds the Sight, as it proceeds on the convex Surface of the Earth from A to C.

2. The Reason why we see not this Convexity in the Surface in Fluids of small Extent, as in Vessels, Ponds, &c. is because the Superficies of a Sphere so large as that of the Earth will so nearly co-incide with a Plane, for a certain Space, as not to be discernable from it; and therefore the Surface of Fluids within that Space or Extent, will appear plain, or nearly so; and what that Extent of Surface is, as the Reader may be curious to know, I will shew by the following Calculation.

VIII.

VIII. SINCE Fluids press equally every way, the Pressure of each Particle against the Side of a Vessel will be proportional to its Altitude; and consequently the Pressures of the Particles 1, 2, 3, 4, &c. of a perpendicular Column against the Side BC

3. Let AC be the Semidiameter of the Earth = 4000 Miles, or 21120000 Feet; (for 5280 make one Mile) AE a Part of the Earth's Surface, AF a Tangent thereto at the Point A. Now it is found by Experience, that nothing is distinctly visible to the Eye Pl. XIV that does not subtend an Angle of one Minute at least. Fig. 5. Let BD then be the least Distance between the Convex Surface AE, and the Tangent Line AF, that is discernable to the Eye at A; then will DAB, contain'd between the Tangent and the Chord AB, be one Minute. If now on AB we let fall the Perpendicular CG, then in the right-angled Triangle AGC, we have all the Angles given, and the Side AC, to find the Side AG. For since DAC is a Right Angle, and DAB = 1 Minute, the Angle GAC = $89^{\circ} 59'$; and therefore the Angle ACG = 1 Minute also. Therefore we say,

As Radius 90°	— — — — —	10,000000
Is to the Angle ACG 1 Minute	—	6,463725
So is the Side AC = 21120000	—	7,324694
To the Side AG =	6144	— 3,788420

4. But $2 AG = AB = 12288$ Feet, which is a little more than two Miles and a Quarter; and therefore, unless the Surface of Water be more than two Miles extent, it will not appear different from a Plane; the Distance DB being in any less Extent insensible. Hence a Ship on the Sea will depart about two Miles and a Quarter before we sensibly lose Sight of any Part, because till then she will seem to be sailing on a Plane.

will be as a Series of Numbers in Arithmetical Progression, whose first Term is 0; therefore *the Sum of all the Pressures is equal to the Number of Pressures multiplied by half the greatest Pressure*: But the Number of Pressures is as the Number of Particles, or Altitude of the Fluid BC; also the greatest Pressure is as the same Altitude: *Wherefore the total Pressure against the Side of a Vessel is as the Square of the Altitude of the Fluid* (LV).

(LV.) 1. That the perpendicular Pressure of Fluids on the Bottoms of Vessels is estimated by *the Area of the Bottom multiplied by the Altitude of the Fluid*, is evident, I suppose, to every Reader. For suppose a Vessel were 2 Feet wide, 3 Feet long, and 4 Feet deep, and fill'd with Water to the Brim; then the Area of the Bottom is $2 \times 3 = 6$ square Feet, and every square Foot being press'd by the Column of Water, containing 4 cubic Feet, 'tis plain the whole Bottom will be press'd by $6 \times 4 = 24$ cubic Feet, that is, by the whole Number of cubic Feet which the Vessel contains.

2. But the Quantity of the lateral Pressure, or that against the Side of the Vessel, is not so obvious; yet among the several Ways of computing it, I think none so easy, natural, and universal, as that I have mention'd, viz. by considering it in a single Column of Particles, as a Series of Quantities in *Arithmetical Progression*, because this depends entirely on the single Principle already established, viz. *that the Action or Pressure of Fluid Particles is every Way equal*. By which I shall endeavour farther to illustrate this Matter, and facilitate the Method of Computation.

3. Upon the uppermost Surface of the first Particle 1, which coincides with the Surface of the Fluid, there is

IX. HENCE

IX. HENCE if the Vessel AC be of a ^{Pl. XV.} Cubical Form, the Pressure against a Side ^{Fig. 1.}

no Gravity, and consequently no Pressure; therefore the Beginning of the Series, or first Term is 0, or nothing. The Body of the first Particle presses the second Particle 2 with its own Weight which is as 1, and it presses the Side of the Vessel with the same Force, and therefore the *second Term* of the Series will be 1. The second Particle presses the third Particle 3 with the Force of its own Weight, and the Weight of that above it, that is, with a Force as 2; and since it presses the Side of the Vessel with the same Force, the *third Term* of the Series will be 2. After the same Manner it is shewn, that the *fourth Term* will be 3, the *fifth Term* 4, and so on. Whence 'tis evident the several Pressures will be as the Series, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, &c.

4. Now that the Sum of such a Series is equal to the greatest Term multiplied by half the Number of Terms, is known to every Person versed in common Arithmetick, and may be easily shewn by an Example. For suppose the Series were 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, then the Number of Terms is 12, and the half thereof 6; also the greatest Term is 11; but $6 \times 11 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 = 66 =$ the total Pressure against the Side of the Vessel. Now this is manifestly but half the Pressure upon a Line of the same Length in the Bottom of the Vessel, *viz.* a Line of 12 Particles, for since each Point sustains the Pressure of 11 Particles, the whole must be $12 \times 11 = 132$.

5. It may here be objected that I have taken 12 Particles in a Line at the Bottom, whereas there is but 11 at the Side, and therefore the Length of the Side and Bottom is not the same, as in the Supposition we make it. But it is to be consider'd, that when the Particles are suppos'd indefinitely small and numerous, as is the real Case of Fluids, the Difference of Length occasion'd by one single Particle will be infinitely small, and therefore will make no Error in the Computation.

BC is half that upon the Bottom CD; and consequently, *the total Pressure against*

6. Let the Sum of all the Pressures, or the total Pressure against the Side of a Vessel, be represented by S, the Number of Terms in the Series by N, and the greatest Term by G; then since $S = \frac{1}{2} NG$, and since N and G are ever proportional to each other, we shall have S as $\frac{1}{2} NN$, or 2 S as N^2 ; and since Halves are in the same Ratio with their Wholes, we have S as N^2 ; that is, *the Sum or total Pressure will be as the Square of the Number of Terms, or Altitude of the Fluid.*

7. For Example, in the above Series, if we take the Sum of 4 Terms, it will be $0+1+2+3=6$; and then of 8 Terms, the Sum will be $0+1+2+3+4+5+6+7=28$, which is 4 Times 6, and 4 over. Now these Sums ought to be as 1 to 4, because the Altitudes were as 1 to 2; and tho' the Sum of any Number of Terms will always be a little more than 4 Times the Sum of half that Number of Terms in this Way of computing, yet when the Number comes to be exceeding great, the Excess will become indefinitely small, and therefore may be neglected in the Case of Fluids.

8. Or yet more clearly and accurately in Symbols thus; since S is always as $\frac{1}{2} GN$ or as GN ; and in the Series above adapted to Fluids, $G=N-1$, therefore $GN=NN-N$; and so S will always be as $NN-N$; but when N is indefinitely great, N^2 will be infinitely greater than N, which therefore will vanish in that Case in the Expression $NN-N$; and so S will ever be as N^2 , that is, *the Sum or total Pressure will be as the Square of the Altitude of the Fluid.*

9. This Way of considering the Quantity of lateral Pressure by the *Arithmetical Series* is universal, whereas the common Method restrains it to the Property of an equicrural right-angled Triangle, and to a Vessel of a cubical Form; which I shall here give for the sake of such as would see the Demonstration of a Thing in several Ways. ABCD is a Vessel of a cubical Form, that is, whose Side BC is equal to the Length of the Bottom CD; if then the Diagonal BD be drawn, we shall have the Lines $1a=B_1$, $2b=B_2$, $3c=B_3$, $d4$
the

the Sides and Bottom is equal to three times the Weight of the Fluid on the Bottom of such a Vessel.

$=B_4$, &c. but B_1, B_2, B_3, B_4 , &c. being as the Altitudes of the Fluid, will represent the lateral Pressures in the Points 1, 2, 3, 4, &c. therefore also the Lines a_1, b_2, c_3, d_4 , &c. will represent the same lateral Pressures; hence when the Distances B_1, B_2, B_3, B_4 , &c. are indefinitely small, the Lines a_1, b_2, c_3, d_4 , &c. will be infinitely near each other; and so all those Lines drawn in the Triangle BCD will make the Area of that Triangle. Therefore the Sum of all the lateral Pressure against the Side BC, will be as the Area of that Triangle. But the Area of the Triangle BCD is as the Square of the Side BC, as we know from the Elements of *Geometry*. Consequently, the Sum of all the lateral Pressures is as the Square of the Altitude of the Fluid BC. Q. E. D.

10. Now the Pressure on the Bottom of the Vessel CD being equal to $CD \times BC$, and the Area of the Triangle BDC being equal to $BC \times \frac{1}{2} CD$; 'tis plain, the Sum of all the lateral Pressures against any one Side, is equal to but half the Pressures on the Bottom of such a Vessel; and so the Sum of all the Pressures against the four Sides is twice that upon the Bottom; and that the whole Force of the Fluid is equal to three Times the Force of its Gravity.

11. Hence we see how very differently Fluids act from Solids, which act upon each other by their Gravity only, whereas Fluids act by Gravity and by Pressure jointly. Solids act only downwards, but Fluids every way equally. Solids act with a Force proportional to their Quantity of Matter, but the Force of Fluids is not according to their Quantity, but their Altitude only.

12. The Quantity of Pressure on the Bottom of Vessels, or Planes oblique to the Horizon, is easily estimated from what has been said above; for let AK be such an inclined Plane, and AE the Surface of the Water; then will the Pressure upon the Points A, G, H, I, K, be as the Altitudes BG, CH, DI, EK. If now on the said Points we erect the Perpendiculars Gb, Hc,

Pl. XV.
Fig. 2.

X. THE *Weight, Pressure, or Effect of a Fluid upon the Bottom DE of any Vessel*

Id, &c. equal to those Altitudes respectively, and suppose the same Thing done for every other Point in the Line *AK*, 'tis evident the Triangle *AeK* will be as the whole Pressure on the Line *AK*. But the said Triangle is equal to $AK \times \frac{1}{2} eK$; and supposing $AG = GH = HI = IK$, and *cf* drawn parallel to *AK*; we shall have $\frac{1}{2} eK = fK = cH = CH$; therefore $AK \times CH$, will be as the total Pressure on the Line *AK*.

13. If *AK* were the Section of a Plane, then the Surface of that Plane multiplied by *CH*, will be the Expression for all the Pressure on that Plane, provided *CH* be the Depth of the Centre of Gravity *H*, from the Surface of Water. But to give a more general Theorem of the above Rule, let *a*, *b*, be two Weights hanging from an horizontal Plane, at the Distances *ac*, *bd*; and join their Centres by the Line *ab*, and let *x* be their common Centre of Gravity, and *xo* its Distance from the Plane *cd*, perpendicular to which draw *ay* and *bz*; then since $a:b :: bx:ax$, by the Property of the common Centre of Gravity; and by similar Triangles we have $bx:ax :: xz:xy$. Therefore $a \times xy = b \times xz$; but $xy = xo - yo = xc - ac$; and $xz = zo - xo = bd - xo$; therefore $a \times xc - ac = b \times bd - xo$; that is, $a \times ac + b \times bd = a + b \times xo$. That is, in Words, *The Product of the Weights multiplied by their Distances from the Plane is equal to their Sum multiplied by the Distance of their common Centre of Gravity from the Plane.*

14. Now that this holds true in Lines and Planes is evident, because the indefinitely small Particles of Lines and Planes may be consider'd as very small Weights, and as what has been demonstrated of two, holds equally for all, therefore the above Rule is applicable to all Sorts of Surfaces, or the Pressure upon the Bottoms of Vessels, however posited or figured, may be exactly computed thereby.

15. I thus suppose *ABCDEF* represent a Vessel of a Prismatic Form, whose Bottom is an oblique trian.

ACDEF,

ACDEF, is proportional to the Altitude AE only, and not to the Quantity of the Fluid

gular Plane DEF, in which let FG be drawn perpendicular to the Base DE, then if we make $Fx : xG : 2$ Pl. XIV. : 1, the Point x will be the Centre of Gravity of the Fig. 7. Plane, and xo its perpendicular Depth from the Surface of the Fluid ABC, when the Vessel is fill'd with Water. Now suppose $AF = 1$ Foot; $FG = 6$ Feet; $ED = 2$ Feet; $CE = HG = 3$ Feet; and let the Plane Fbc be parallel to the Surface of the Fluid ACB. Then we have $FG : Fx :: Gb : xa$; that is $6 : 4 :: 2 : 1\frac{1}{3} = xa$; therefore $xa + ao = xo = 2\frac{1}{3}$ Feet, the Depth of the Centre of Gravity of the Triangle FED whose Area is $FG \times EG = 6 \times 1 = 6$ square Feet. Therefore $6 \times 2\frac{1}{3} = 14$ Cubic Feet of Water, whose Weight is equal to the Pressure on the Bottom of the Vessel.

16. Thus in case of a Cylindric Vessel whose Bottom is either an Ellipsis, a Cone, a Segment of a Sphere, &c. if the Quantity of the Surface, and Centre of Gravity be found, the Content of the Vessel is then known by the Rule above. Thus also if the Surface of the Side of any Vessel in any Form or Position be multiplied by the Depth of the Centre of Gravity, the Product will be the Number of Cubic Feet, whose Weight is equal to the Pressure on such a Side or Surface. And since a cubic Foot of Water weighs 1000 Ounces, or $62\frac{1}{2}$ lb. Averdupoise Weight, the Quantity of Pressure may be easily express'd in any Denomination of Weight, as will be more particularly shewn hereafter.

17. One Thing more the Reader should be appriz'd of, and that is the *Centre of Pressure*, which is that Point in which we may conceive the whole Pressure to be concenter'd or united; or to which if a Force were applied equal to the whole Pressure, and acting in a contrary Direction, it would exactly balance or restrain Fig. 8. the Effect of the Pressure; or it would keep the Plane (otherwise free to move) in *Equilibrio* with the total Pressure, unequally distributed over all its Parts. Thus if ABCD be a Vessel of Water, and the Pressure on the

in the Vessel: For every Column of Particles *GH*, which presses downwards on

Side *AB* be equal to twenty Pounds; if *a* be the Centre of Pressure, and a String *abc* were fix'd thereto going over the Pulley *b*, and sustaining a Weight *c* of twenty Pounds, then would the Side *AB* be kept in *Equilibrio* by those two equal and opposite Powers.

Fig. 5.

18. Now the *Centre of Pressure* is the same as the *Centre of Percussion*, as is evident from hence, that in the Line *AK* the percussive Force of every Point *G, H, I, K*, is as the Velocity, that is, as the Distance *AG, AH, AI, AK* from the Point *A* considered as the Point of Suspension. But those Distances are as the Altitudes *GB, HC, ID, KE*, which are as the Pressures on the foresaid Points: Therefore since the Pressures and percussive Forces are always alike, the Centre of one must be the same with that of the other, but the Centre of Percussion of the Line *AB* is two Thirds of its Length distant from the Point of Suspension *A*, therefore also the Distance of the Centre of Pressure *a* is $\frac{2}{3}$ of the Side *AB* below the Surface of the Water. See *Annot. XXIX.*

SCHOLIUM.

19. The Reason why a greater Force of Pressure is allow'd to an oblique Plane or Bottom of a Vessel than what is equal to the Gravity of the Fluid, is this, that every Point in the Line *AK* is press'd with a *perpendicular* and *lateral Force* at the same Time, and by both *obliquely*; and of those two oblique Forces, or Pressures, one *direct Force* is compounded, which is greater than either, and acts in a Direction perpendicular to the Plane. But since both the oblique Pressures are always as the Depth of the Fluid, the direct compound Pressure will be in the same Ratio likewise. Hence as there is a greater Number of Points in the Line *AK* than in the Line *AE*, and each is press'd perpendicularly with a Force proportional to the Depth, 'tis necessary that we estimate the whole Force by the Area of the Triangle *Aek*, and not of *AEK*, as we must have done if Fluids press'd like Solids by their Gravity only.

the

the Side of the Vessel EF, has its Force destroyed by the equal Re-action of the subjacent Particle H in the Side, and so cannot at all affect the Bottom of the Vessel. Again; the Pressure of any Column of Particles LM upwards, against the Side of any Vessel CD, is equally re-acted by the Particle of the Vessel over it, and so its Force or Pressure on the Bottom must be the same as that of another Column of Particles AB of equal Altitude with the Fluid: Whence the Proposition is evident (LVI.)

(LVI.) 1. There is nothing in the Explication of the Hydrostatic Paradox difficult to be understood, but the Manner how the Point M comes to be pressed with the same Force as the Point B, which has the whole Height of the Fluid above it. But this Difficulty will soon vanish when we consider *that the Pressure among the Particles of a Fluid at the same Depth is every Way equal*; therefore if LH be parallel to the Bottom DE, 'tis plain there will be the same Pressure upwards in every Particle H, I, Q, L, as there is downwards, and every where as the Altitude AI. The Particle at L therefore presses upwards against a Particle in the Side of the Vessel with a Force equal to the Weight of the Column of Particles AI, which Force is destroyed by the equal Re-action of that Particle in the Side of the Vessel. The same Fluid Particle L also presses the Particle immediately beneath it in the Column ML with the same Force, which added to the Weight of the Column LM makes the same Pressure on the Point M, as there is on the Point B arising from the Pressure on the two Parts AI+IB.

2. That the Pressure upward at the Point L is equal to the Weight of the Column AI, is evident from hence, that if it be removed (that is, if a small Hole be there made) the Fluid will from thence be thrown

XII. IN

Pl. XV.
Fig. 2.

XII. HENCE *a very small Quantity of a Fluid, as APRS, may be made to counter-*

up in Form of a *Jet d'Eau* to the Height of T nearly, which is in the same horizontal Line with the Point A, or Level of the Fluid. All which well consider'd is, I presume, sufficient to evince that the Pressure on the Point M and every other Point in the Line D E is the same with that on the Point B.

3. From what we have said, 'tis also plain, that the Top of the Vessel C N R F is press'd upwards with a Force every where proportional to the Altitude of the Fluid AP, which Force of Pressure upon the whole Surface is equal to the Weight of a Body of Fluid of the same Base, and whose Altitude is AP. The Sides also are press'd every where outwards with a Force proportional to the Altitude of the Water, and in the same Manner as they would be at the Depth were the Fluid every where of the same Bulk as in the Body of the Vessel, and of the same Altitude as at A.

Pl. XIV.
Fig. 9.

4. The Consequence of this will be, that if an Instrument be contriv'd with a Bottom AB and Top CD connected with pliant prepared Leather EF, so as to be Water-tight, and if in the Middle of the upper Part at G be inserted a small Tube GH, and Water pour'd in so as to raise the upper Part DC that it may float freely on the Surface of the contain'd Water, then any Weight I, laid thereon will cause the Water to rise so high in the small Tube as will be equal to the Height of a Cylinder of Water of the same Base D E, and whose Weight will be equal to that laid on; and if another Weight K be added, it will raise the Water to twice the Height in the Tube; if a third Weight L be added, it will rise to three Times the first Height, and so on. Whence it appears that since the Bore of the Tube may be exceeding small, the greatest Weight may be sustain'd or counterbalanced by the least assignable Quantity of Water in the Tube.

5. But this Matter will be best explain'd by Calculation and Example. Suppose then D C = 6 Inches in Diameter, the Area of the circular Surface will be
balance

balance or be equivalent to the Weight or Force of any given Quantity T K G V, how great soever.

28,27 Square Inches; and since a Square Inch of Water weighs $\frac{5578}{1000}$ of an Ounce Averdupois, therefore a Pound will contain 27,7 Cubic Inches; consequently as this Number is but half a Cubic Inch less than the other, 'tis evident that a Cylandric Column of Water whose Base is 6 Inches in Diameter, and Height one Inch, will weigh nearly one Pound; if therefore I be one Pound, the Water by its Pressure will rise one Inch in the Tube; if K be another Pound, then I+K will raise the Water two Inches, and so on. If the Bore of the Tube be $\frac{1}{10}$ of an Inch Diameter, the Quantity in the Tube one Inch high is to that in the Vessel one Inch high, as the Square of $\frac{1}{10}$ to the Square of 6, that is, as $\frac{1}{100}$ to 36, as 1 to 3600; and in that Proportion are their Weights also; whence 'tis plain the Water in such a Tube will sustain 3600 Times its own Weight.

6. In like Manner we may consider how great an Effect might be produced by only blowing with one's Breath thro' the Tube H G into the Vessel ABCD. By this Means, if the Dimensions of the Vessel were large, and the Tube long and small, a Man with the Breath of his Mouth might raise any given Weight, how great soever. Suppose, for Instance, the Area of the Top DC were 1000 square Inches, and the Distance between the Top and Bottom AD= $\frac{1}{4}$ of an Inch, then will there be contain'd in the Vessel 250 Cubic Inches of Air, of the same Density with the external Air. If now more Air be blown thro' the Tube into the Vessel, it will condense the internal Air, and by increasing its Spring cause it to raise the moveable Top DE, though charged with a very great Weight.

7. Thus since the Weight of Air presses upon every Square Inch with a Force of 15 lb. (as will be shewn hereafter) if we place on the said Top DC, $1000 \times 15 = 15000$ lb. and then blow into the Instrument 250 Cubic Inches of Air, it will double the Density of the internal Air, and consequently its Spring, which

XIII. WHEN

XIII. WHEN any Body is immerfed in a Fluid, *it lofes juft fo much of its Weight as is equal to the Weight of an equal Bulk of the Fluid* ; but *the Weight loft by the Body is*

will then hold the great Weight in *Equilibrio*, and if ever fo little more be blown in it will raife it ; for in that Cafe the Air will prefs upwards with a greater Force than that of the Weight downwards.

8. If 500 Cubic Inches of Air were blown into the Instrument it would fustain 30000 *lb.* and fo on. But in fuch Cafes we muft fuppofe the Parts EF fufficiently ftrong to bear fuch a Prefsure from within ; for fince the Air is a Fluid it preffes every Way equally. Now becaufe the Bore of the Tube is fmall (fuppofe the 100th Part of a fquare Inch) a Perfon blowing through the fame would meet with no more Refiftance than the Prefsure of the Air on the Area of the Bore, which when the Air is of a double Density, or when 250 Cubic Inches are crowded in, will be but $\frac{15}{1000}$ of a Pound which may be eafily overcome by the Force of the Mufcles in blowing, efpecially if we ufe a Stop-cock to fhut off the Air in the Tube every Time we take in frefh Air to our Mouths.

9. If the Top of the Instrument DC were fix'd, and a Wire paffing thro' the Tube GH were fix'd to the Bottom, and fufpended at the End of a Balance, then if the Bottom be nicely balanced by Weights in the Scale of the other End, and after that Water pour'd in till the Top and Bottom were feparated to their utmoft Dif- tance ; to balance this Water will require one Pound Weight for every Inch in Height in the Veffel, and it will after that require one Pound Weight for every Inch that it rife in the Tube, which plainly fhews that the Force of Fluids is always proportional to the Height, and not the Quantity of the Fluid. *Note*, I have fup- pofed in this Article that the Diameter of the Veffel is fix Inches, as before in *Art.* 5. Another remarkable Inftance of this Hydroftatic Paradox, I fhall give when I come to confider the *Swimming of an heavy Body in a*
I gain'd

gain'd by the Fluid, which will be so much heavier than before (LVII.)

lighter Fluid, and which will be a very clear Illustration of the Matter: See Annot. LXI.

(LVII.) This is the fundamental Principle of every Hydrostatic Process, particularly of the whole Doctrine of *Specific Gravities*; which therefore cannot be made too plain and easy to be understood. To this End, let ABCD be a Vessel fill'd with Water to the Height EF; and let I be a cylindric Body (heavier than Water) to be immersed therein, as at L. By this Immersion of the Body I, a Quantity of the Fluid *abcd*, equal in Bulk to the Body, will be displaced by the superior Force or greater Gravity of the Solid: And this Quantity of Fluid must ascend, (as being confined towards the Bottom and Sides) and so raise the Surface of the Liquor from EF to GH; and then will the Quantity EFGH be equal to the Bulk of the immersed Solid *abcd*.

Pl. XIV.
Fig. 10.

2. But as the Solid comes to enter the Fluid, each Particle of the Fluid by its *Vis Inertiæ* will resist the Solid, or endeavour to oppose its Descent with all its Power; and so the whole Body of the Fluid that is removed or displaced by the Solid will resist it by the united Force of all the Particles: But this Force is equal to the Gravity of the Fluid removed, as is evident from hence, that the Fluid so removed is obliged to ascend or move in a Direction quite contrary to Gravity; *Wherefore the Solid in its Descent will be resisted by a Force equal to the Gravity of an equal Bulk of the Fluid.*

3. A Consequence of this will be, that, since Action and Re-action are equal, there will be just as much of the Gravity of the descending Solid destroy'd, as is equal to the renitent Force, that is, to the Gravity or Weight of an equal Bulk of the Fluid. Whence 'tis evident that all Bodies immersed in Fluids will weigh lighter if suspended therein at the End of a Balance, than in the Air; and lighter in the Air than in *Vacuo*, where only the true, real, or absolute Weight of Bodies can be shewn or known.

XIV. IF any Body E could be found without Weight, *it would, if placed on the*

4. And since the Force which resists the Descent of Solids is proportioned to their Bulk only, it follows, that equal Bodies immersed in Fluids lose equal Parts of their Weight; and therefore a lighter Body loses more of its absolute Weight, than a heavier one of the same Bulk. And consequently if two Bodies of unequal Bulk are *in Equilibrio* in the Air, that Equilibrium will be destroyed on their being immersed in the Fluid; because that which has the largest Bulk will lose most Weight in the Fluid.

5. Again; 'tis plain the Weight of the Fluid is augmented in the same Proportion, as that of the immersed Solid is diminished; for the Force or Action of the Fluid on the Bottom of the Vessel CD is before Immersion to that afterwards as the Altitudes CF to CH, or to the Bulks of the Fluid EFCD and GHCD. And since those Bulks act only by their Gravity, 'tis plain the Action of the Fluid is increased only by the additional Gravity of the Quantity GHFE, which is equal to that which the Solid loses by Immersion, as was before shewn in Art. 2. All which Cases answer very exactly by Experiments.

6. From what has been premised we may easily apprehend what is meant by *Specific Gravity*, viz. *that which is peculiar to any Sort or Species of Matter or Body, when consider'd or compared in any given Bulk or Magnitude, as a Cubic Inch, for Instance.* Thus we say, the specific Gravity of common Water is to the Specific Gravity of Lead, as the Weight of a Cubic Inch of Water to the Weight of a Cubic Inch of Lead. And as the *Absolute Weight* of Bodies is their whole Power of Gravity *in Vacuo*, so their *Relative Gravity* is that which they have in any resisting Medium, as Air, Water, &c. and is equal to the Absolute Weight diminished by the Weight of an equal Bulk of the Medium.

7. What relates to the *Absolute and Specific Gravities*, Pl. XIV. the *Magnitude, Density, &c.* of Bodies, will best be Fig. 11. understood by symbolical Computation; in order to
Surface

Surface of a Fluid AB, float thereon without any Part immersed; for being devoid of

to which let A and B be two Bodies of equal Bulk, but different Quantities of Matter: and let B and C be two other Bodies with equal Quantities of Matter, but of different Bulks.

And let $\left\{ \begin{array}{l} D = \text{Density} \\ B = \text{Bulk} \\ M = \text{Quantity of Matter} \end{array} \right\}$ in the Body A:

Also $\left\{ \begin{array}{l} D = \text{Density} \\ B = \text{Bulk} \\ M = \text{Matter} \end{array} \right\}$ in the Body B:

And $\left\{ \begin{array}{l} d = \text{Density} \\ b = \text{Bulk} \\ m = \text{Matter} \end{array} \right\}$ in the Body C.

8. Then, because the Density of any Body is proportional to the Quantity of Matter under equal Bulks, we shall have $D : d :: M : m$; and, because when the Quantities of Matter are equal, the Bulks must be reciprocally as the Densities, therefore we have $D : d ::$

$b : B$: Whence $D = \frac{D M}{M} = \frac{d b}{B}$; consequently $D M B$

$= d b M$. But $B = B$, and $M = m$; therefore $D B m = d b M$. Whence we have $D : d :: b M : m B$; and $B : b :: d M : D m$; and $M : m :: D B : d b$.

9. The *Specific Gravity* of Bodies being as the Weights, that is, as the Quantities of Matter, in equal Bulks, will be as the Density: Therefore $D : d :: S : s$; and by Substitution of Ratios we have the general Theorem above become $S B m = s b M$. And since the Absolute Weights (A, a,) of any two Bodies are as the Quantities of Matter, we have $S B a = A s b$. Wherefore $S : s :: A b : a B$; that is, the Specific Gravities will be as the Absolute Weights directly, and the Bulks inversely, or as the absolute Weights divided by the Bulks.

10. Also $A : a :: S B : s b$; that is, the Absolute Weights of Bodies are in the compound Ratio of their Specific Gravities and Bulks. Or the Absolute Weight of any Body is had by multiplying its Bulk and Specific Gravity together.

Gravity, it could have no Force to displace any Particles of the Fluid, and sink therein.

XV. IF any heavy Body F, lighter than an equal Bulk of the Fluid, be placed on its Surface, it will sink or descend therein, till it has removed or displaced so much of the Fluid whose Weight is equal to that of the Body: For then the Pressure upwards and downwards on the under Surface of the Body is equal; and consequently the Body will be there quiescent, or in *Equilibrio* with the Fluid. Hence the whole Solid is to the immersed Part, as the specific Gravity of the Fluid to that of the Solid (LVIII).

II. Again; because $B : b :: A s : a S$, it appears that the Bulk or Magnitude of Bodies will be as the Absolute Weights directly, and Specific Gravities inversely. Or the Magnitude of any Body is had by dividing its absolute Weight by its Specific Gravity.

PL. XIV. (LVIII.) 1. This Case is not strictly true but *in Vacuo*: for in the Air such a Body may be consider'd as sustain'd in two Mediums, viz. Air and Water; in one of which it will sink or descend, and in the other rise. And therefore to represent the true State of this Matter universally. we must raise a general Theorem in the Manner following: Let ABCD be a Vessel fill'd first with an heavy Fluid to the Level EF, and from thence with a lighter Fluid to AB. Suppose the Solid X sustain'd by those two Fluids; let the Part in the heavier be call'd A, and that in the lighter B; and let the specific Gravities of the heavier and lighter Fluids be as a and b .

2. Then since the Part A displaces a Bulk of the Fluid equal to A, the absolute Weight of that Bulk of Fluid will be Aa , (by *Annot. LVII. Art. 10.*) and for

XVI. IF

XVI. *If a Solid, as G, equal in Weight to an equal Bulk of the Fluid, be immersed therein, it will take any Situation indifferently in any Part of the Fluid, as at G, H, I, without any Tendency to ascend or descend therein:* For being totally immersed, it must remove a Parcel of the Fluid of equal Bulk and Weight; and consequently the Pres-

the same Reason the absolute Weight of a Bulk of the lighter Fluid equal to B will be Bb . Let c be the specific Gravity of the Solid X ; then the Sum of the Weights of the two Portions of the Fluids must be equal to the Weight of the Solid; otherwise it could not be sustain'd by them: Therefore $Aa + Bb = Xc = A + B \times c$. Hence $Aa - Ac = Bc - Bb$. Consequently, $A : B :: c - b : a - c$; and compounding, $A : A + B (=X) :: c - b : a - b$.

3. These two Theorems are thus express'd in Words:

1. As the Part of the Solid within the heavier Fluid is to the Part contain'd within the lighter: So is the Difference between the specific Gravity of the Solid and lighter Fluid, to the Difference between the Specific Gravity of the Solid and the heavier.
2. The Part of the Solid in the heavier Fluid is to the whole Solid, as the Difference between the specific Gravity of the Solid and lighter Fluid, to the Difference between the specific Gravity of the two Fluids.
4. Hence, if $b = c$, we have $A : X :: c : a$; that is, the Part immersed is to the whole Solid, as the specific Gravity of the Solid to the specific Gravity of the Fluid. And if the two Fluids were Water and Air, Water and Oil, or any other, and their specific Gravities given, with that of the Solid, it will be easy to find the Parts of the Solid contained in either Fluid by the Theorems above mentioned.

ture upwards is equal to the Tendency downwards on the lower Surface every where ; and therefore it can have no Power to sink : Also the Pressure downwards must be equal to the Pressure upwards on the upper Surface, whence it can have no Tendency to rise or swim ; it will therefore remain at Rest in any Position, G, H, I, wheresoever in the Fluid (LIX.)

XVII. LASTLY, IF a Body K or L, *heavier than an equal Bulk of the Fluid, be immersed therein, it will descend by the Excess of its Gravity above that of the Fluid*: For, when immersed, it will be resisted by the

(LIX.) I have shewn, that while a Body is suspended in a Medium, its absolute Gravity is diminished by the Resistance of the Medium ; it is therefore only the *relative or residual Gravity* of the Body that we find in such a Case : And this being **equal** to the Difference between the specific Gravities of the Body and the Medium, 'tis plain, where that Difference vanishes, that is, where the specific Gravities of the Body and Medium are equal, there the relative Gravity will become Nothing ; whence such a Body suspended by us in such a Fluid has no sensible Weight. And this is the Reason why a Bucket of Water, while in the Water, seems to have no Weight ; because the specific Gravities of the Water in the Bucket, and of the Wood of the Bucket, being the same with that of the Water in which it is suspended, there can be no relative Gravity of the Bucket of Water experienced, and therefore no Gravity or Weight at all. The Want of considering this has been the Occasion of many absurd Positions and puerile Conclusions in Philosophy ; as *absolute Levity, a Diminution of absolute Gravity, &c.*

Force

Force of an equal Bulk of the Fluid, which therefore will destroy just so much of the Gravity of the Solid; and consequently, the Residue or Excess of Gravity in the Solid is that alone by which it must descend (LX.)

(LX.) 1. As those Bodies swim which are specifically lighter than Water, as above explain'd; so others sink or descend in a Fluid by being specifically heavier, that is, by their relative Gravity. Thus, if the specific Gravity of the Solid be to that of the Fluid as 3 to 1, then $3 - 1 = 2$ is the relative Gravity by which it descends. If the specific Gravities are as 7 to 1, then $7 - 1 = 6$ is the relative Gravity. Whence we observe,

2. That the Descent of Solids in a fluid Medium is the very same with that of Bodies descending on an inclined Plane, because in both Cases the absolute Gravity is only diminished, by the Resistance of the Medium in one Case, and by that of the Plane in the other; and therefore all the Properties of the Motion of a Body falling freely, belong to this Motion thro' a Medium likewise.

3. This relative Gravity of Solids, by which they sink or swim, is usually illustrated by the Descent and Ascent of Glass Images and Bubbles included in a Jar of Water cover'd over with a Bladder, so as to include a small Quantity of Air between the Bladder and Water. The Images, &c. have small Holes in the Bottoms of their Feet, thro' which some Water is put into their Bodies, and that in such Quantities as will render them but very little specifically lighter than Water, but some more so than others, that they may not begin to move all together. Pl. XIV.
Fig. 13.

4. The Images being thus put to float in Water, and the Bladder tied down, if the Hand be laid on the Bladder, and gently compresses the Air beneath, the Air by its Spring will act upon the Water, and cause it to compress the Air in the Bodies of the Images, by which means more Water will be driven into

FROM what has been premised of the Nature of Fluids, it will be easy to understand,

their Bodies ; and when so much is got in as will make them specifically heavier than the Water, then they will begin to descend one after another ; and by varying the Degree of Pressure you may keep them suspended in any Part of the Fluid as you please.

5. It is to be observed, that the Matter of which the Bodies of those Images and Bubbles are made ought to be specifically heavier than Water, that they may sink when fill'd with Water, which otherwise they could not do : They are therefore made with Glass, which is about three Times heavier than Water. Also the Holes in the Feet ought to be very small, lest the Water should run out into the Air.

6. On the contrary, if the Images were but just heavy enough to sink to the Bottom of the Jar, if then the Bladder, instead of being press'd down, were lifted up from the Surface of the Water, the Air would expand itself, and have a less Spring ; and therefore the Air also in their Bodies would exert its Spring, (which is now greater than that under the Bladder) and drive some Water out of their Bodies, by which Means they will become specifically lighter than the Water, and rise in it to the Top.

7. The Image or Bubble will sink or swim without the Artifice of a Bladder, if nicely managed ; for if Care be taken to have just so much Water in the Image as will render it but very little higher than the Water, it will then swim at the Surface ; if then you put it into a proper Depth, the natural Pressure of the Fluid upwards will force so much Water more into it, as to make it heavier than Water, and from that Place, it will of its own Accord, sink down to the Bottom. Whence it appears the same Body will sink or swim in the same Medium, according to the different Circumstances it is under from the Medium, and not from any Thing in itself.

Pl. XIV.
Fig. 14.

8. If any Vessel AB be fill'd half full with Salt Water to C, and a Bubble at B be made just heavy enough to sink therein to the Bottom ; and after that an l-

that

that the lightest Body P may be depress'd in the heaviest Fluid, by any Contrivance to keep the said Fluid from pressing on the under Surface of the light Body, by which means only light Bodies are made to swim. Thus Cork or Wood will abide at the Bottom of a Vessel fill'd with Quick-silver.

AGAIN : *On the other Hand, the heaviest Body M may be made to swim in the lightest Fluid, by keeping the said Fluid from pressing on its upper Surface, by means of the Tube NO : For when by this means it is immerfed so deep as to keep off an equal Weight of the Fluid, the Pressure then of the Fluid acting upon its under Surface upwards will be equal to the Weight of the Solid tending downwards; and therefore if the Solid be sunk ever so little deeper, it must*

mage at C, having so much Water put into its Body that when the Hole in its Foot is sealed up, it shall be just light enough to swim in the said salt Water : Things being thus prepared ; if so much hot fresh Water be put into the Jar AB as will fill it to the Top, the Consequence will be, that the Fluid being now lighter, will not sustain the Image C which will sink to the Bottom ; and the Heat of the Water will rarify the Air in the Bubble B, by which means the Water in it will be in Part extruded, and the Bubble thereby becoming lighter, will rise to the Top. Thus by this Artifice, the Image and Bubble will spontaneously (as it were) change Places ; which pleasant Experiment, or Hydrostatic Problem, was first proposed by Mr. John Caswell, Astronomy Professor at Oxford, near 40 Years ago.

swim by the superior Force or Pressure of the Fluid upwards (LXI.)

HENCE also the Reason of *trying the different Gravity, Density, or Strength* (as it is commonly call'd) of *divers Fluids or spirituous Liquors by the HYDROMETER or Water-Poise*: For, since the stronger any Fluid is, the greater will be its Resistance to any Solid im-

pl. XV.
Fig. 3.

(LXI.) Thus for Instance, if the Body M be 5 times heavier than Water of an equal Bulk, and if by means of the Tube NO, placed on its upper Surface, the Water be kept from pressing thereon, that it be immersed to 7 times its Thickness below the Surface of the Water, 'tis plain the Pressure on the under Surface upwards will be as 7, but downwards only as 5; and therefore since there is the Excess of two Degrees of Pressure upward, 'tis plain the Body cannot descend; but may very properly be said to swim on the Water, and this will be the Case of all Bodies, however large or heavy, if placed at a proper Depth in the Water, with the above mentioned Circumstances.

2. But what is most remarkable, and at the same time the clearest Proof of the Hydrostatic Paradox, is, that if the Body M be immersed to the Depth of 10 times its Thickness below the Surface, with a Tube to keep off the Water from its upper Surface, then whatever be the Form of the Tube upwards, water pour'd into the same will in all Forms thereof, at the Altitude of 5 times the Thickness, make the Pressure on the under Surface upwards and downwards equal, and consequently, if the Altitude be ever so little increased, the Body will descend. If the Body be immersed to a Depth, equal to 12 times its Thickness, then the Altitude of the Water pour'd in must be equal to 7 times its Thickness, because $7+5=12$, and so on in all other Depths, let the Vessel NO be ever so great or small, provided its Base be equal to the Surface of the Body; the Pressure of the Water pour'd in being proportional to the Altitude only, and not at all to the Quantity thereof.

mersed,

mersed, 'tis evident the Hydrometer cannot sink so far into the heavy or strong Fluids, as into those which are lighter or weaker. The several Degrees of Strength, therefore, are easily shewn by the graduated Neck of this Instrument (LXII.)

(LXII.) 1. The HYDROMETER is one of the most useful Instruments of the Philosophical Kind; for tho' the *Hydrostatic Balance* be the most general Instrument for finding the specific Gravities of all Sorts of Bodies, yet the Hydrometer is best suited to find those of Fluids in particular, and with the greatest Ease, Conveniency and Expedition, as will appear from the following Account, and Description thereof.

2. This Instrument consists of four principal Parts, viz. (1.) A large round hollow Ball or Sphere ABCD which is made of Ivory, Glass, or Metal, as Copper, Brass, &c. (2.) An hollow Glass-Ball EC, of a smaller Size, partly fill'd with Quick-silver screw'd on to the lower Part of the former, in order to render it but little specifically lighter than Water, that it may nearly sink in it. (3.) A long small Stem or Shank A F fix'd into it on the upper Part, which by its Weight causes the Body of the Instrument to descend in the Fluid with Part of the said Stem. (4.) A small Weight F, screw'd on upon the Top of the Stem or Wire A F, to cause the Instrument to sink so far, that the Surface of the Fluid may always cut the Stem in its middle Point G.

3. Tho' the above be the usual Structure and Composition of this Instrument, it is certainly not the best; for if the Body or Globe ABCD be of Ivory, it will imbibe Part of the Liquor, and so its specific Gravity will be alter'd, and consequently the Experiment cannot be just. If it be of Glass, it will be subject to break, and will stand in need of Screws, Solder, Leather, &c. to make the Parts tight, which will be found also both inaccurate and inconvenient. The best Way therefore is, to have it made of Metal, viz. of Copper; with a Brass Stem above, solder'd on; and

THE

THE HYDROSTATIC-BALANCE is also an Instrument invented on the same Prin-

a Brass Ball below to screw on with a very nice Shoulder ; then every Part will be durable, and very secure from any Alteration of its Weight.

4. When this Instrument is swimming in the Liquor, the Part of the Fluid displaced by it will be equal in Bulk to the Part of the Instrument under Water, and equal in Weight to the Weight of the whole Instrument. Suppose the Weight of the Whole were 4000 Grains, then 'tis evident we can by this means compare together the different Bulks of 4000 Grains of various Sorts of Fluids. For if the Weight F be such as shall cause the Hydrometer to sink in *Rain Water*, till its Surface comes to the middle Point of the Stem G ; and if after this, it be immerfed in common *Spring Water*, and the Surface observed to stand $\frac{1}{10}$ of an Inch below the middle Point G, 'tis evident that the same Weight of each Water differs in Bulk only by the Magnitude of one Tenth of an Inch in the Stem.

5. Now suppose the Stem were 10 Inches long, and weigh'd 100 Grains ; then every 10th of an Inch would be 1 Grain Weight, and since the Stem is of Brass, and Brass is about 8 times heavier than Water, the same Bulk of Water will be equal to $\frac{1}{8}$ of a Grain ; and consequently to the $\frac{1}{8}$ of $\frac{1}{40000}$ Part, that is a 32000th Part of the whole Bulk, which is a Degree of Exactness as great as can be desired. Yet the Instrument is capable of still greater, by making the Stem or Neck to consist of a flat thin Slip of Brass, instead of one that is round or cylindrical : By this means we increase the Surface, which is the most requisite Thing ; and diminish the Solidity, by which the Instrument is render'd more exact.

6. In order to adapt this Instrument to all Sorts of Uses, there ought to be two different Stems to screw on and off in a small Hole at A. One Stem should be such a nice thin Slip of Brass, or rather of Steel, like a Watch-Spring set strait, as I have just mentioned, on one Side of which ought to be the several Marks, or Divisions, to which it will sink in various Sorts of Waters, as *Rain-Water*, *River-Water*, *Spring-Water*,
ciple.

ciple. By it we have a most useful and ready Method of finding the various comparative or specific Gravities of Fluids and solid Bodies,

Sea-Water, Salt-Spring-Water, &c. And on the other Side you mark the Division to which it sinks in various lighter Fluids, as *Hot-Bath-Water, Bristol-Water, Lincomb-Water, Cheltenham-Water, Port-Wine, Mountain, Madeira,* and various other Sorts of Wine. But in this Case, the Weight F on the Top must be a little less than before, when it was used for the heavier Waters.

7. But in Case of trying the Strength of spirituous Liquors, a common cylindric Stem will do best, because of its Strength and Steadiness; and this ought to be so contrived, that when immersed in what is call'd *Proof-Spirit*, the Surface of the Spirit may be upon the middle Point G, which is easily done by duly adjusting the small Weight F on the Top, and making the Stem of such a Length, that when immersed in Water, it may just cover the Ball, or rise to A; but when immersed in pure Spirit, it may rise to the Top at F; then by dividing the upper and lower Parts G A, G F into ten equal Parts each, when the Instrument is immersed into any Sort of spirituous Liquor, it will immediately shew how much it is above or below Proof.

8. This Proof Spirit consists of half Water, and half Alcohol, or pure Spirit, that is, such as when pour'd upon Gun-powder, and set on Fire, will burn all away, and permit the Powder to take Fire, which it will, and flash as in the open Air. But if the Spirit be not so highly rectified, there will remain some Phlegm or Water which will make the Powder wet, and unfit to take Fire. This Proof Spirit of any Kind weighs seven Pounds twelve Ounces *per* Gallon.

9. The common Method of shaking the Spirits in a Vial, and by raising a Crown of Bubbles, to judge by the Manner of their rising and breaking away, whether the Spirit be Proof or near it, is very precarious, and capable of great Fallacy. There is no way so easy, quick, certain, and philosophical, as this by the *Hydrometer*, which will demonstrate infallibly the Difference of

to

*to the last Degree of Accuracy ; especially in the New Structure and Method of using it, as represented in Plate XV. Fig. 4. The Parts of which are as follow ; AB the Foot on which it stands ; CD, a Pillar supporting a moveable Brass Plate E F, fastened thereto by the Screw in the Knob e. In the End of this Plate is fix'd an upright Piece I K, supporting another Plate GH, which slides backwards and forwards thereon, and is moveable every way about it. In the End of this Plate, at H, is fix'd (by a Nut beneath) a Wire LM, tap'd with a fine Thread from one End to the other ; upon this moves the Swan-Neck Slip of Brass N O, to which a very exact Balance is hung at the Point N ; to one of whose Scales P is appended the heavy Body R, by a fine Horse-hair or Piece of Silk S : The Weight of the said Body R in the Air is express'd by the Weights put into the Scale Q to make an *Equilibrium* therewith, which being destroy'd by immersing the Solid in the Fluid T V, contain'd in the Glass WV, is again restor'd by Weights put into the Scale P. So that the Weights in the Scale Q compared with those in the Scale*

Bulks, and consequently specific Gravities, in equal Weights of Spirits, to the 30, 40, or 50 thousandth Part of the Whole, which is a Degree of Accuracy, beyond which nothing can be desir'd.

P, *shew*

P, *shew at once the specific Gravity of the Solid R to that of the Fluid TV* (LXIII).

THE specific Gravity of Fluids is readily determined by weighing one and the

(LXIII.) 1. The Reason why I have given no Account or Figure of the *Common Hydrostatic Balance*, is because it is every where to be found in Books of this Sort ; but principally, because I would advise Gentlemen to the Use of that exhibited in the Lectures, which they will find far more expeditious and exact, and therefore much better fitted to answer the End of such an Instrument. For this Method of suspending the Solid to be weigh'd by a Horse-Hair, or fine Silk, requires not the large heavy Glas-Bucket (as in the *Common Hydrostatic Balance*) to find the specific Gravities of Solids ; or the heavy Glas-Bubble to find those of Fluids ; and therefore the Weight being reduced, the Balance may be smaller and nicer, and consequently such as will turn with a lesser Difference of Weight.

2. The different Gravities of Solids being first compared with that of Water, are then easily compared with each other ; for let S be the Weight of one Solid, s the Weight of any other, and W the Weight of Water, all of equal Bulks ; and let $S : W :: 5 : 1$, and $s : W :: 7 : 1$; therefore $5W = S$, and $7W = s$; consequently $S : s :: 5W : 7W :: 5 : 7$; that is, the specific Gravities of the Solids S and s are as 5 to 7. And in the same Manner the specific Gravities of Fluids are compared, being first compared with that of any one Solid.

3. As the very Notion of *specific Gravity* implies Comparison, so there must be some Sort of Body fix'd upon, whose Gravity must be made a Standard for the Gravities of other Bodies of equal Bulk to be compared with. This Standard Body must have two Properties ; first, it must be easy to be had or come at upon all Occasions ; and, secondly, it must be of a fix'd unalterable Nature, or at all Times the same, that there may be no Variation of its Gravity in equal Bulks, in different Times or Places. Now 'tis certain such a Body must be of the Fluid Kind, because the best Way of finding specific Gravities is by Immersion.

same

same solid Body in them severally; for since we suppose the Balance *in Equilibrio*

4. Among Fluids there are none which promise the Requisites for a Standard so fairly as Water. Yet here we find various Kinds, and of different Gravities, and none which are quite unexceptionable. *Rain-Water* is the most so of any, (but this is not always at hand) its specific Gravity being so nearly always the same, that could it be always had, it would answer all our Purposes very well. However, *common Water*, by means of the Hydrometer, might be always made a Standard in the following Manner.

5. Let there be a Quantity of Salt dissolved in Spring-Water to give it a Body or Density, a little greater than any Water of that Kind can be supposed to have naturally. Then let a very exact Hydrometer be set to float therein, and observe at what Division of the Neck the Surface of the Water stands; and that will be the Point, to which the Hydrometer ought to sink in the Water design'd for these Experiments, and which it may easily be made to do by the Solution of Salt therein, or a Mixture of Salt-Water therewith. And a better Method than this, for procuring a *Standard Fluid*, I am not able to think of.

6. By this Method I only propose to fix a Standard for very nice Enquiries; but for common Uses, *common Water* will do, whose Gravity must be represented by Unity or 1, or, (in case of constructing Tables with great Accuracy) by 1,000, where three Cyphers are added to give room to express the Ratios of other Gravities in larger Numbers in the Table. In doing this, we have a twofold Advantage; the first is, that by this Means we can express the specific Gravities of Bodies to much a greater Degree of Accuracy and Exactness. The second is, that the Numbers of the Table do also express the Ounces *Averdupois* contain'd in a cubic Foot of every Sort of Matter therein specified, because a cubic Foot of common Water is found, by Experiment, to weigh very nicely 1000 Ounces.

7. Now an Ounce *Averdupois* weighs $437\frac{1}{2}$ Grains,
with

PRICE

Fig. 1. p. 287.

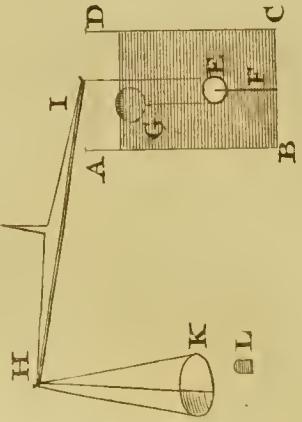


Fig. 2. p. 288.

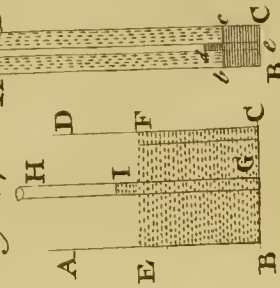


Fig. 4. p. 289.

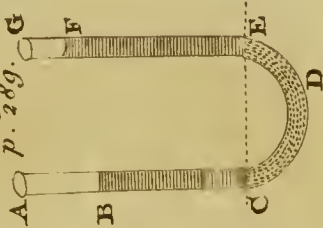


Fig. 3. p. 288.

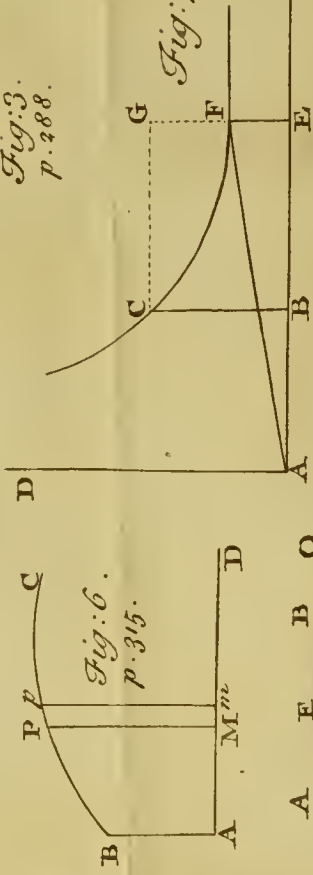


Fig. 6. p. 315.

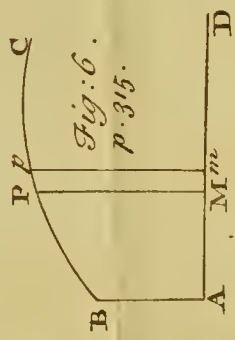


Fig. 7. p. 326.

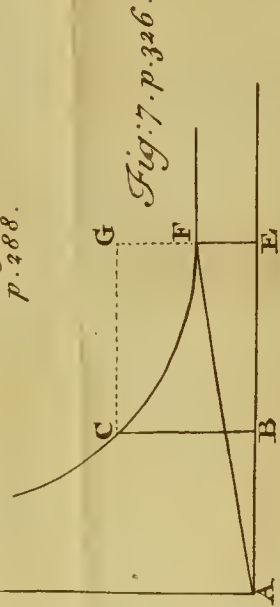


Fig. 5. p. 305.

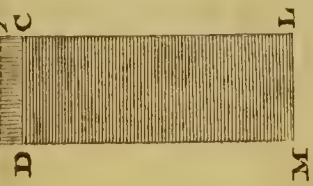


Fig. 8. Vol. II. p. 6.

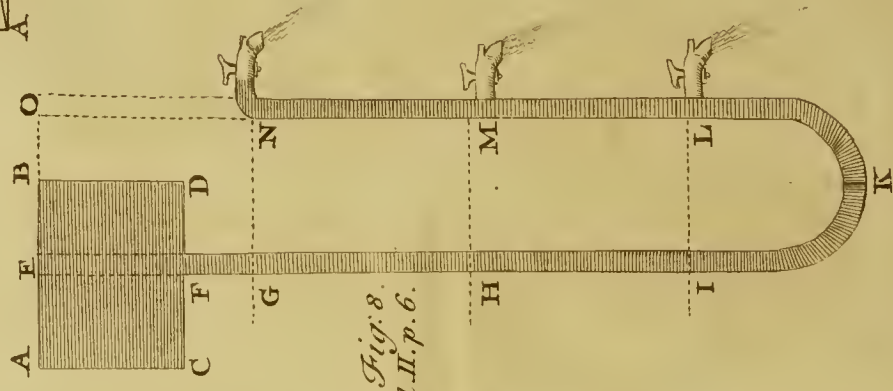


Fig. 9. Vol. II. p. 6.

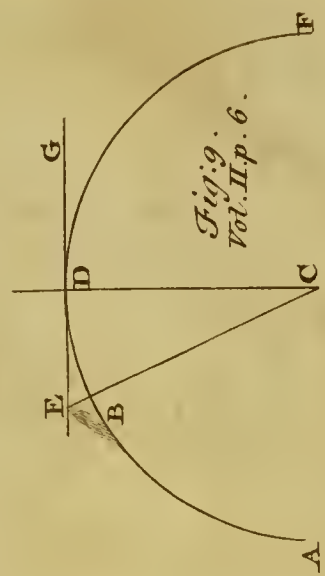


Fig. 10. Vol. II. p. 8.

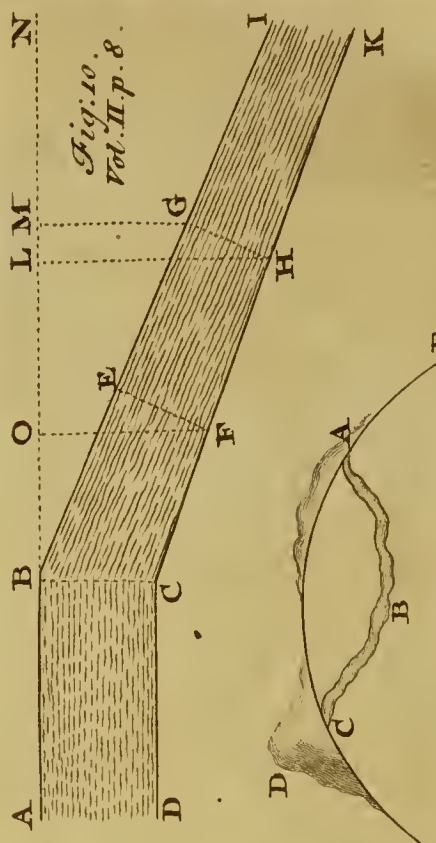
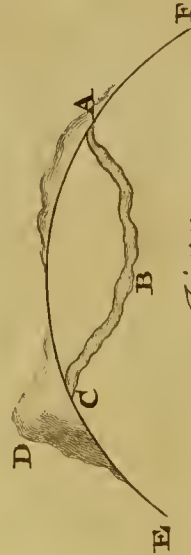


Fig. 11. p. 10.



with the Body suspended in the Air, the Equilibrium will be destroyed when the

and an Ounce *Troy* contains 480 Grains, therefore the *Averdupois* Ounce is to the *Troy* Ounce as $437\frac{1}{2}$ to 480, or, as 51 to 56 nearly. The *Averdupois* Pound is to the *Troy* Pound as $437\frac{1}{2} \times 16 = 7000$, to $480 \times 12 = 5760$, that is, as 17 to 14 nearly; so that a cubic Foot of Water weighs $62\frac{1}{2}$ lb. *Averdupois*, and nearly 76 lb. *Troy*. And hence it is that such a Table becomes so useful in the Solution of various Physical and Mathematical Problems, and Science of Geometry, extended by Hydrostatic Principles.

8. From what has been said, nothing can be easier to be understood, than the Method or Praxis of the Hydrostatic Balance, for finding the specific Gravities of all such Solids as are heavier than Fluids, and will sink in them; but for those which are lighter, as Wood, Cork, &c. and will not be totally immersed, a different Method must be taken, *viz.* the lighter is to be connected to an heavier, and so both together to be immersed into the Fluid, as one compound Body, and its specific Gravity thus determined; then having the specific Gravity of the heavier Body, and of the compound, that of the lighter Body will be easily found.

9. For suppose to a Piece of Copper, weighing 18 Grains, be tied a Piece of Elm Wood, weighing 15 Grains, then will the Compound weigh 33 Grains: Again, suppose the Copper alone in Water weighed 16 Grains, and the Compound only 6 Grains; the Loss of Weight in the Compound will be 27 ($= 33 - 6$) which is the Weight of an equal Bulk of Water: If from this we subtract 2 (the Loss of Weight in the Copper, and Weight of an equal Bulk of Water) the Remainder 25 will be the Weight of a Bulk of Water, equal to that of the Elm. But the Weight of the Elm was 15 Grains: The specific Gravity therefore of Water is to that of Elm-Wood, as 25 to 15, or, as 1 to 0,6.

10. Another Way more simple to find the specific Pl. XV. Gravities of light Bodies (without tying them to heavy Fig. 1. ones) is as follows, A B C D is a Vessel of Water,

Solid

Solid is immerfed in the Fluid, and muft be then reftored by Weights put into that

in which is placed a fmall Pulley E; on a Foot F; G is a light Body floating on the Surface of Water; HI is a Balance, I E G a Horfe-Hair going round the Pulley, and connecting the light Body G to the End of the Balance. Now let the Veffel be placed in fuch a Manner, that the Body G refusing to go under Water, may draw the Balance HI out of an horizontal Pofition; then fuch a Weight L, placed in the Scale K, as will draw the Body G under Water, and reftore the horizontal Pofition, or Equilibrium of the Balance, will difcover the fpecific Gravity of the Body G.

11. For fince Bodies afcend, as well as defcend; by the Differences of fpecific Gravities, or relative Weight; 'tis plain the Weight L, that detains the Body under Water, muft be equal to the Excefs of the Weight of the Fluid, above that of the Solid under equal Bulks; and therefore the Weight L, added to that of the Body, will give the Weight of an equal Bulk of Water, and of Courfe the fpecific Gravity. For Example, let the Body G be a Piece of Elm-Wood weighing 36 Grains; and the Weight L fufficient to detain or keep it under Water, will be found equal to 24 Grains; then $36 + 24 = 60 =$ the Weight of Water, equal in Bulk to the Elm. Confequently the fpecific Gravity of Water is to that of Elm, as 60 to 36, that is, as 1 to 0,6; the fame as was found before. *Note*, the Weight of the fcale K muft be added to the Weight L, becaufe 'tis all the Force upon the Point H that keeps the Body under Water.

12. There is yet another Way, and that a very good one, for finding the fpecific Gravities of fuch Liquors as will not mix with each other, as Water and all Kinds of Oil; it is thus: Let A B C D be a Veffel of Water to the Height E F, and G H a Tube dipt in Oil firft, for the Oil to rife a fufficient Height therein, when the upper Orifice H is to be ftopped clofe with the Finger, and thus immerfed into the Water of the Veffel; then the Finger being taken away, the Oil

Scale

Scale to which the Body is appended.
These Weights will severally express the

will subside and stand at the Altitude GI in the Tube. Now since this Column of Oil is balanced or sustain'd by a Column of Water of an equal Base and Altitude CF , and the Densities of Fluids are reciprocally as the Bulks, the specific Gravities (which are as the Densities) will be as the Bulks, that is, the Altitudes reciprocally, (because the Quantity of Matter is the same in both) therefore if $GI = 100$, and $CF = 87$, the specific Gravity of the Oil will be to that of the Water, as 87 to 100, or that of the Water to that of the Oil, as 1,000 to 0,87.

13. In like Manner, if $ABCD$ be a long Tube with a little Mercury pour'd in to a small Height bc , and then a small Tube ae , open at both Ends, be put into it; and lastly, if Water be poured into the large Tube upon the Mercury to the Height of 14 Inches; it will by its Pressure raise the Mercury to the Height b of one Inch in the small Tube above the Surface of that in the large one; which shews that the specific Gravity of Mercury is to that of Water nearly as 14 to 1. As will appear by the Experiment if accurately made.

14. Once more; another Way, which in some Cases may do very well, to find the specific Gravities of any different Liquids, is by Means of a recurved Tube ADG , in which if Quicksilver be pour'd first to fill the Bottom Part, and then one Liquid into one Leg, and another Liquid into the other Leg, in such Manner that they press on the Quicksilver on each Side equally, which will appear by the Surface on each Side being in the same horizontal Line CE ; then will the specific Gravity of the Liquid in the Leg AC be to that in the Leg EG , as the Altitude of the latter EF to the Altitude of the former CB .

15. By the Table of specific Gravities it appears that Gold is the heaviest Body in Nature, and that Mercury is next to it in Gravity; and consequently Gold

Gravities of an equal Bulk of the respective Fluids; and consequently, they may

only will sink in Mercury; therefore by weighing Gold in Mercury and Water, the specific Gravities of these two Fluids may be determined by the Hydrostatic Balance: But since Mercury will readily adhere to Gold, it will create us some Trouble in getting it off again, which must be done by Fire, or *Aqua Fortis*; as the latter is not always at hand, 'tis common to put Gold into the Fire, but Care must be taken not to put it into a Coal Fire, because the acid Spirit of the Sulphur will lessen the Cohesion of the Particles of the Gold, and so render it very brittle, and apt to break, especially when laid on a cold Stone to cool; the best way therefore to find the specific Gravities of these Fluids is that above directed in *Art. 13*.

16. *A TABLE of SPECIFIC GRAVITIES.*

Of METALS.

Fine or pure Gold,	19,640
Gold of a Guinea of <i>George II.</i>	17,150
Gold of a Moidore,	17,140
Silver fine or pure,	11,091
Silver of a Shilling of <i>George II.</i>	10,000
Lead,	10,130
Copper,	9,000
Brass cast,	7,850
—— wrought,	8,000
Steel temper'd,	7,704
Iron,	7,645
Tin,	7,550

17. *MINERALS, ORES, &c.*

Copper Ore,	3,775
Lead Ore,	6,800
Bismuth,	9,700
Turbith Mineral,	8,235
Antimony from <i>Germany</i> ,	4,000

thus

thus be compared with each other, or all of them with the Gravity of *common Wa-*

Antimony from <i>Hungary</i> ,	4,700
Speltar,	7,065

18. STONES, FOSSILS, &c.

Adamant or Diamond,	3,400
A Pseudo-Topaz,	4,270
A Pseudo-Hyacinth,	2,631
A Pseudo-Jasper,	2,666
A <i>Bohemian</i> Granate,	4,360
<i>Swedish</i> Granate,	3,978
Onyx-Stone,	2,510
A Cornelian,	2,568
An <i>English</i> Agate,	2,512
<i>Turcois</i> Stone,	2,508
Sardrachates,	3,598
A golden Marcasite,	4,589
Rock Crystal,	2,659
Island Crystal,	2,720
Lapis Nephriticus,	2,894
Lapis Lazuli,	3,054
Lapis Hæmatites,	4,360
Lapis Calaminaris,	5,000
Lapis Judaicus,	2,500
Lapis Manati,	2,270
Lapis Amianthus, or Asbestos from <i>Wales</i> ,	2,913
— ditto, from <i>Italy</i> ,	2,360
Glass of the common Sort,	2,666
Flint,	2,542
Black <i>Italian</i> Marble,	2,704
White <i>Italian ditto</i> ,	2,707
A fine Marble,	2,700
Another <i>ditto</i> of <i>Italy</i> ,	2,718
A pellucid Pebble,	2,641
A Selenitis,	2,322
Mundick, or Gold Spar,	4,430
Kidney Stone,	3,600
Blue Stone,	2,740

ter, as usual, and disposed in a proper Table; making that of Water 1,000,

Star Stone,	3,450
Hard Paving Stone,	2,460
Burford Stone,	2,049
Alabaster,	1,875
Rag-Stone,	2,470
Rotten Stone,	1,980
Copperas Stone,	4,300
Chalk,	2,370
Slate,	2,740
Oil-Stone,	2,380
An Hone,	2,388
China,	2,270
Piece of Brown Stone Bottle,	1,777
Piece of White Stone Mug,	2,250
Talc,	2,657
—— of Venice,	2,780
—— of Jamaica,	3,000
Armenian Bole, or Earth,	2,727
Common Sea Coal,	1,272
Magnet, or Loadstone,	1,840
Piece of Stonehenge, very hard,	2,618
—— ditto, of a softer Sort,	2,500
Bristol Stone,	2,510

19. ANIMAL SUBSTANCES.

Bone of an Ox,	1,656
Ivory,	1,826
The Tip of a Rhinoceros's Horn,	1,242
—— of an Ox Horn,	1,840
—— of a Stag's Horn,	1,875
Calculus humanus,	1,700
Ditto,	1,240
Ditto,	1,433
Ditto,	1,660
Oyster Shell,	2,092
Murex Shell,	2,590

IN the same Manner, if divers Solids
are first weighed in Air, and then after-

A Cockle Shell,	2,520
Mother of Pearl,	2,480
A Piece of hard Fish Skin,	1,621
A Piece of dried Flesh of Fish,	1,129
The Quill Part of a Feather,	1,330

20. VEGETABLE SUBSTANCES.

Dry Box-Wood,	1,030
Dry Oak,	0,925
Dry Elm,	0,600
Dry Ash sappy,	0,734
<i>Ditto</i> more dry, about the Heart,	0,845
Dry Maple,	0,755
Dry Fir,	0,546
Dry Cedar,	0,600
Dry Walnut-tree,	0,631
Dry Yew,	0,760
Beach meanly dried,	0,854
Crab-tree meanly dried,	0,765
Lignum Vitæ,	1,327
Lignum Nephriticum,	1,200
Lignum Aloes,	1,777
Lignum Brazilicum,	1,030
Lignum Rhodium,	1,125
Lignum Asphaltum,	1,179
Lignum Guaiacum,	1,337
Sassafras Wood,	0,482
Red Wood,	1,031
Red Santalum Wood,	1,128
White <i>Ditto</i> ,	1,041
Citrine <i>Ditto</i> ,	0,809
Speckled Wood of <i>Virginia</i> ,	1,313
Mastic Wood,	0,849
Ebony,	1,177
Cork,	0,240

wards immerfed in the fame Fluid, as Water, for Instance, the *Equilibrium* will be

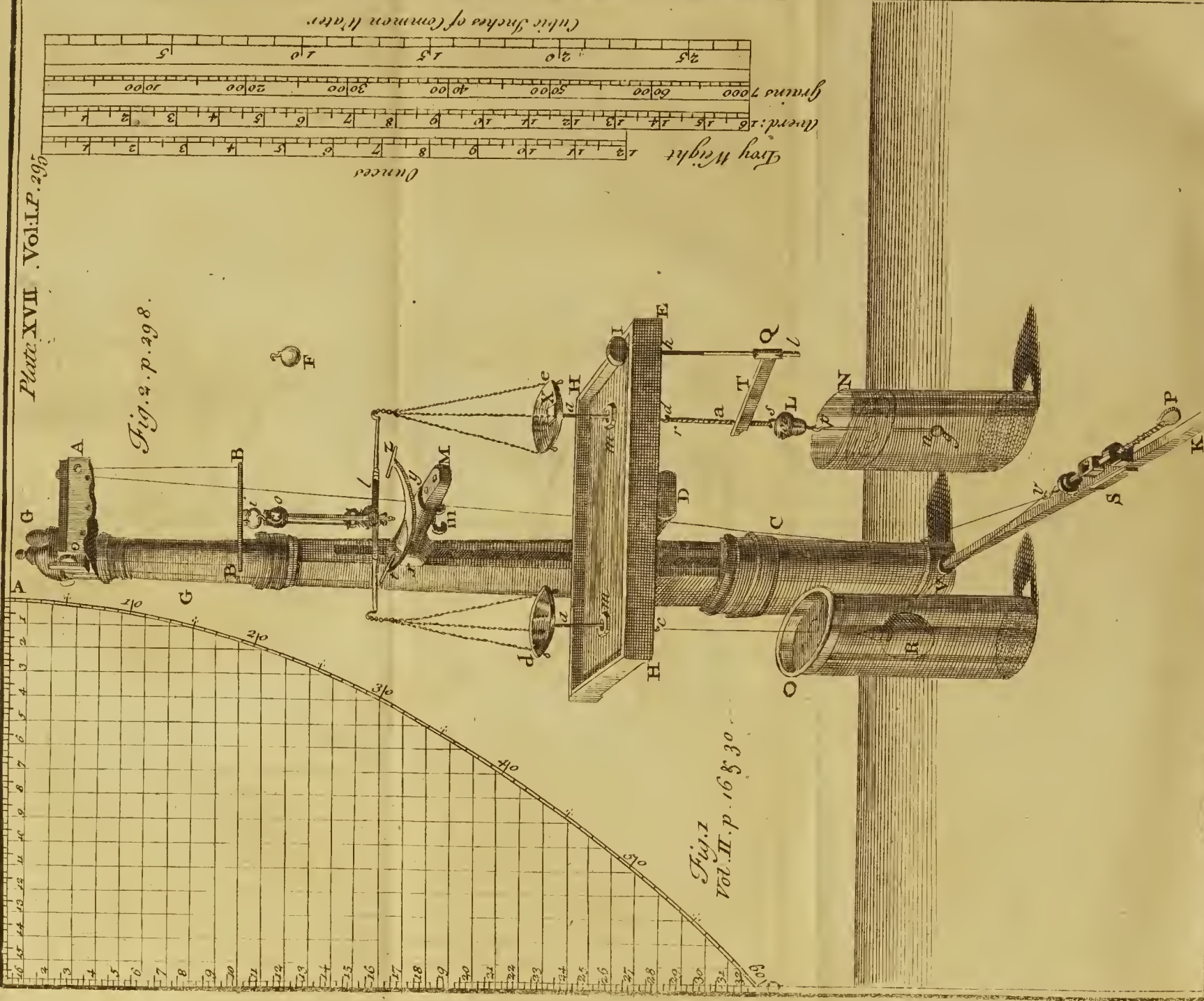
Good Wheat of the laft Year,	0,757
White Oats,	0,472
Blue Peafe,	0,795
White Peafe very dry,	0,807
Barley of the laft Year,	0,658
Malt made of the fame,	0,485
Field Beans very dry,	0,807
Wheaten Meal unfifted,	0,495
Rye Meal unfifted,	0,454
Wood-Afhes,	0,930

21. MISCELLANEOUS SUBSTANCES.

Amber,	1,040
Jet,	1,238
Bezoar Oriental,	1,530
—— Occidental,	1,500
Sulphur Common,	1,800
—— Vivum,	2,000
Borax,	1,720
Wood petrified,	2,341
Coral Red,	2,689
—— White,	2,500
Corallachates,	2,605
Cinnabar Natural,	7,300
—— Artificial,	8,200
—— of Antimony,	6,044
The reputed Silver Ore of Whales,	7,464
The Metal thence extracted,	11,087
Cerufe,	3,156
Tartar Common,	1,849
—— Emetic,	2,246
—— Vitrioli,	2,298
Cream of Tartar,	1,900
Camphire,	0,995
Merçury Crude,	13,593

deftroy'd;

Fig.2.p.298.



destroy'd; which will be restored, as before, by putting in so much Weight as is

Mercury Distilled once,	13,570
—— Sublimed 511 Times,	14,110
Glass of Antimony,	5,280
Vitriol of <i>Dantzick</i> ,	1,715
—— <i>English</i> ,	1,880
—— White,	1,900
<i>Sal Gemma</i> ,	2,143
—— <i>Prunellæ</i> ,	2,148
—— <i>Polychrestum</i> ,	2,148
—— <i>Ammoniacum</i> ,	1,453
—— <i>Mirabile Glauberi</i> ,	2,246
Salt of Hartshorn,	1,496
Salt of Vitriol,	1,900
Alum,	1,714
Nitre,	1,900
Gum <i>Arabic</i> ,	1,375
Gum Tragacanth,	1,333
Myrrh,	1,250
Verdigrease,	1,714
Opium,	1,363
Litharge of Gold,	6,000
—— of Silver,	6,044
Bees-Wax Yellow,	0,960
Bees-Wax White,	0,865
Pitch,	1,190
Tutty,	4,615
Honey,	1,450
Rosin,	1,100
<i>Crassamentum</i> of the human Blood,	1,126
<i>Serum</i> of the human Blood,	1,030
Piece of petrified Bone,	1,895

22. Of FLUIDS.

Rain Water,	1,000
Distilled Water,	0,993

U 4

equal

equal to the Weight of the same Bulk of Water: The Gravity, therefore, of every

Well or Spring Water,	0,999
River Water,	1,009
Sea Water,	1,030
<i>Aqua Fortis</i> ,	1,300
<i>Aqua Regia</i> ,	1,234
Oil of Vitriol,	1,709
Oil of Clove Gilliflowers,	1,034
Oil of Amber,	0,978
Oil of Aniseed,	0,994
Oil of Caraway-feed,	0,940
Oil of Linseed,	0,932
Oil of Mint,	0,975
Oil of Olives,	0,913
Oil of Orange,	0,888
Oil of Origany,	0,949
Oil of Rosemary,	0,934
Oil of Sassafras,	1,094
Oil of Spikenard,	0,936
Oil of Turpentine,	0,792
Spirit of Turpentine,	0,874
Spirit of Wine rectified,	0,849
Ethereal Spirit of Wine,	0,732
Spirit of Vitriol,	1,203
Spirit of Amber,	1,039
Spirit of Hartshorn,	1,073
Spirit of Urine,	1,120
Spirit of Honey,	0,895
Spirit of Nitre,	1,315
—— <i>Ditto</i> rectified,	1,610
Spirit of Sea-Salt,	1,130
Spirit of Tartar,	1,073
Tincture of Antimony,	0,866
Butter of Antimony,	2,470
Balsam of <i>Tolu</i> ,	0,896
Lixivium of Salt of Tartar,	1,550
<i>Burgundy</i> Wine,	0,953

Solid

Solid is thus compared with Water, and consequently with each other; as in the Table below.

Canary,	1,033
Red Wine from <i>Pontac</i> ,	0,993
White Wine Vinegar,	1,011
Distilled Vinegar,	1,030
Milk of Goats,	1,009
Cow's Milk,	1,030
Urine,	1,030

23. Since all Bodies are subject to expand with Heat, and be condensed with Cold, it must follow, that the specific Gravities of Bodies cannot be precisely the same both in Summer and Winter. This was first observed in Experiments by M. *Homberg*, and after him by M. *Eisenfchmid*, who found the absolute Weight of a cubic Inch of several Sorts of Bodies in Summer and Winter were sensibly different in each other, but upon examining the Numbers in his Table I find they are very erroneous, and have left them out in this Edition, and shall only observe, that this Affair may be render'd more apparent by taking the *specific Gravities* of these Liquors, (1.) when they are very cold; (2.) when they are pretty warm; and (3.) when they are very hot; for in these several Cases the Subject will discover a much less Density or Weight in equal Bulks, either by the Common or Hydrostatic Balance. But there is no Method in this Case so accurate as examining the different Bulks and Gravities of Fluids by the Divisions of *Farenheit's* Thermometer from the Degree of Cold in a *freezing Mixture*, to Heat of *boiling Water*, and tabulating the Observations.

24. And as in this, and many other Cases, it is required to be very exact in weighing Bodies, even beyond what is attainable by the nicest Mechanism of the Instrument itself, I shall here give the Reader an Account of an Improvement of the Common Balance in this Respect, from the late learned *S'Gravesande*; and

THE

THE *Application of Hydrostatics* to the several Uses of Life will be evident from the

it will be the more pertinent in this Place, as it depends on an hydrostatic Principle. This Instrument serves equally for Exactness in common as in hydrostatical Matters.

25. The Figure of the Machine represents the Balance in its hydrostatic Use. I shall first describe the Machine; then shew the new-contrived Artifice for Exactness; and, lastly, give an Instance of its universal Use. VCG is the Stand or Pillar fixed in the Table. From the Top at A hangs, by two silken Strings, the horizontal Piece or Bar BB; from which is suspended, by a Ring at *i*, the fine Beam of a Balance *l*, which is kept from descending too low on either Side by the gentle springing Piece *txyz*, fixed on the Supporter M. The Harness is annulated at *o*, to shew distinctly the perpendicular Position of the *Examen* by the small pointed Index fixed above it.

Pl. XVII.
Fig. 2.

26. The Strings by which the Balance is suspended passing over two Pulleys, one on each Side the Piece at A, go down to the Bottom on the other Side, and are hung over the Hook at *v*; which Hook, by means of a Screw P, is moveable about $1\frac{1}{4}$ Inch backwards and forwards, and therefore the Balance may be raised or depressed as much. But if a greater Elevation or Depression be required, the Sliding-Piece S, which carries the Screw P, is readily moved to any Part of the square Brass Rod VK, and fixed by means of a Screw.

27. The Motion of the Balance being thus provided for, the rest of the *Apparatus* is as follows: HH is a small Table fixed upon a Piece D, under the Scales *d* and *e*, and is moveable up and down in a long Slit in the Pillar above C, and fastened at any Part with a Screw behind. At the Point in the Middle of the Bottom of each Scale is hung by a fine Hook a Brass Wire *ad*, *ac*. These pass through two Holes *m*, *m*, in the Table; and to the Wire *ad* is suspended a curious cylindric Wire *rs*, perforated at each End for that Pur-

follow-

following Instances; having first premised, (what is found by Experiment) *that a Cubic Foot of common Water weighs*

pose. This Wire *rs* is covered with Paper graduated by equal Divisions, and is about five Inches long.

28. In the Corner of the Table at *E* is fixed a Brass Tube, in which a round Wire *hl* is so adapted as to move neither too hard nor too freely by its flat Head *I*. Upon the lower Part of this moves another Tube *Q*, which has Friction enough to cause it to remain in any Position required; to this is fixed an Index *T*, moving horizontally when the Wire *hl* is turn'd about, and therefore may be easily set to the graduated Wire *rs*.

29. To the lower End of the Wire *rs* hangs a Weight *L*, and to that a Wire *pn* with a small Brass Ball *g*, about $\frac{1}{4}$ of an Inch in Diameter. On the other Side, to the Wire *ac* hangs a large Glass Bubble *R* by a Horse-hair. Let us at present suppose the Weight *L* taken away, and the Wire *pn* suspended from *S*; and on the other Side let the Bubble *R* be taken away, and the Weight *F* suspended in its Room at *c*. This Weight *F* we suppose to be such as will keep in *Equilibrio* with the several Parts appended to the other Scale, at the same Time that the middle Point of the Wire *pn* is in the Surface of the Water in the Vessel *N*.

30. The Wire *pn* is to be of such a Size, that the Length of one Inch shall weigh four Grains. Hence it is evident, since Brass is eight Times heavier than Water, (see *Art. 16.*) that for every Inch the Wire sinks in the Water, it will become half a Grain lighter, and half a Grain heavier for every Inch it rises out of the Water: Consequently, by sinking two Inches below the middle Point, or rising two Inches above it, the Wire will become one Grain lighter or heavier.

31. And therefore, if when the middle Point is at the Surface of the Water in *Equilibrio*, the Index *T* be set to the middle Point *a* of the graduated Wire *rs*, and the Distance on each Side *ar* and *as* contain 100 equal Parts; then, when in weighing Bodies the

very

very exactly 1000 Ounces *Averdupois*, or 62 Pounds and an half; which may be reduced

Weight is desired to the *hundredth Part* of a Grain, it may easily be had by proceeding in the following Manner. Let the Body to be weigh'd be placed in the Scale *d*, and put the Weights in the Scale *e*; and let these be so determined, that one Grain more shall be too much, and one Grain less too little. Then the Balance being gently moved up or down by the Screw *P*, till the Equilibrium be nicely shewn at *a*; and then if the Index *T* be at the middle Point (*a*) of the Wire *rs*, it shews the Weights put into the Scale *e* are just equal to the Weight of the Body.

32. But if the Index *T* stand at any Part between *a* and *r*, it shews the Number of Grains in the Scale *e* were more than equal to the Weight of the Body in the Scale *d*, because the Wire *pn* is now made lighter by sinking below the middle Point. Thus, suppose the Weights put into the Scale *e* were 1095 Grains, and the Index *T* cuts the 36th Division above *a*, it shews that 36 hundredth Parts of a Grain are to be added, or that the Weight of the Body is 1095,36 Grains.

33. On the other Hand, had the Index stood at 36, the Division below *a*, it would have shewn the Weights in the Scale *e* were more than equal to the Weight of the Body by 36 Hundredths of a Grain; and that there the Weight of the Body was 1094,64 Grains. By this Method we find the absolute Weight of the Body; the relative Weight is found by weighing it hydrostatically in Water as follows.

34. Instead of putting the Body in the Scale *d*, as before, let it be appended with the Weight *F* at the Hook *c*, by a Horse-hair as at *R*, supposing the Vessel of Water *O* were away; then the Equilibrium being made, the Index *T* standing between *a* and *r*, at the 36th Division, shews the Weight of the Body 1095,36 Grains. As it thus hangs, let it be immersed in the Water of the Vessel *O*, and it will become lighter by much; the Scale *e* will descend till the Beam of the

to

to Troy Weight, by considering, that the *Averdupois* Pound is to the *Troy* Pound as 17 to 14, and the *Averdupois* Ounce to the *Troy* Ounce as 51 to 56.

HENCE, to find the Quantity of Pressure against the Sluice or Bank that pens the Water, we have this Rule: Multiply the Area of the Sluice under Water by the Depth of the Centre of Gravity in Feet, and that Product again by $62\frac{1}{2}$; the Product will be the Number of Pounds required. Example: Ad-

Balance rests on the Supporter *z*. Then suppose 100 Grains put into the Scale *d* restored the Equilibrium precisely, so that the Index *T* did again point to the 36th Division above *a*: It is plain the Weight of an equal Bulk of Water would in this Case be exactly 100 Grains.

35. But if the 100 Grains in the Scale *d* cause it to preponderate a little, then by turning the Screw *P* the Balance may be raised, till the Wire *pn* becoming heavier restores the Equilibre. Let now the Index *T* cut the 6th Division above *a*; then $36 - 6 = 30$, which shews that the Wire *pn* is now $\frac{3}{10}$ of a Grain heavier than before; therefore the Weight of the Water is only 99,7 Grains: Whence its Gravity to that of the Body is as 99,7 to 1095,36; as required.

36. After a like Manner may this Balance be applied to find the specific Gravities of Fluids; which will not be difficult to those who apprehend what has been already said. In Practice, it will be necessary to use great Precaution in every Particular; the Wire *pn* should be oil'd, and then wiped as clean as possible; enough will remain to prevent the Water adhering thereto: Also the Balance ought to be raised very gently, and when come to an Equilibrium should be gently agitated, to see if it will come so again.

mit

mit the Length of the Sluice be 20 Feet, the Depth of Water 5; then will the Area under Water be 100 square Feet; which multiplied by $2\frac{1}{2}$, the Depth of the Centre of Gravity, gives 250 cubic Feet; which again multiplied by $62\frac{1}{2}$, gives 15625 *lb.* equal to 7 Tons nearly (LXIV.)

AGAIN: *Since the Weight of Bodies is always as the specific Gravity in equal Bulks,* it follows, that the Numbers in the Table do also express the Number of *Averdupois*

(LXIV.) 1. That the Area under Water multiplied by the Depth of the Centre of Gravity gives the Number of solid Feet of Water pressing against the Sluice, is evident from *Annot. LV. Art. 13, 14, 15, &c.* And this Product is to be multiplied by $62\frac{1}{2}$, because so many Pounds is the Weight of one solid or cubic Foot of Water. See *Annot. LXII. Art. 7.*

2. This Example gives the Numbers which I took from a Pen that was made across a River; and though it may seem wonderful that all the Water in the River should affect the Pen with no Force of Pressure, yet this will be found consistent with the Laws of the Action of Fluids, when that is understood which we have delivered in *Annot. LVI.* For the Water of the River does no otherwise influence the Pen, than as it sustains a Plane of fluid Particles to a certain Height in immediate Contact with the Pen. This single Plane of Particles it is that gives all the Pressure on the Pen; and they would do the same, could they be otherwise supported against it, if all the rest of the Water in the River were taken away. Thus the Waters of the Sea or Ocean lying against any Dam or Bank will press it with no greater Force than the least Quantity of Water standing to the same Height, over the same Extent of the Dam.

Ounces

Ounces contain'd in a cubic Foot of each respective Sort of Matter therein mention'd. Therefore, *if the Magnitude of any Body be multiplied by the specific Gravity, the Product will be its absolute Weight.* Thus, suppose I would know what Weight of Lead will cover a Church whose Area is 30000 Feet, and the Thickness of the Lead $\frac{1}{100}$ of a Foot: Then *per Rule*, $(30000 \times \frac{1}{100} =) 300 \times 11325 = 339750$ Ounces, or $97\frac{2}{3}$ Tons; the Weight required.

ANOTHER useful Problem is, to *find the Magnitude of any Thing, when the Weight is known*; which is done by dividing the Weight by the specific Gravity in the Table, the Quotient is the Magnitude sought. For Instance, What is the Magnitude of several Fragments of Coral whose Weight is 7 Ounces; Divide 7 by the specific Gravity 2690, the Quotient is $\frac{26}{100000}$ of a cubic Foot: then $\frac{26}{100000} \times 1728 = 4\frac{1}{2}$ cubic Inches, very nearly the Magnitude required.

ALSO, by knowing the Magnitude and Weight, we can find the specific Gravity, by dividing the Weight by the Magnitude in cubic Feet. Thus suppose a Piece of Marble contain 4 cubic Feet, and weighs 603 lb. or 10800 Ounces; then $\frac{10800}{4} = 2700$, the specific Gravity required, as *per Table*.

HAVING

HAVING given the specific Gravity of Gold to Silver as 19 to 11, and suppose any compound thereof, as King *Hiero's* Crown, whose specific Gravity is 16; to determine the Proportion and Weight of the Gold and Silver employed in making it, say, *As the difference of the specific Gravities of the Compound and the lighter Ingredient, viz. 5, is to the Difference of the specific Gravities of the heavier Ingredient and the Compound, viz. 3, so is the Bulk of Gold to that of Silver made use of.* That is, if the whole Crown were divided in 8 Parts, the Gold would consist of 5, and the Silver of 3: Then the Magnitudes 5 and 3, multiplied by the specific Gravities 19 and 11 severally, will give the Numbers 95 and 33, which express the *Proportion of the Weight of the Gold to that of the Silver* (LXV.)

(LXV.) 1. The Rules here given for solving the three first Problems are only those Theorems express'd in Words which you find in *Annot. LVII, Art. 9, 10, 11.* But that which concerns the fourth, relating to King *Hiero's* Crown, I shall here demonstrate as follows:

2. Let A and B be any two sorts of Matter, and a and b denote their specific Gravities; the Compound made of those two Bodies will be $A + B$; and let c be the specific Gravity thereof. Then since the absolute Weight of a Body is equal to its Bulk multiplied by its specific Gravity (See *Annot. LVII. Art. 10.*) we shall have $A.a =$ Weight of the Body A, and $B.b =$

SINCE

SINCE Bodies of different specific Gravities, equiponderating each other in Air,

Weight of the Body B, and $\overline{A+B} \times c =$ Weight of the Compound. But $Aa + Bb = \overline{A+B} \times c = Ac + Bc$; whence $Aa - Ac = Bc - bB$; consequently $A : B :: c - b : a - c$, which is the Rule above laid down where A is the Quantity of Gold, and B that of Silver, and $A + B$ the Composition of *Hiero's* Crown.

3. When King *Hiero* (suspecting the Workmen had allay'd the Gold with more Silver than was necessary) sent to *Archimedes* to examine into, and detect the Fraud, if there were any; this great Mathematician was long at a loss to think of any Method of doing it; till one Day getting into a Tub full of cold Water to bathe himself, he observed. that as he entered the Tub the Water ran out, and he immediately saw it must follow, that if the Tub were full, the Water which ran out upon his Immersion, must be equal in Bulk to his Body.

4. Hence the Philosopher began thus to reason: if I immerse the Crown in a Vessel full of Water, it will protrude as much as is equal to its Bulk. If after this I immerse the same Weight of pure Gold, and pure Silver, I shall get Bulks of Water equal to each; consequently having the Bulks of Gold, Silver, and Crown of equal Weight, I have the specific Gravities, which must be as the Bulks inversely. Then I proceed to find the Ratio, or Proportion of Gold to the Silver in the Crown as follows.

5. Suppose AMLB be a Vessel fill'd with Water to the Height DC, and that the Mass of Gold equal in Weight to the Crown, being immersed into the Water, raises the Surface thereof to E; and after that, the Mass of Silver of the same Weight immersed raises the Surface to G; then if the Height of the Vessel above C be divided into equal Parts, and $DF = 11$, and $DG = 19$; it is plain the Bulks of the Gold and Silver will be as DF to DG, and the specific Gravities as DG to DF. Lastly, if the Crown be immer-

Pl. XVI.
Fig. 5.

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upon

upon being immerfed into Water, will have the *Equilibrium* immediately destroy'd

ed, it will raife the Surface to E, fo that $DE = 10$. Whence the Proportion of the Bulks of the Gold and Silver in the Crown may be determined.

6. For fince the Difference of fpecific Gravities of the Gold and Silver is $DG - DF = FG = 8$; if the Bulk of the Crown be divided into 8 equal Parts, it is evident, that fince the fpecific Gravities of the debafed and pure Gold Crowns will be as the Bulk inverfely; that is, as DF to DE , we can eafily find the Point H, which will exprefs the fpecific Gravity of the former; for $DE : DF :: DG : DH$. Now the Point H always divides the Difference FG into two Parts GH , HF , which have the fame Proportion as the Parts of Silver in the Crown have to the Parts of Gold; for as the Point E afcends, the Point H defcends, and when E coincides with G, H falls upon E, and the Crown becomes all Silver; as on the contrary, when E defcends to F, and H afcends to G, the Crown becomes all Gold; therefore every where FH will be to HG as the Parts of Gold to the Parts of Silver in the Crown.

7. Therefore, in the prefent Cafe, becaufe the Crown, when immerfed, raifes the Water to the Height DE , and H is 3 Divifions below G, it fhews that 3 of the 8 Parts of the Crown are Silver, and confequently the other 5 Parts are Gold, as H is 5 of the Divifions above F. Hence the Bulk of Gold in the Crown is to that of the Silver as 5 to 3, as before determined. That *Archimedes* took fome Method like this is certain; and it is faid, he offered to *Jupiter* a Hecatomb of Oxen for infpiring him with the Thought.

8. To the Problems in the Lecture, I fhall add the following. Let there be two Bodies A and B, one heavier, the other lighter than Water; let their fpecific Gravities be a and b , and let the Weight of A be given, and denoted by W ; and let it be required to find what Weight (w) of the lighter Body B muft be connected with the heavier, that fo the Compound fhall have the fame fpecific Gravity with Water. Now be-

by

by the greater Resistance of the Fluid, and consequently the greater Loss of Weight in

cause $A = \frac{W}{a}$ and $B = \frac{w}{b}$, therefore $A + B = \frac{W}{a} + \frac{w}{b} = \frac{Wb + wa}{ab}$ Bulk of Water, which multiplied by its specific Gravity (c) gives the Weight of Water $\frac{Wbc + wac}{ab} = W + w$ the Weight of both the Bodies.

Whence we have $Wbc + wac = Wab + wab$; and so $Wbc - Wab = wab - wac$; consequently $\frac{Wbc - Wab}{ab - ac} = w$; or if $c = 1$, then we have $\frac{Wb - Wab}{ab - a} = w$; or $\frac{b - a}{ab - a} \times W = w$ Weight of the Body B required.

9. For Example; suppose the specific Gravity of a Man's Body, Water, and Cork be as 10, 9, 2 $\frac{1}{2}$; then $a = 10$, $c = 9$, and $b = 2\frac{1}{2}$; let $W = 150$ lb. = Weight of the Man's Body; then will $\frac{Wab - Wbc}{ab - ac} = w = 5$; therefore if such a Man were to take 5 lb. of Cork with him into the Water, he would be of the same specific Gravity of the Water, and ever so little more would make it impossible for him to sink; whence the Art of learning to swim might by this Problem be greatly facilitated.

10. To find the Weight of a Globe of Water one Inch in Diameter. We shall here chuse Troy Weight, where one Cubic Foot weighs 76 lb; and therefore one cubic Inch weighs $\frac{76}{12 \times 12 \times 12}$ of a Pound, or $\frac{76 \times 12 \times 20 \times 24}{12 \times 12 \times 12}$ Grains, that is, $\frac{76 \times 40}{12}$ or $\frac{760}{3}$ Grains. But in Geometry we prove the Cube is to its inscribed Sphere as 1 to 0,524; therefore as 1 : 0,524 :: $\frac{760}{3}$: 132 Grains = the Weight of a Globe of Water one Inch Diameter.

11. To find the Diameter of a Globe by weighing it

X 2

the

the lightest and most bulky Body ; therefore it follows, that in *buying Gold*, which

hydrostatically. Let it first be weighed in Air, and then in Water, and the Difference of those Weights will be the Weight of an equal Bulk or Globe of Water, which call W ; now the Weights of homogeneous Bodies are as their Bulks, which (in Spheres) are as the Cubes of their Diameters. But the Weight of a Globe of Water 1 Inch in Diameter is 132 Grains (by *Art.* 10.) Therefore we say as $132 : W :: 1^3 : D^3$; Whence $\frac{W}{132} = D^3$, consequently $\sqrt[3]{\frac{W}{132}} = D = \text{Diameter of the Globe required.}$

12. *To find the solid Content in Cubic Inches of any irregular Body.* Let $W =$ Difference of its Weight in Water and Air $=$ Weight of an equal Bulk of Water.

Then as the Weight of a cubic Inch of Water $\frac{760}{3} : W$

$:: 1 : \frac{3}{760} \times W = \text{solid Content of the Body required.}$

13. *To find the Proportion of Magnitude between any two Bodies proposed.* Let each be weighed in Air and Water, and the Losses of Weight they each sustain will be the Weights of two Bulks of Water, and equal to those Bodies respectively ; and consequently will express the Ratio of the Magnitudes of those Bodies.

SCHOLIUM.

14. The Excellency of this Hydrostatic Method, above all others, appears from hence, that it is universal, and equally adapted to all Sorts and Shapes of Bodies, which common Geometry is not. How would it puzzle a Geometrician to exhibit the just Dimensions or Bulk of a Fish, for Instance ? How does the awkward Figure of his Body, the Appendage of Fins and Tail, elude the Principles of his Art ? Whilst the Philosopher immediately, and without any Trouble, gives the Answer by his *Hydrostatic Balance*. Again, were a Geo-

is heavier than Brass, we should chuse the lightest Air, i. e. when the Mercury in the Barometer stands lowest; but in buying precious Stones, Pearls, &c. which are lighter than Brass, the best Time to do it in is when the Air is heaviest and most buoyant, viz. when the Quicksilver stands highest in the Barometer: But in selling Gold or Jewels, the contrary Rules are to be observed in regard to the Gravity of the Air (LXVI.)

meter ask'd, which is biggest, a Guinea or a Shilling, and what the Difference? He would find it vain to consult his Art, and must borrow his Aid and Answer from this most useful Science, which excels equally in Accuracy as Universality.

(LXVI.) 1. Since a cubic Inch of Gold weighs $10,36\frac{6}{100}$ Ounces Troy, and a cubic Inch of Air $\frac{2}{7}$ of a Grain, when the Air is of a mean Gravity; if we say,

As $10,36 : \frac{2}{7} :: 12 : \frac{24}{72,52} =$ the Parts of a Grain

which 12 Ounces or a Pound of Gold will lose when the Air is in a mean State. Now since the Air, when it supports a Column of Mercury 28 Inches high, is lighter by $\frac{1}{10}$ Part than when it sustains a Column of 30

Inches Height; therefore $\frac{24}{72,52} + \frac{1}{20} = \frac{552,52}{1450,4} =$

the Parts of a Grain which so much Air weighs when heaviest, as is equal in Bulk to a Pound of Gold; and

twice the Quantity, viz. $\frac{1105,04}{1450,4} =$ the Weight of

Air equal in Bulk to two Pounds of Gold. But this is a little less than one Grain; let us suppose it one Grain.

2. Then two Pounds of Gold will lose of its Weight one Grain; and since Gold is twice as heavy as Brass that it is weighed against, the Brass Weights will have twice

ONCE more : Since *the Goodness of Mineral Waters, Drugs, Metals, precious Stones, &c. is best shewn by their specific Gravity*, it will at once appear of what vast Importance the Hydrostatic Balance is, and how absolutely necessary in the Hands of every judicious Dealer in any such Kind of Commodities (LXVII.) (LXVIII.)

the Bulk, and will there lose two Grains of its absolute Weight. If an Equilibrium be now made, when the Air is lightest, or let's by one Tenth of its former Weight, the Gold will lose one Tenth of a Grain, and the Brass will lose two Tenths of a Grain; consequently the Buyer will get one Tenth of a Grain of Gold in this last State of the Air, more than he would have had in the former, or heaviest State; because now one Tenth of a Grain more must be added to the Brass to make an Equilibrium. And this in Value is the fifth Part of a Penny, reckoning Gold at Two-pence per Grain; or one Part in 115200.

3. Since the specific Gravity of Diamond is to that of the Brass Weights as 1 to 3, if we make an Equilibrium between the two Pounds of Brass and Diamond, when the Brass loses two Tenths of a Grain, the Diamond will lose six Tenths of a Grain; consequently four Tenths of a Grain must be subducted from the Brass for an Equilibrium in the lightest State of the Air, and so much the Buyer will lose of Diamond more than when the Air is heaviest of all; which is about one Part in 28800 of the Whole. Whence it appears, that a Regard to the State of the Air is a Matter of more Curiosity than Importance, the Advantage being so very inconsiderable in either Case.

(LXVII.) 1. How great the Usefulness and Importance of Hydrostatic Knowledge is to *Physicians, Chemists, Apothecaries, Jewellers, Goldsmiths, &c.* will appear by reading Mr. Boyle's excellent *Medicina Hydrostatica*; in which

which Book the skilful Author proposes the following Uses to be made of Hydrostatic Knowledge, viz.

2. *First*, To explore the Nature and Difference of Fossils by finding their specific Gravities. For since the most pure and homogeneous Kind of Stones are in Gravity to Water as about $2\frac{1}{2}$ to 1; and Tin, the lightest of Metals, is to Water in Gravity as about 7 to 1; if a stony Substance be found to have a greater Proportion of Gravity than that of $2\frac{1}{2}$ to 1, it must be probable that it has in it some adventitious Matter of a metalline Nature, or is at least commix'd with some mineral Body more heavy than pure Stone, and may therefore very probably be usefully applied to some medicinal Purposes. For Instance of this Kind, he mentions *Lapis Hæmatites* or Blood-stone, *Lapis Lazuli*, the Load-stone, and *Lapis Calaminaris*; all which have their Uses in Physic.

3. *Secondly*, He proposes this Method as very certain to determine whether a Body, supposed to be a Stone of the mineral Kind, be so indeed. Thus Coral, which, says he, some take to be a *Plant*, and others a *Lithodendron*, but most reckon it among precious Stones, is in Gravity to Water as 2,68 to 1, which favours the last Opinion. Thus a *Calculus humanus* and a *Bezoar* were found as 1,7 and 1,5 to 1, and therefore too light to be of the same Species with common Stone.

4. A *Third Use* which he proposes is to discover the Resemblance or Difference between Bodies of the same Denomination, and thereby to collect and ascertain the several Degrees of Goodness respectively. Whence he argues the Necessity of this Sort of Knowledge to Physicians, Chemists, Apothecaries, Druggists; to the Goldsmith, the Merchant, the Miner, &c.

5. A *Fourth Use* is to discern genuine Stones from counterfeit ones, which may be of great Help to Jewellers. Here he gives Instances of factitious Coral and factitious Gems, and a Bezoar, which he found out that Way not to be genuine, though a great Price was set on the latter.

6. Hence Mercury is found to have a different Gravity, being sometimes but $13\frac{1}{2}$, and sometimes above 14 Times heavier than Water; and hence a notable Difference may arise in two Weather-glasses at the

same Time, and in the same Place, even to a whole Inch, from the different Gravity of the Mercury in one and the other. Therefore those who publish Registers of the Weather ought to find out and declare to the World the specific Gravity of the Quick-silver they use in their Barometers.

7. These Uses he enumerates over and above what we have taken Notice of, of a Mechanical and Geometrical Nature: And to let us know the high Value he had for this Science, he thus expresses himself: "As little Skill as I have in Hydrostatics, I would not be debarred from the Use of them for a considerable Sum of Money, it having already done me acceptable Service, and on far more Occasions than I myself first expected, especially in the *Examen* of Metals and mineral Bodies, and of several mineral Productions." With much more to the same Purport.

(LXVIII.) 1. To render this Lecture on *Hydrostatics* more compleat, I shall here subjoin what relates to the Resistance of Mediums, or to the Motion of Bodies moving in a resisting Medium: and to determine rightly in this Case, I shall consider every particular Circumstance by itself, *ceteris paribus*, and then represent the Whole connected together. We shall suppose the Case of *a Globe moving in a Fluid of an uniform Density for a given Time*.

2. First, Let the same Globe move with the same Velocity, first in a denser Fluid, and afterwards in a rarer; then it is plain, the denser the Fluid is, the more Particles the Globe will meet with, and strike in a given Time; and therefore the greater will be the Re-action or Resistance of the Fluid: Therefore in this Case *the Resistance of the two Fluids will be as their Densities*.

3. In the next Place, suppose two different Globes, of the same Quantity of Matter, to move in the same Fluid with the same Velocity. As these Globes move, 'tis evident the Number of Particles they meet and strike, and therefore by which they are resisted, will be as their Surfaces, or rather as their Half-Surfaces; and they are as the Squares of their Diameters: Consequently,

the Resistance which these Globes meet with will be as the Squares of their Diameters.

4. Lastly, Let the same Globe move in the same Fluid with different Velocities. Now it is easy to conceive, that if one Globe moves with a Velocity double to that of the other, it will strike twice the Number of Particles in the same Time, and its Resistance will be twice as great on that account. Again: The Globe which moves with a double Velocity will strike each Particle with a Force twice as great as the other does with half the Velocity; the Re-action therefore of the Particles on the former will be twice as great as upon the latter. Therefore the Resistance to the Globe moving with the double Velocity will be four Times as great: And since this will be the Case universally, *the Resistance to a moving Body is as the Square of its Velocity.*

5. Now putting all these Cases together, the Resistance to a Globe moving uniformly in an uniform Fluid will be in the compound Ratios of the Density of the Medium, the Square of the Diameter, and the Square of the Velocity. So that if A and B are two Globes, whose Diameters are C and d, and Velocities V and v, moving in Mediums whose Densities are N and n; then will the Resistance of the Globe A be to that of B, as $N \times D^2 \times V^2$ to $n \times d^2 \times v^2$.

6. In this Computation we have consider'd only that Resistance which arises from the Re-action of Matter by its *Vis Inertiae*: But there is some small Resistance arising to Bodies moving in a Fluid from two other Causes, viz. one from the Friction of the Body against the Parts of the Fluid as it moves, the other from the Tenacity of the Particles of the Fluid, which arises from their Attraction of Cohesion. But both these are very small when compared with the other, and quite inconsiderable to Bodies moving with any considerable Swiftnefs. The Resistance peculiar to Air, arising from its Elasticity, will be consider'd in another Place.

7. In the above Cases we have suppos'd the Bodies moving in the Fluid to be homogeneous, or of the same Matter and Density; but if they are heterogeneous, or of different Densities, then will the Resistance be further variable; for it will be inversely as the Densities,

or

or Quantities of Matter which the Bodies contain ; because the greater the Quantity of Matter, the greater will be the *Momentum* of the moving Body, and the less the Resistance of the Medium compared therewith, and so will be the more easily overcome by it. Hence, if the two Globes mentioned in *Art. 3.* have not equal Quantities of Matter, (as is there supposed) the Resistance will be as their Surfaces directly, and inversely as the Quantities of Matter ; that is, as $D^2 \times \frac{1}{D}$ to $d^2 \times \frac{1}{d}$,

or as $\frac{1}{D}$ to $\frac{1}{d}$, or as d to D . That is, *the Resistance to Globes of unequal Diameters and Quantities of Matter will be (ceteris paribus) inversely as their Diameters.*

8 Since a Body moving in a resisting Medium must every Moment have its *Momentum* or Quantity of Motion ($Q=MV$) abated or diminished by the Reaction of the Fluid ; and since in a given Time this Loss or Decrement of Motion is always as the Reaction, that is, as the Resistance (R) of the Fluid : But the momentary Increase or Decrease of Motion is as its Fluxion $\dot{Q}$ or $\dot{V}M$. Wherefore the Resistance of the Fluid for any equal Parts of Time will be $R=\dot{Q}=\dot{V}M$; or when the Mass of Matter M is given, we have $R=\dot{Q}=\dot{V}$.

9. But when the Moments of Time are not given, that is, not equal, the Resistance R will as the Decrement of Motion $\dot{Q}$ directly, and the Moment or Fluxion of the Time ($\dot{T}$) inversely ; for twice the Resistance will destroy the same Quantity of the Motion in half the Time, and three times the Resistance will destroy it in one

Third of the Time. Therefore $R = \left(\frac{\dot{V}M}{\dot{T}} \right) \frac{\dot{Q}}{\dot{T}}$; consequently, when the Mass M is given, we have $R = \frac{\dot{V}}{\dot{T}}$; that is, the Resistance is directly as the Fluxion

or momentaneous Decrement of the Velocity, and inversely as the Fluxion of the Time.

10. We have before shewn, (*Annot. XXI. Art. 5.*) that in uniform Motions *the Velocity is as the Space directly,*

reely, and the Time inversely, or $V = \frac{S}{T}$; and because

in every Kind of Motion, whether in *Vacuo* or in any Sort of resisting Medium, the momentaneous Velocity may be consider'd as equable or uniform, or proportional to the Space described each Moment of

Time; and therefore $V = \frac{\dot{S}}{\dot{T}}$, and so $V \dot{T}$

$= \dot{S}$, and $T = \frac{\dot{S}}{V}$; and, taking the Fluents, we shall

have the Sum of all the Rectangles $V \dot{T} = \dot{S} =$ the Space described, and the Sum of all the Quotients $\frac{\dot{S}}{V} = T =$

the Time.

11. Therefore if the Curve BPC be so described that its Ordinates MP, mp , (perpendicular to the Axis AD) expound the Velocity V, and the Abscissæ (from the fix'd Point A) AM, Am , the Time T; the Perpendicular AB being erected in the Point A, and meeting the Curve in B, the Area ABPM will express the Space S described in the Time T. Let mp be infinitely near to MP, then will $Mm = T$; therefore the Fluxion of the Area ABPM will be the Areola MP $pm = V \dot{T} = \dot{S}$. Pl XVII.
Fig. 6.

12. In like Manner, if the Absciss AM expounds the Space described = S, and the Ordinate MP be as the Velocity inversely, that is, if MP be as $\frac{1}{V}$, the

Area ABPM will expound the Time T in which AM is described; for the Fluxion of the Time being

$\dot{T} = \frac{\dot{S}}{V} = \dot{S} \times \frac{1}{V} = Mm \times MP$, the Sum of all

these Fluxionary Areolas MP pm will be the Area ABPM expressing the Time.

13. Because when the Mass of Matter in the moving Body is given, we have $R = \frac{\dot{V}}{\dot{T}}$ (Art. 9.); therefore

$R \dot{T} = \dot{V}$, or the Fluxionary Decrement of Velocity is as the Resistance and the Fluxion of the Time conjointly. But the Fluxionary Increment of the Space will be as the Velocity and its Fluxionary Decrement directly, and the Resistance

distance inversely, that is, $\dot{S} = \frac{V \times \dot{V}}{R}$; for as above

$T = \frac{\dot{V}}{R}$, and $\dot{S} = VT$ (Art. 10.); therefore $S = V$

$\times \frac{\dot{V}}{R}$ or $\dot{S} = \frac{\dot{V} \times V}{R}$.

14. Hence $R = \frac{V \times \dot{V}}{\dot{S}}$, or the Resistance is as the

Velocity and its Fluxionary Decrement directly, and the Fluxion of the Space inversely. Also $R\dot{S} = V \times \dot{V}$; that is, the Velocity multiplied by its Fluxionary Decrement is as the Resistance and Fluxionary Increment of the Space conjointly. Because the Velocity has been hitherto consider'd as decreasing, it ought to have the Fluxion $\dot{V}$ express'd negatively in the above Equations; that is, $R\dot{T} = -\dot{V}$, and $R\dot{S} = -\dot{V} \times V$.

15. We have also hitherto consider'd the Resistance R as comprehending all that retards the Motion of the Body in the Medium. But this retarding Power may arise from different Principles, as the Density of the Medium, or its particular Resistance; and also from a centripetal Force, as that of Gravity: Thus a Stone thrown upwards is retarded by the Resistance of the Air, and Power of Gravity. Let the Centripetal Force be C , the Resistance of the Medium R ; then will $R = C + R$ when these Forces conspire, and $R = C - R$ when they act contrarily.

16. Thus in the Motion of an ascending Body, where $R = C + R$, we have $R\dot{T} = C\dot{T} + R\dot{T} = -\dot{V}$; and $C\dot{S} + R\dot{S} = -\dot{V} \times V$. But in the Descent of the Body, where the Centripetal Force C is greater than that of the Resistance, we have $C - R$; and then the above Theorems will be $C\dot{T} - R\dot{T} = \dot{V}$, and $C\dot{S} - R\dot{S} = \dot{V} \times V$; for now the Velocity is increasing, and its Fluxion positive. But if the Central Force be less than the Resistance, we have $R - C$; then it will be $R\dot{T} - C\dot{T} = -\dot{V}$, and $R\dot{S} - C\dot{S} = -\dot{V} \times V$.

17. If in these Theorems we suppose $R = 0$, they will be changed into others, by which the Motions of Bodies in non-resisting Mediums may be determined. Thus
for

for the Ascent they will be $C \dot{T} = -\dot{V}$, and $C \dot{S} = \dot{V} \times V$; and for the Descent $C \dot{T} = \dot{V}$, and $C \dot{S} = \dot{V} \times V$. And thus the Motion of a Body in a resisting Medium may be compared with the Motions of the same Body in a non-resisting Medium, or in *Vacuo*.

18. If the Resistance be equal to the central Force, that is, if $R=C$, then in descending Bodies we have $C \dot{T} - R \dot{T} = \dot{V} = 0$; consequently, as the Fluxion of the Velocity is equal to Nothing, the Velocity can neither increase nor decrease, but is in that Case uniform or equable.

19. Since, till this happens, the Velocity is continually increasing, (because its Fluxion $\dot{V}$ is always as $C-R$ for any given Time) therefore when $C=R$, or $\dot{V} = 0$, the Velocity of the descending Body is a *Maximum*, or the greatest possible: But this Velocity Bodies descending in a Fluid, though they constantly approach to it, will never attain; as will be seen in the Sequel hereof.

20. In order to estimate the Velocity, the Time, and the Space described by Bodies falling in an uniform Fluid, such as Waters, whose Parts have no considerable Tenacity, and where the Retardation of the Body is the Effect only of the Resistance it meets with from the *Vis Inertiæ* or Re-action of the Fluid; it will be necessary first to observe, that when any Body moves in a Fluid with a given Velocity, the Re-action of the Fluid upon the Body is the very same as its Action upon it would be, were the Body to be at Rest in the Fluid moving against it with the same Velocity: Because the Magnitude of the Stroke is always as the relative Velocity, which is here the same in both Cases. (See *Annot.* XXIII. 8.)

21. Also that a circular Plane, a Cylinder, a Globe, P. XVIII. a Spheroid, a Cone, &c. whose Breadth is the Fig. 6. same, and against which the Fluid moves in a perpendicular Direction, are equally acted upon by the Fluid; or, if they move in the Fluid, are equally resisted by it. For let P Q be the Diameter of any of those Bodies placed in a Canal E F T S, thro' which the Water flows from the large Vessel A B C D, into another below fill'd with stagnant Water, touching
the

the Bottom of the former CD. Then since the Capacity of the Canal is alike straiten'd or diminish'd by each of those Bodies, it is plain the Fluid flowing through it must suffer the same Retardation from all of them, and therefore must act alike upon each; consequently, were the Fluid at Rest, and these Bodies moving through it, they would all be equally retarded.

22. It must be farther observed, that all Resistance arising from the Asperity of the Surfaces of Bodies, is not here consider'd; also it is supposed, that the Fluid is continuous, infinitely compress'd, and whose Parts have no Elasticity, Tenacity, nor Friction; and then the Case will be very nearly the same with *very deep Water*.

23. Now Sir *Isaac Newton* has shewn, that if the Vessel ABCD be indefinitely large or wide, the Velocity of the Fluid in the annular Space about the circular Plane PO will be equal to that which a Body acquires in falling through the Height GH; because it would be so, were the Plane PQ placed in the Orifice EF, the Motion of the Fluid through the Canal being uniform, and not accelerated by Reason of the stagnant Water beneath.

24. He farther shews, that if this Plane were placed in the Orifice EF at the Bottom of the Vessel, as at pq , it would sustain a Portion of the descending Fluid, or Cataract AEFB, like to that represented by pHg , which is less than two Thirds of a Cylinder of the same Base pq , and Altitude GH, but greater than one third Part thereof: And that therefore it is nearly an Arithmetical Mean between them, or equal to Half of the said Cylinder.

25. This is upon Supposition that the Plane pq is exceeding small; but if it be augmented till it becomes equal to the Hole EF, it will then sustain the Weight of the perpendicular Column of Water over it, that is, it will sustain the Weight of a Cylinder of Water of the same Base and Altitude GH: And that therefore in all other Cases the Weight which the Plane pq sustains is to the Weight of a Cylinder of Water, whose Base is equal to the Plane and Altitude $\frac{1}{2}$ GH, as EF^2 to $EF^2 - \frac{1}{2} pq^2$.

26. The Resistance (R) therefore to the Plane pq moving in the Orifice EF, and consequently of the Plane

Plane PQ moving in the Canal, will be in all Dimensions of the Canal and Plane compared with the Weight (W) of a Column of Water whose Base is equal to the Plane, and Altitude $\frac{1}{2}$ GH, as the Area of the transverse Section of the Canal to the Excess of this Area above $\frac{1}{2}$ the Area of the Plane; that is, $R : W :: EF^2 : EF^2 - \frac{1}{2} PQ^2$.

27. Let now the Orifices of the Canal EF and ST be closed up, and let the Plane PQ ascend in the Fluid compress'd on every Side; and in its Ascent let it oblige the superior Fluid to descend by the annular Space between it and the Canal; then will the Velocity (V) of the ascending Plane be to the Velocity (v) of the descending Water, as $EF^2 - PQ^2$ to PQ^2 ; that is, $V : v :: EF^2 - PQ^2 : PQ^2$; whence $V : V + v :: EF^2 - PQ^2 : EF^2$.

28. But since the Motion of the Water is contrary to that of the Plane, the relative Velocity, with which they act on each other, is the Sum of both, viz. as $V + v$. (See *Annot.* XXIII.) Now let this relative Velocity be equal to that with which the Water before descended in the annular Space by the Plane PQ at Rest, (as in *Art.* 23.) viz. that which is acquired by falling through GH. Then will the Action of the Water upon the Plane be the same as before at Rest, and consequently we have here also $R : W :: EF^2 : EF^2 - \frac{1}{2} PQ^2$; and $V : (V + v) :: EF^2 - PQ^2 : EF^2$.

29. If now we suppose the Canal to be augmented in Width *ad infinitum*, PQ will then be infinitely small in respect of EF; whence PQ^2 and $\frac{1}{2} PQ^2$ will vanish in the Terms of the above Ratios, which will then become equal; and so $R = W$, and $V = v$. That is, the Resistance of the Plane will now become equal to the Weight of a Cylinder of Water, whose Base is equal to that of the Plane, and Altitude $\frac{1}{2}$ GH: Also, the Velocity of the Plane is equal to that which such a Cylinder would acquire by falling through the Height GH.

30. Let $A =$ Area of the Plane PQ, then will $\frac{1}{2} GH \times A =$ the Cylinder of Water, whose Weight W is equal to the Resistance R of the Plane. But the Cylinder $\frac{1}{2} GH \times A$, moving with the same Velocity V of the Plane, will have the same Resistance. The Space
through

through which it must descend to acquire that Velocity is twice its Length, viz. GH; and with that Velocity, by an uniform Motion, it would pass over a Space equal to $2 \text{ GH} = 4$ times its Length $\frac{1}{2} \text{ GH}$. *Wherefore the Resistance of the Cylinder is equal to the Force (or Weight W) by which (in falling freely) a Motion equal to that of the Cylinder may be generated, (in acquiring the Velocity V) in the Time in which the said Cylinder will describe a Space equal to four times its Length.*

31. Then because the Motion M of the Cylinder is as its Length L, its Base A, its Density D, and its Velocity V, that is, $M : L \times A \times D \times V$; (see *Annot. LVII. 8.*) therefore when (as in this Case) the Quantities A, D, V, are given, we have $M : L$; and since the Time T (in which the Motion M is generated, or 4 L is described) is also as L, therefore both M and T will vary with L equally, and consequently will be proportional to each other. But the Force which generates Quantities of Motion proportional to the Time is in itself constant and immutable, and is therefore equal to the Resistance of the Cylinder, which also remains unchanged under all Degrees of Motion.

32. If the Density D of the Cylinder be variable, so will the Quantity of Motion M be likewise, and also the Force W, by which an equal Quantity of Motion in the same Time may be generated or destroyed. Therefore, let $d =$ Density of a Cylinder of Water, $D =$ Density of a Cylinder of Lead of the same Base, $w =$ Force which would generate the Motion of the Cylinder of Water in the Time it would describe four Times its Length, and W the Force that would generate the Motion of the Cylinder of Lead in the same Time: Then $d : D :: w : W$. But we have shewn $w = R$, the Resistance of each Cylinder: Consequently, $R : W :: d : D$, that is, *the Resistance of a Cylinder of Lead is to the Force which will generate its whole Motion while it describes four Times its Length, as the Density of Water to the Density of Lead.*

33. Hence, since the Resistance of a Globe and Cylinder moving in a Fluid is the same, and since a Globe is $\frac{2}{3}$ of its circumscribing Cylinder, the Force which will generate or destroy the whole Motion of the Cylinder while it moves over 4 times its Length, will generate or destroy the whole Motion of the Globe while

while it moves uniformly through $\frac{2}{3}$ of 4 Diameters, or $\frac{8}{3}$ of its Diameter.

34. From what has been said, if the Velocity of a Globe or Cylinder moving in Water be known, and also its Diameter; then the Resistance it meets with from the Fluid, while it moves uniformly through, is known also. Thus let its Velocity $V=16$ Feet *per* Second, and its Diameter three Inches. Then a Body falling through the Space of 4 Feet acquires that Velocity, (see *Annot.* LXXV. 6.) whence $GH=4$, and $\frac{1}{2}GH=2$ Feet, the Length of a Column of Water, whose Base being 3 Inches in Diameter will weigh about 6 lb. and is equal to the Resistance of the Globe.

35. Hence also it is easy to find the Height from which if the Globe falls *in Vacuo*, it shall acquire the Velocity which shall be the greatest it can possibly acquire by descending in the Fluid by the Force of its *relative Weight*. Which Height will be to $\frac{8}{3}$ Parts of its Diameter, as the Density of the Globe to the Density of the Fluid.

36. For, let $R =$ Resistance, $D =$ Density of the Globe, $d =$ that of the Fluid, and $F =$ Force which will generate the Motion of the Globe in the Time it moves over $\frac{8}{3}D$, where $D =$ the Diameter of the Globe; then we have $R : F :: d : D$ (in *Art.* 33.) whence $F = \frac{DR}{d}$.

Now let $2S =$ Space which the Globe describes uniformly in the Time of the Fall, and with the Velocity acquired thereby. Then will $2S : \frac{8}{3}D :: D : d$ (per *Art.* 32) $:: S : \frac{1}{2}D$. But T and t , the Times in which uniform Spaces are described, are as those Spaces, *viz.* $T : t :: 2S : \frac{8}{3}D$; whence $T : t :: D : d$. Again, since in the Times T and t equal Quantities of Motion are produced, *viz.* the whole Motion of the Globe; for F produces that Motion in the Time t , and in the Time T it is produced by the Force or Relative Weight of the Globe, which call P . Then, since the greater the Time is, the less will be the Force acting uniformly to produce a given Effect, we have $F : P :: T : t$; hence $F : P :: D : d$; and so $F = \frac{PD}{d} = \frac{RD}{d}$; whence $P = R$, that is, the Resistance being equal to the Weight of the Body, it can be no longer accelerated.

37. As Sir *Isaac Newton* was the Author of this Theory, so he instituted a Series of Experiments to confirm the same, for which Reason, as well as to exemplify the foregoing Method of Computation, I shall here repeat some of the Principal, made with Globes of Wax and Lead included, let fall in Water through the Depth of 112 Inches. His first Experiment was with a Globe $\frac{84}{100}$ Parts of an Inch Diameter = D , whose Weight in *Vacuo* was $156 \frac{1}{3}$ Grains, and in Water 77 Grains, therefore $156 \frac{1}{3} - 77 = 79 \frac{1}{3}$ Grains = the Weight of an equal Bulk of Water. Wherefore $d : D :: 79 \frac{1}{3} : 156 \frac{1}{3} :: \frac{8}{3} D (=2,24597 \text{ Inches}) : 2S=4,4256 \text{ Inches}$; and so $S=2,2128 \text{ Inches}$.

38. The Globe falling in *Vacuo*, with its whole Weight of $156 \frac{1}{3}$ Grains, will in one Second of Time describe $193 \frac{1}{2}$ Inches. And by its relative Weight of $79 \frac{1}{3}$ Grains in Water, it would in the same Time by falling in *Vacuo* describe 95,219 Inches. For the Spaces described in the same Time will be as the accelerating Forces or Weights of the Bodies. (See *Annot. XXVII 2, 3.*)

39. But the Time T , in which the Globe by descending in *Vacuo* with its relative Weight of $79 \frac{1}{3}$ Grains, will describe the Space $S=2,2128$, is to the Time $t=1$ Second, in which it described the Space 95,219, in the subduplicate Ratio of those Spaces; that is, $T : t :: \sqrt{2,2128} : \sqrt{95,219}$, wherefore $T = \sqrt{\frac{2,2128}{95,219}} = 0'', 15244$ = the Time in which the Globe by descending in *Vacuo* acquires the greatest Velocity V with which it can descend in the Water.

40. In the next Place, to find the Velocity and the Space described in the Fluid in the whole Time of the Fall (F) Sir *Isaac* has this Method; let the Number N be found for the Logarithm $0,4342944819 \frac{2F}{T}$; then is the Velocity of the Globe at the End of the Fall = $\frac{N-1}{N+1} \times V$. Again, let L be the Logarithm of the Number $\frac{N+1}{N}$; then will the Space described be $\frac{2SF}{T} -$

138629S+4,60517 LS. These Velocities and Spaces described are calculated and disposed into a Table by our Author for any Number of Seconds to 10 T. The Demonstration of the above Rules is too long and intricate for this Place, but may be seen in the Commentary on the *Principia* by *Le Scur* and *Jacquier*. This Table will be inserted at the End of this Note.

41. By this Table you see how soon Bodies acquire their greatest Velocity in descending in Water; for Instance, in $5T=0'',76=46'''$, the Globe has acquired a Velocity which is to the greatest as 99990920 to 100000000. And in $10T=1'' : 30'''$ the Velocity of the Globe is to the greatest Velocity as 99999999 $\frac{1}{2}$ to 100000000. Which Difference is insensible to the most critical Eye; and from thence (having fallen through 37 Inches) it descends with an uniform Velocity to Appearance, tho' it really for ever increases.

42. If the Time of Descent in the Water be less than 10 T, the Distance descended is shewn by the third Column; but if it be greater, as in the present Experiment it is 4'', then say, as $T : 2S :: 4'' : \frac{8S}{T} = 116,1245$

Inches, because Spaces described with an uniform Velocity are as the Times. This uniform Space must be diminished by subtracting the constant Number 1,38629 S. Therefore $116,1245 - 1,38629 S = 113,0569$ Inches, which the Globe should describe in the Water in 4'' by the Theory.

43. But in the Experiment it described only 112 Inches, which Difference of 1 Inch in 112 is inconsiderable, and would have given no Disturbance to any but Sir *Isaac Newton*. But he being as nice in Practice as skilful in Theory, attempted another Set of Experiments in a deeper Vessel, viz. of 15,3 Feet or 183,6 Inches. I shall here give the Reader a View of the Event of his several Experiments in both Vessels in the following Table.

In the Vessel 112 Inches deep.								
Experiment.	Weight in Air.	Weight in Water.	Number of Globes.	Times of their Descent in Experiments.				Time by the Theory.
1	7 $\frac{1}{4}$	5 $\frac{1}{8}$	3	15	15	15		14:48
2	121	1	3	46	47	50		40
In the Vessel 183 Inches deep.								
3	139 $\frac{1}{4}$	7 $\frac{1}{8}$	4	9 $\frac{1}{2}$	50	51	53	50
4	54 $\frac{3}{8}$	21 $\frac{1}{2}$	4	28 $\frac{1}{2}$	29	2 $\frac{1}{2}$	30	29
5	212 $\frac{3}{8}$	79 $\frac{1}{8}$	5	15	15 $\frac{1}{2}$	16	17	15
6	293 $\frac{1}{8}$	35 $\frac{1}{8}$	4	29 $\frac{1}{2}$	30	30 $\frac{1}{2}$	31	28
7	139	6 $\frac{1}{8}$	4	50	50 $\frac{1}{2}$	51	52	52
8	273 $\frac{1}{4}$	140 $\frac{3}{4}$	4	12	12 $\frac{1}{2}$	12	13	11 $\frac{1}{3}$
9	384	192 $\frac{1}{2}$	4	17 $\frac{1}{4}$	18	18 $\frac{1}{2}$	19	15 $\frac{5}{8}$
10	48	4 $\frac{1}{8}$	3	4 $\frac{1}{2}$	45	46		46 $\frac{1}{2}$
11	141	4 $\frac{1}{8}$	3	63	64	65		64 $\frac{1}{2}$

44. In all the Experiments of the deep Vessel, the Times are expressed in *Half Seconds*, that is, in all but the two first. And the small Differences from the Theory, Sir *Isaac* very easily accounts for from the oscillatory Motion with which those which descended most slowly were found to move, as in the 5th, 8th, and 9th Experiments. But in those which descended more swiftly, the Times very nearly correspond with the Theory. These Experiments were made in Water; others were made in Air, to shew that the Phænomena agree with the Theory in an elastic and rare Medium, as well as in a non-elastic and dense one.

45. In order to this the Density of Air to that of Rain Water is assumed as 1 to 860; and 5 Globes form'd of Hog's Bladders, blown in a wooden Mould, while green, and taken out when dry, were let fall from the Dome of St. *Paul's* to the Pavement, from the Height of 272 Feet, and the Time measured by Half Second Pendulums; the Event of all the five Globes is shewn in the following Table. I shall here subjoin a Calculation for one of them, viz. the 3d, as follows.

46. The

46. The Diameter of this Globe was $5,3=D$, and it weigh'd 137,5 in Air. A Globe of Air of the same Diameter weighs 23 Grains, very nearly; therefore $137,5+23=160,5$ Grains, the Weight of the Globe in *Vacuo*. Wherefore, as $23 : 160,5 :: \frac{8}{3} D : 2 S = 98,626$. Again, as $160,5 : 137,5 :: 193\frac{1}{3} : 165,628 =$ Inches the Bladder would fall in the Air in one Second. But with the same relative Weight $137\frac{1}{2}$, it will in *Vacuo* descend thro' the Space $S=49,313$ in the Time $0'',5456=T$, and acquire the greatest Velocity with which it can descend in Air. The Time of the Fall was $18\frac{1}{2}''$, in which with that Velocity the Globe would describe 278 Feet 8 Inches by an uniform Motion; for $0'',5456 : 98,626 \text{ Inches} :: 18\frac{1}{2}'' : 278\frac{3}{4} \text{ Feet}$. Lastly, subduct $1,3863 S = 5\frac{1}{2}$ Feet, and there remains 273 Feet, though in the Table, by a more accurate Calculation, it is but 272 Feet, 7 Inches, and by the Experiment it was 272 Feet.

47. The following Table shews, in the first Column, the Weight of each Globe or Bladder; the second its Diameter; the third the Time in which they severally descended through the Height of 272 Feet; the fourth shews the Spaces they should have descended through by the Theory; and the last Column shews the Difference between the Theory and Experiments.

The Weight in Grains.	Diameter in Inches.	Times of the Fall.	Spaces per Theory.		Difference.	
			Feet.	Inch.	Feet.	Inch.
128	5,28	19"	271	— 11	0	— 1
156	5,19	17	272	— $0\frac{1}{2}$	0	— $0\frac{1}{2}$
$137\frac{1}{2}$	5,3	$18\frac{1}{2}$	272	— 7	0	— 7
$97\frac{1}{2}$	5,26	22	277	— 4	5	— 4
$99\frac{1}{8}$	5	21	282	— 0	10	—

48: According to this Theory, having found the Resistance which a Globe meets with in descending in any resisting Medium, agreeing to its greatest Velocity, it is easy then to find its Resistance for any lesser Velocity proposed; because the Resistance for any given Velocities, will be as the Squares of those Velocities.

49. The Globe has been hitherto supposed to begin its Motion in the resisting Medium from a State of Rest; but we may suppose it also projected into such a Medium, or to enter it with a certain Degree of Motion or Velocity; and in this Case we may determine the Velocity, the Resistance, and the Space described in the Medium, from any given Time by knowing the Density of the Globe and Medium, and the Velocity with which it enters it; as also what Part of its Motion the Globe shall have lost in that Time.

Pl. XVI.
Fig. 7. 50. But in order to this, we must take to our Assistance the *Hyperbola*. Therefore let $AB =$ Time in which the Globe would lose its whole Motion by the Resistance it meets with (upon entering the Fluid,) if uniformly continued. In the Points A and B erect the Perpendiculars AD and BC ; and let BC represent the Motion or Velocity, and also the Resistance of the Globe at its Entrance into the Fluid; thro' the Point C , let the Hyperbola be described to the Asymptotes AD and AB continued to any Point E ; in E erect the Perpendicular EF meeting the Hyperbola in F ; complete the Parallelogram $CBEH$; and draw AF cutting BC in H .

51. Then if the Globe in any Time BE describes in *Vacuo* the Space represented by the Parallelogram $BCHE$ by its first Velocity BC uniformly continued, it will in a resisting Medium describe the hyperbolic Space or Area $BCFE$; and its Motion or Velocity at the End of the Time BE will be represented by the Ordinate EF , and the Part lost by FG . Lastly, its Resistance at the End of that Time will be BH , and the Part lost HC .

52. For $BC^2 : EF^2 :: BC : \frac{EF^2}{BC} =$ the Resistance at the End of the Time BE , as BC is that at the Beginning. And from the Nature of the Hyperbola, $BC : EF :: AE : AB$; and by similar Triangles $AE : AB :: EF : BH$; wherefore $BH = \frac{EF^2}{BC}$, the Resistance at the End of the Time BE .

53. Hence if the Globe in any Time T loses its whole Motion M by the Resistance R uniformly continued;

tinued; the same Globe shall in a resisting Medium, by the Resistance R decreasing in the duplicate Ratio of the

Velocity, in the Time t lose the Part $\frac{tM}{T+t}$ of its

whole Motion M ; and then $M - \frac{tM}{T+t} = \frac{TM}{T+t}$ will

be the Part of the Motion remaining. For let m be the residual Motion, then $T:t :: AB:BE$ (by *Art.* 50, 51.) and hence $T+t:T :: AE:AB$; and moreover $M:m :: CB:FE :: AE:AB$; whence $T+t:T$

$:: M:m$; consequently $m = \frac{TM}{T+t}$.

54. Lastly, the Space (S) described in the resisting Medium in the Time t will be to the Space ($2S$) described by the uniform Motion M , as the Logarithm

of the Number $\frac{T+t}{T} \times 2,302585$ to $\frac{t}{T}$. For let AB

$=a$, $BC=b$, $BE=x$, and $AE=a+x$, and $L=\text{Logarithm}$. Then (*per* Conics) the Area $BCEF = ab \times L \frac{a+x}{a}$, and the Rectangle $BCGE = bx$, whence

$s:2S :: ab \times L \frac{a+x}{a} : bx$; and dividing by ab , we

have $s:2S :: L \frac{a+x}{a} :: \frac{x}{a} :: L \frac{T+t}{T} : \frac{t}{T}$; but the

Expression $L \frac{T+t}{T}$, as it here stands, is an hyperbolic Logarithm, which is to the common tabular Logarithm as 2,302585 to 1. Consequently the tabular

$L \frac{T+t}{T} \times 2,302585 = \text{Logarithm sought}$.

55. The Spaces, Times, and Velocities of Bodies moving either by their *Vis insita* alone, or conjointly with an accelerating Force, as Gravity; in resisting Mediums of various Density and Ratio of Resistance, may be analytically investigated as follows. Let the initial Velocity, or that with which it begins its Motion in the Fluid be V , the Time t , the Space described s , and the Velocity at the End of the Time v , the Density of

the Medium d , and the Resistance r as $\frac{dv^n}{a^n}$. Then

(by *Art.* 14.) $r \dot{s} = -\dot{v}$, wherefore $\frac{dv^n \dot{s}}{a^n} = -\dot{v}$; and so $d\dot{s} = a^n \dot{v} v^{1-n}$.

56. Whence if the Density be uniform, as of Water, then $d=1$. And if $n=2$, then $\dot{s} = a^2 \dot{v} v^{-1} = -\frac{a^2 \dot{v}}{v}$. Now s is the Fluent of the Fluxion $\dot{s}$, and the

Fluent of $-\frac{a^2 \dot{v}}{v}$ is $Q - a^2 L v$, where Q is some constant Quantity, to find the Value of which, suppose $s = Q - a^2 L v = 0$, then $Q = a^2 L v$, but when $s=0, v=V$; therefore $Q = a^2 L V$. Wherefore $s = a^2 L V - a^2 L v = a^2 L \frac{V-v}{1}$.

57. The Time t is found by the *Formula* in *Art.* 10. where we had $\dot{t} = \frac{\dot{s}}{v}$, but because when d is given,

$\dot{s} = a^n \dot{v} v^{1-n}$ (*Art.* 55.) therefore $\dot{t} = \frac{\dot{s}}{v} = \frac{a^n \dot{v} v^{1-n}}{v} = a^n \dot{v} v^{-n}$. Whence taking the Fluents we have $t = \frac{Q - a^n v^{1-n}}{1-n}$; and putting $t=0$, we have $v=V$, wherefore $Q = a^n V^{1-n}$; consequently, substituting for Q its Value, we have $t = \frac{a^n V^{1-n} - a^n v^{1-n}}{1-n}$. Hence, if $n=2$, $t = \frac{a^2 V - a^2 v}{V v} = a^2 \times \frac{V-v}{V v}$.

58. We had above $\dot{s} = a^n \dot{v} v^{1-n}$; and taking the Fluents on both Sides, we have $s = \frac{Q - a^n v^{2-n}}{2-n} = \frac{a^n V^{2-n} - a^n v^{2-n}}{2-n}$. Let $2-n=m$; then $ms = a^n V m - a^n v m$; hence $a^n V m - m s = a^n v m$; therefore $v = \frac{a^n V m - m s}{a^n m}$. Which Theorem therefore gives the

Velocity when the Space s is known.

59. In these Theorems we see the Agreement with the former Methods of determining these Quantities.

Thus the Space by Theorem in *Art.* 56. is $s = a^2 L \frac{V}{v}$

$$= 2,302585 L \frac{T+t}{T} \text{ (in } Art. 54.) = BCFE, \text{ the}$$

Hyperbolic Space in the Figure. Whence, in this Case,

$$a^2 = 2,302585 \times \frac{8}{3} D.$$

60. To determine the Space s which the Globe shall have pass'd by its *Vis infinita* when it has lost half its Velocity or Motion, that is, when $V : v :: 2 : 1$; we

have $L \frac{V}{v}$, or the Logarithm of $\frac{2}{1} = 0,301030$, which

multiplied by 2,302585 gives 0,6931; and this again, multiplied by $\frac{8}{3} D = 2,6 D$, gives 1,84832 $D = s$, which is not quite 2 Diameters.

61. The Time t being always as $a^2 \times \frac{V-v}{Vv}$, or (be-

cause a^2 is constant) as $\frac{V-v}{Vv}$; therefore when $v = \frac{1}{2} V$,

we have $\frac{V-v}{Vv} = \frac{1}{2} = t$, which shews the Time in

which the Globe loses half its Motion is half the whole Time; that is, $BE = AB$ (in the Figure) the Time in which by an uniform Velocity BC it would describe $\frac{8}{3}$ Parts of its Diameter.

62. If the Medium be similar, or of an uniform Density, where $d = 1$; and if the Resistance be in the simple Ratio of the Velocity, then $n = 1$; and by the Theo-

rems above, $s = aV - av$, and $t = a L \frac{V}{v}$, and $v = V -$

$$\frac{s}{a}.$$

63. If in the foregoing Equations the Velocity $v = 0$, then $s = \frac{a^n V^{2-n}}{2-n}$, when n is less than 2. But when n

$= 2$, $s = a^2 L \frac{V}{0} = \text{Infinite}$; if n be greater than 2, then

$$\text{also } s = \frac{a^n V^{n-2} - a^n v^{n-2}}{n-2 \times V^{n-2} \times v^{n-2}} = \frac{a^n}{n-2 \times 0} = \text{Infinite. That}$$

is,

is, there is but one Case in which the Globe moving in the Fluid can lose its Velocity, or come to a State of Rest in describing a finite Space, and consequently in a finite Time, *viz.* when n is less than 2. In all other Cases the Motion will never be totally destroyed. And when $n = 1$, then, though the Space be finite, *viz.* $s = aV$, yet the Time of describing it will be infinite, *viz.* $t = aL \frac{V}{0} = \text{Infinite}$.

64. After a like Manner we may raise Theorems for the Spaces, Times, and Velocities of a Body descending by the Force of Gravity in any resisting Medium. For suppose the Medium of an uniform Density, and the Resistance in any multiplied Ratio of the Velocity, as $r = \frac{v^n}{a^{n-1}}$; then for a descending Body we have (by

Art. 16.) $c \dot{s} - r \dot{s} = v \dot{v}$; and therefore $c \dot{s} - \frac{v^n \dot{s}}{a^{n-1}} = v \dot{v}$; whence $\dot{s} = \frac{v \dot{v} a^{n-1}}{c a^{n-1} - v^n}$.

65. Also because (by *Art. 10.*) $\dot{t} = \frac{\dot{s}}{v}$, therefore we have $\dot{t} = \frac{\dot{v} a^{n-1}}{c a^{n-1} - v^n}$. And in like Manner the Theorems for ascending Bodies are $\dot{s} = \frac{-v \dot{v} a^{n-1}}{c a^{n-1} + v^n}$, and $\dot{t} = \frac{-\dot{v} a^{n-1}}{c a^{n-1} + v^n}$.

66. If the Resistance be as the Velocity, then $n = 1$; and in the descending Body $\dot{s} = \frac{v \dot{v}}{c - v} = -\dot{v} + \frac{c \dot{v}}{c - v}$; the Fluents of which are $s = Q - v - cL \frac{c \dot{v}}{c - v}$, but since when $s = 0$ it is $v = V$, therefore then $Q = V + cL \frac{c}{c - V}$; and so $s = V - v + cL \frac{c - V}{c - v}$. Also the Time is had from the Equation $\dot{t} = \frac{\dot{v}}{c - v}$; of which the Fluents are $t = Q - L \frac{c - v}{c - v} = L$

$$= L = \frac{c-V}{c-v}. \text{ And for ascending Bodies } s = V - v \\ + c L \frac{c+V}{c-v}; \text{ and } t = L \frac{c+V}{c+v}.$$

67. If the Resistance be as the Square of the Velocity, then $n=2$, and $r = \frac{v^2}{a}$. Let V = the greatest Velocity the Body can acquire by descending in the Fluid, and because then the Resistance is equal to its Gravity or Force of its Weight (by *Art.* 36.) we have $c = \frac{V^2}{a}$, (for in that Case $v = V$.) whence $rc = V^2$. Let S = Space described in *Vacuo* to acquire the Velocity V ; then because (by *Art.* 17.) in that Case $c\dot{S} = V\dot{V}$, therefore $cS = \frac{1}{2}VV$, and so $2cS = VV = ac$, consequently $a = 2S$.

68. Therefore since (*Art.* 64.) $\dot{s} = \frac{av\dot{v}}{ac - v^2} = \frac{2Sv\dot{v}}{V^2 - v^2}$ and putting $Vv - vv = xx$, and taking the Fluxions $v\dot{v} = -x\dot{x}$, and therefore $\dot{s} = -\frac{2Sx\dot{x}}{xx} = -\frac{2S\dot{x}}{x}$; and taking the Fluents, $s = Q - 2SL$. $x = Q - SL$. $x^2 = Q - SL \cdot \frac{V^2 - v^2}{V^2 - V^2}$. Wherefore when $s = 0$, we have $v = V$, the initial Velocity, then $Q = SL \cdot \frac{VV - V^2}{V^2 - V^2}$. And consequently $s = SL \cdot \frac{V^2 - v^2}{V^2 - V^2}$.

69. Let $L.b = 1$; then $sL.b = SL \cdot \frac{V^2 - v^2}{V^2 - V^2}$; $\frac{s}{S} = Lb = Lb \frac{s}{S} = L \frac{V^2 - v^2}{V^2 - V^2}$, therefore $b \frac{s}{S} = \frac{V^2 - v^2}{V^2 - V^2}$ whence we get $v^2 = \frac{V^2 b \frac{s}{S} + V^2 - V^2}{b \frac{s}{S}}$. Wherefore

the Velocity v is from thence easily found.

70. The

70. The Times t it obtain'd by the Equation $t =$

$$\frac{a \dot{v}}{c - vv} \text{ (in Art. 65.) } = \frac{2S \dot{v}}{V^2 - v^2} = \frac{\frac{S}{V} \dot{v}}{V + v} + \frac{\frac{S}{V} \dot{v}}{V - v};$$

and taking the Fluents we have $t = Q + \frac{S}{V} L. \frac{V + v}{V - v}$

$$\frac{S}{V} L V - v = Q + \frac{S}{V} L. \frac{V + v}{V - v}; \text{ and putting } t = 0,$$

and so $v = V$, we shall find $Q = -\frac{S}{V} L. \frac{V + V}{V - V}$; and

$$\text{therefore at length we have } t = \frac{S}{V} L \frac{V + v \times V - V}{V - v \times V + V}.$$

71. If the Body descends in the Fluid from a State of Rest, then $V = 0$; and so the Space $s = S L.$

$$\frac{V^2}{V^2 - v^2}; \text{ the Time } t = \frac{S}{V} L. \frac{V + v}{V - v}; \text{ and the Velocity}$$

$$v = \sqrt{\frac{V^2 b \frac{s}{S} - V^2}{b \frac{s}{S}}}. \text{ And in like Manner are found}$$

the Equations for the Spaces, Times, and Velocities of Bodies ascending in a resisting Medium, which are the same with these having only the Signs of V and v changed.

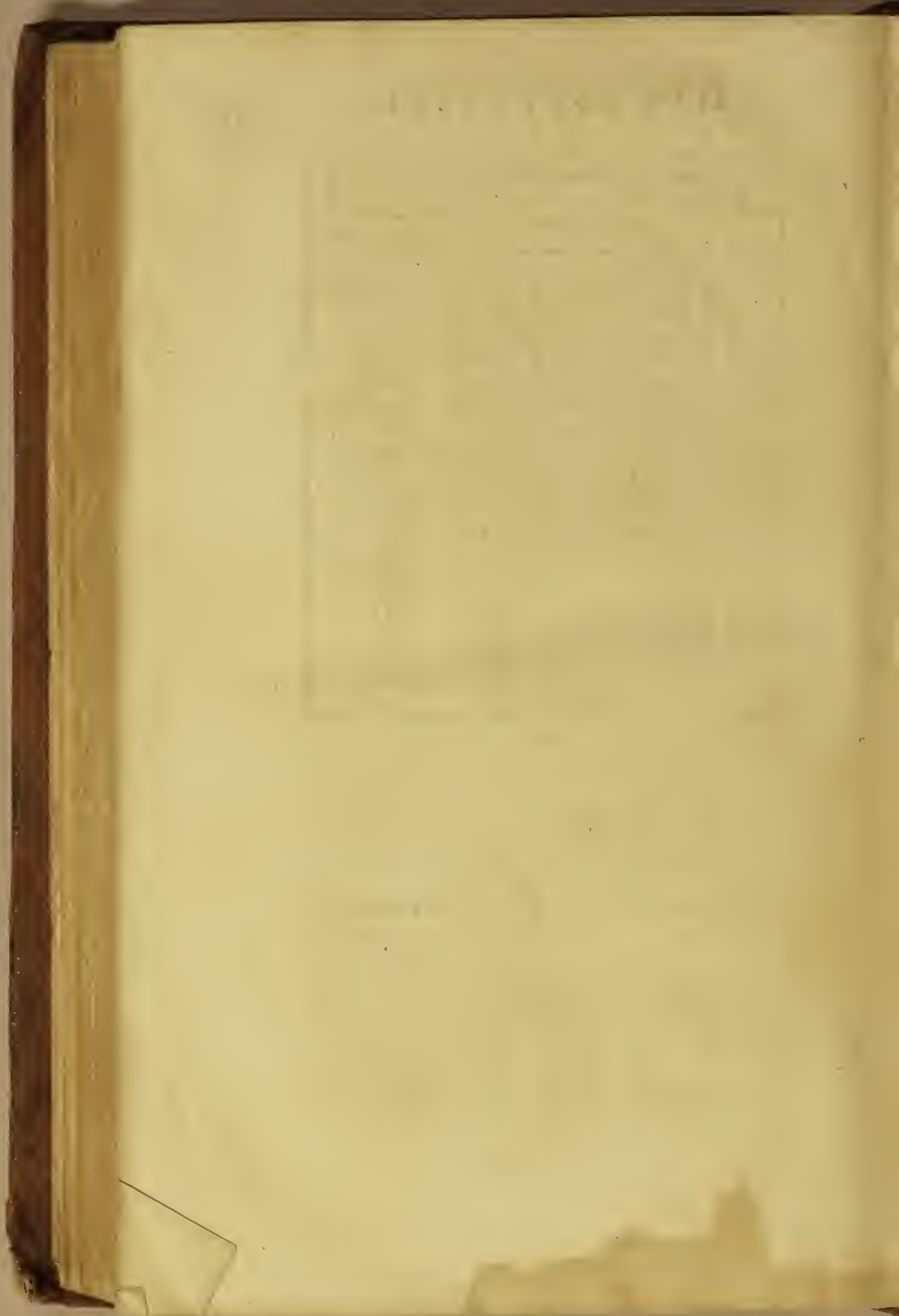
72. Thus I have given a Specimen of the several Ways that have been used to represent and compute the Motion of Bodies moving in resisting Mediums of any Density, and according to any Law of Résistance. The Theorems in the latter Articles express most of the Cases in *Prop. I, II, III, VI, VIII, XI*, of Sir Isaac Newton's second Book of *Principia*; I could have inserted many more, but have already extended this Note to a great Length; and shall refer the Reader to the admirable Commentary on our Author by Messrs. *Le Seur* and *Jacquier*, from whence I have collected most of these Articles.

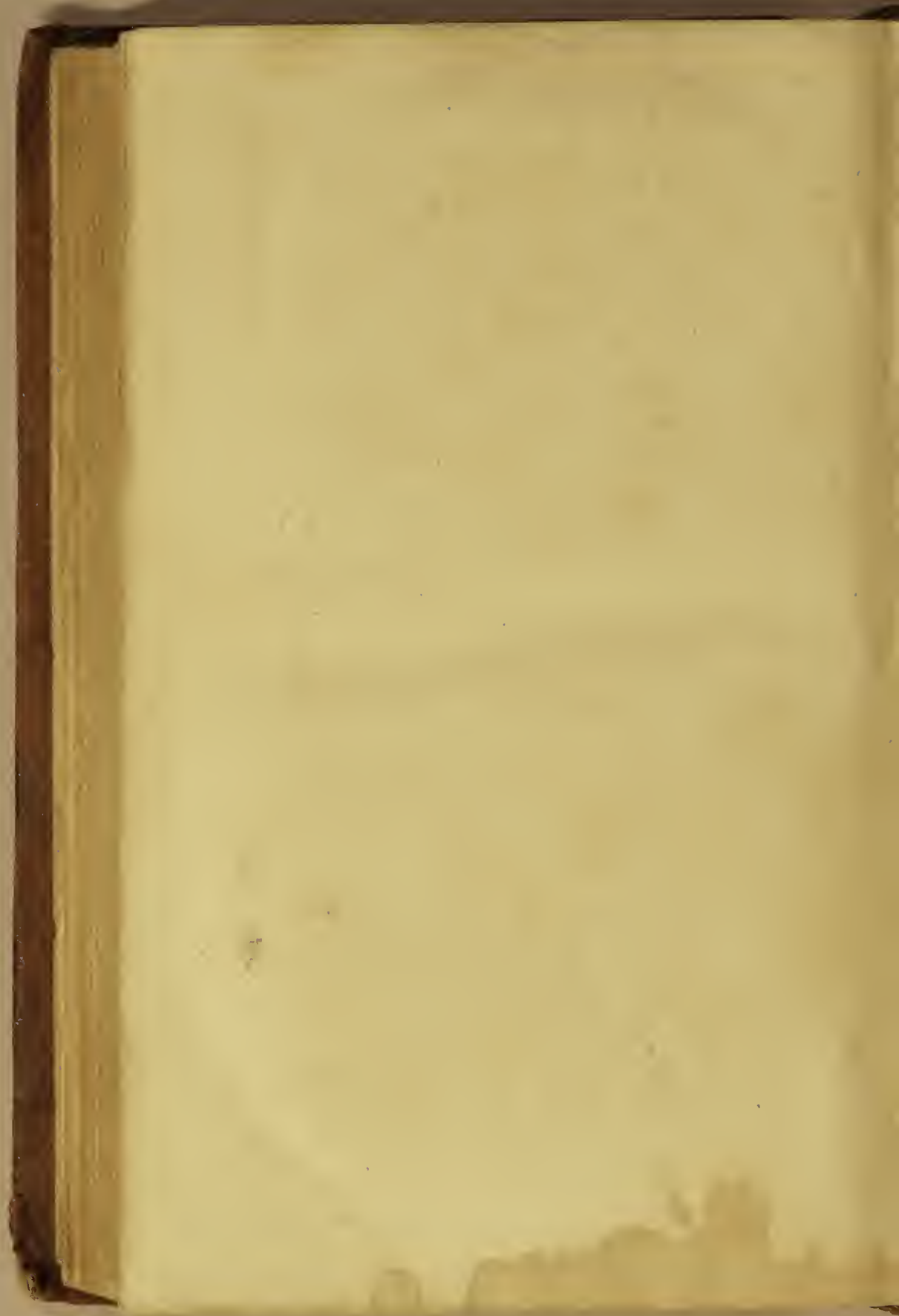
73. I shall conclude with inserting the Table refer'd to in the foregoing Computations, which is as follows.

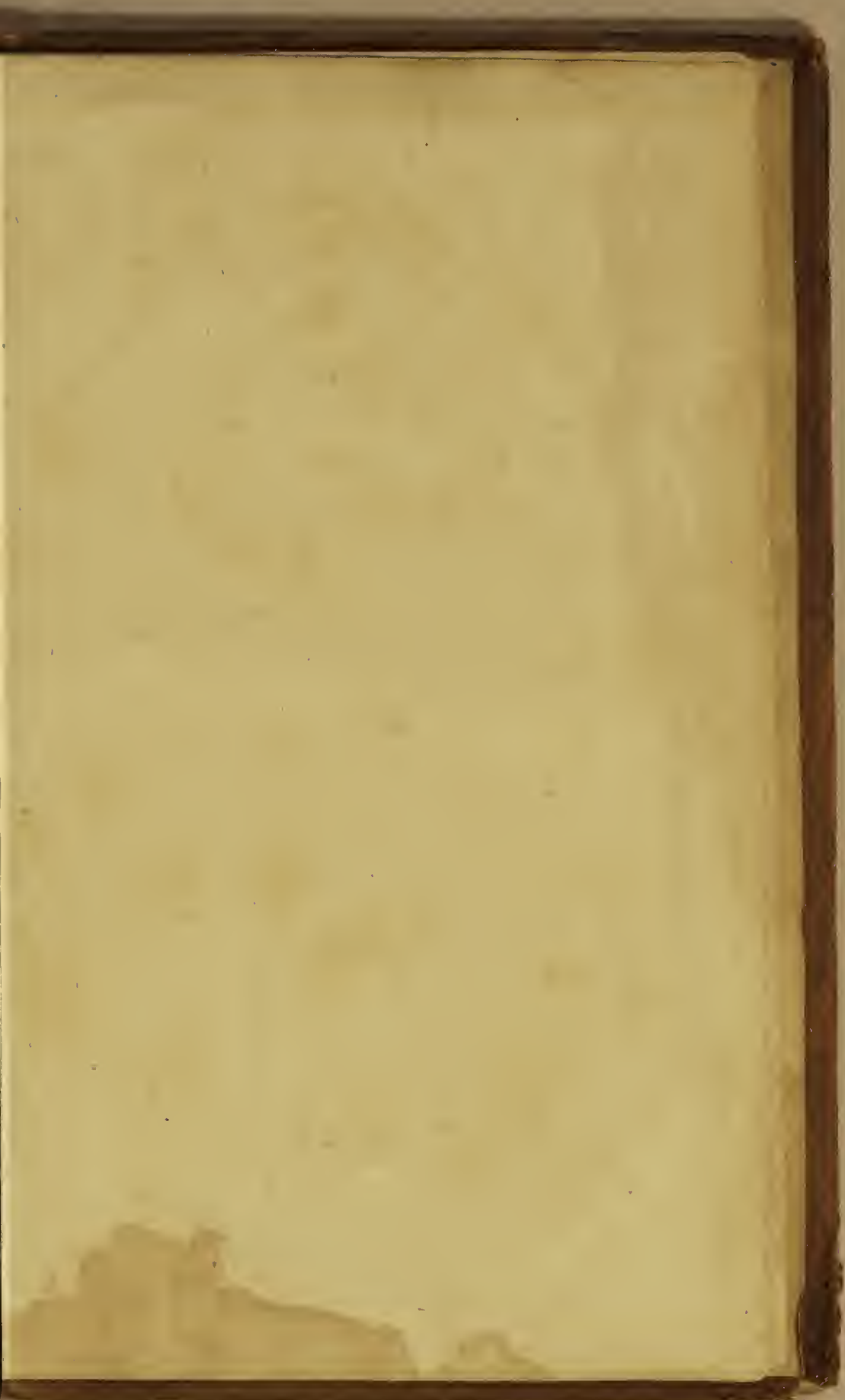
Time

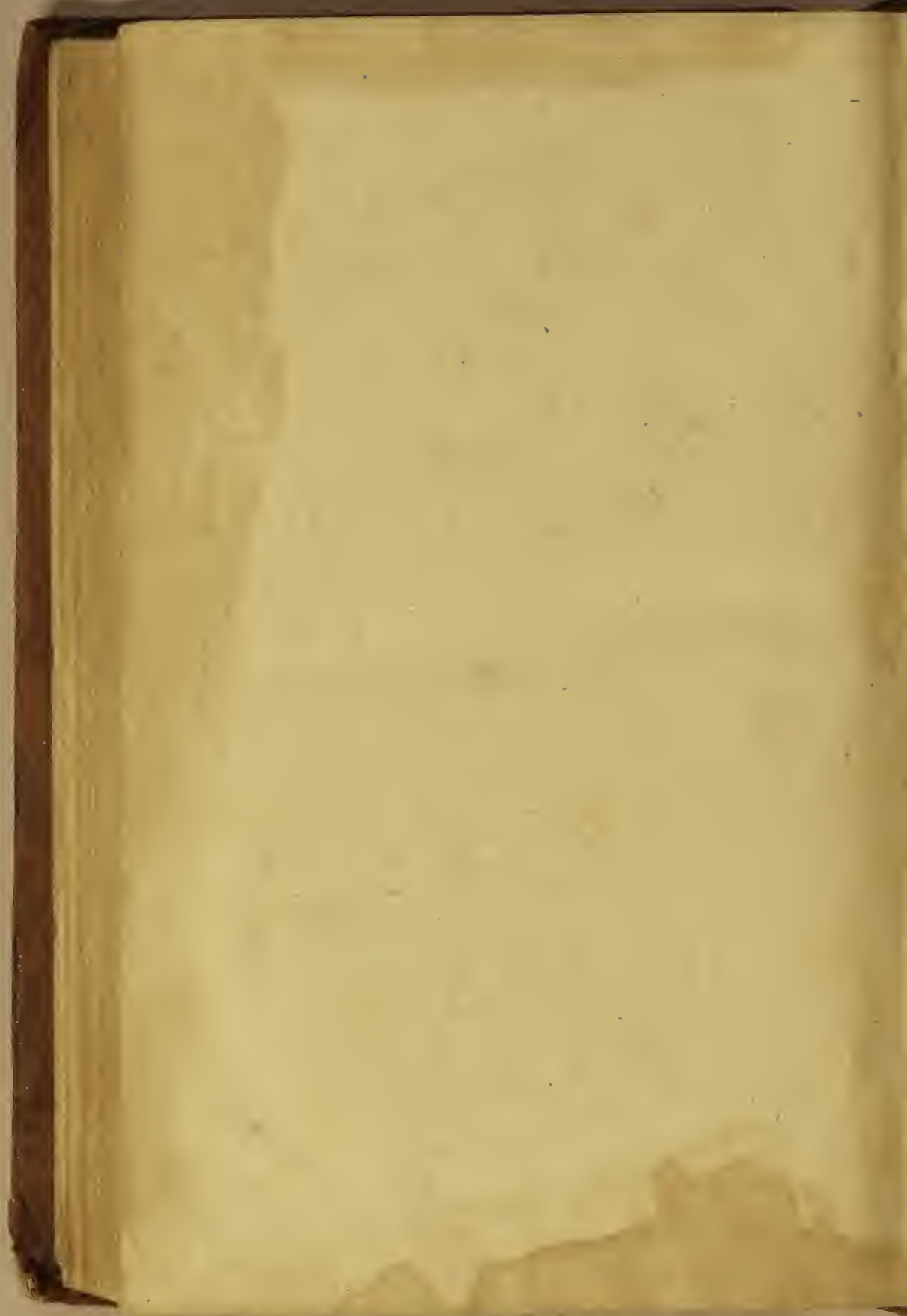
<i>Time of the Fall.</i>	<i>Velocities acquired in the Fluid.</i>	<i>Spaces describ- ed by falling in the Fluid.</i>	<i>Spaces de- scribed with the greatest Velocity.</i>	<i>Spaces de- scribed by falling in Vacuo.</i>
0,001 T	99999 $\frac{2}{3}$	0,000001 S	0,002 S	0,000001 S
0,01 T	999967	0,0001 S	0,02 S	0,0001 S
0,1 T	9956799	0,009983 S	0,2 S	0,01 S
0,2 T	19737532	0,039736 S	0,4 S	0,04 S
0,3 T	29131261	0,088681 S	0,6 S	0,09 S
0,4 T	37994896	0,155907 S	0,8 S	0,16 S
0,5 T	46211716	0,240229 S	1,0 S	0,25 S
0,6 T	53704957	0,340270 S	1,2 S	0,36 S
0,7 T	60436778	0,454540 S	1,4 S	0,49 S
0,8 T	66403677	0,581507 S	1,6 S	0,64 S
0,9 T	71629787	0,719660 S	1,8 S	0,81 S
1 T	76159416	0,867561 S	2 S	1 S
2 T	96402758	2,650005 S	4 S	4 S
3 T	99505475	4,618657 S	6 S	9 S
4 T	99932930	6,614376 S	8 S	16 S
5 T	99990920	8,613796 S	10 S	25 S
6 T	99998771	10,613718 S	12 S	36 S
7 T	99999834	12,613707 S	14 S	49 S
8 T	9999980	14,613706 S	16 S	63 S
9 T	9999997	16,613705 S	18 S	81 S
10 T	9999999	18,613705 S	20 S	100 S

The END of the FIRST VOLUME.









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